

# Actor Critic Methods

CS 224R

# Reminders

Since Wednesday:	Homework 1 is out
Next Monday:	Project survey due
4/19:	Homework 1 due, Homework 2 out

Reminder:

- Provide your AWS account ID if you haven't yet! (see Ed)

# The Plan

Policy gradients recap

Variance reduction continued

Policy gradients tricks

Actor-critic

Case studies: robotics & RLHF

## Key learning goals:

- Practical policy gradient implementation tricks & case studies
- Understanding a generic actor-critic method

# The Plan

Policy gradients recap

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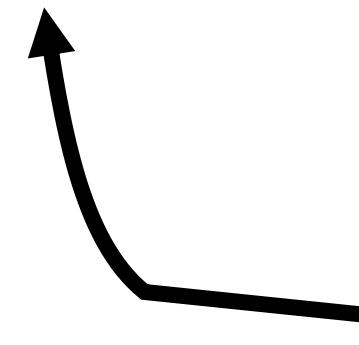
Case studies: robotics & RLHF

# Evaluating the objective

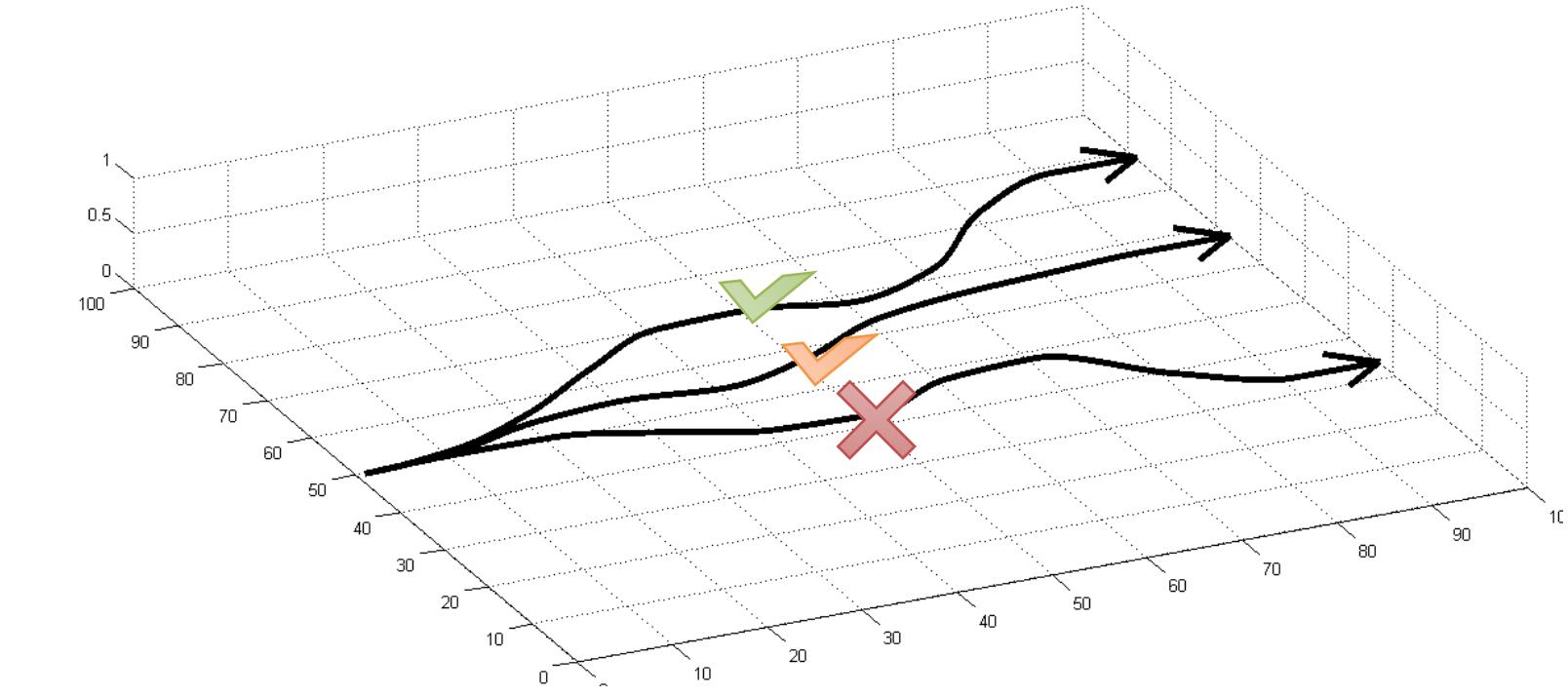
$$\theta^* = \arg \max_{\theta} E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$J(\theta)$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



sum over samples from  $\pi_{\theta}$



# Policy gradients

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left( \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \underbrace{\sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\tau_i) r(\tau_i)}_{\sum_{t=1}^T \nabla_{\theta} \log_{\theta} \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})}$$

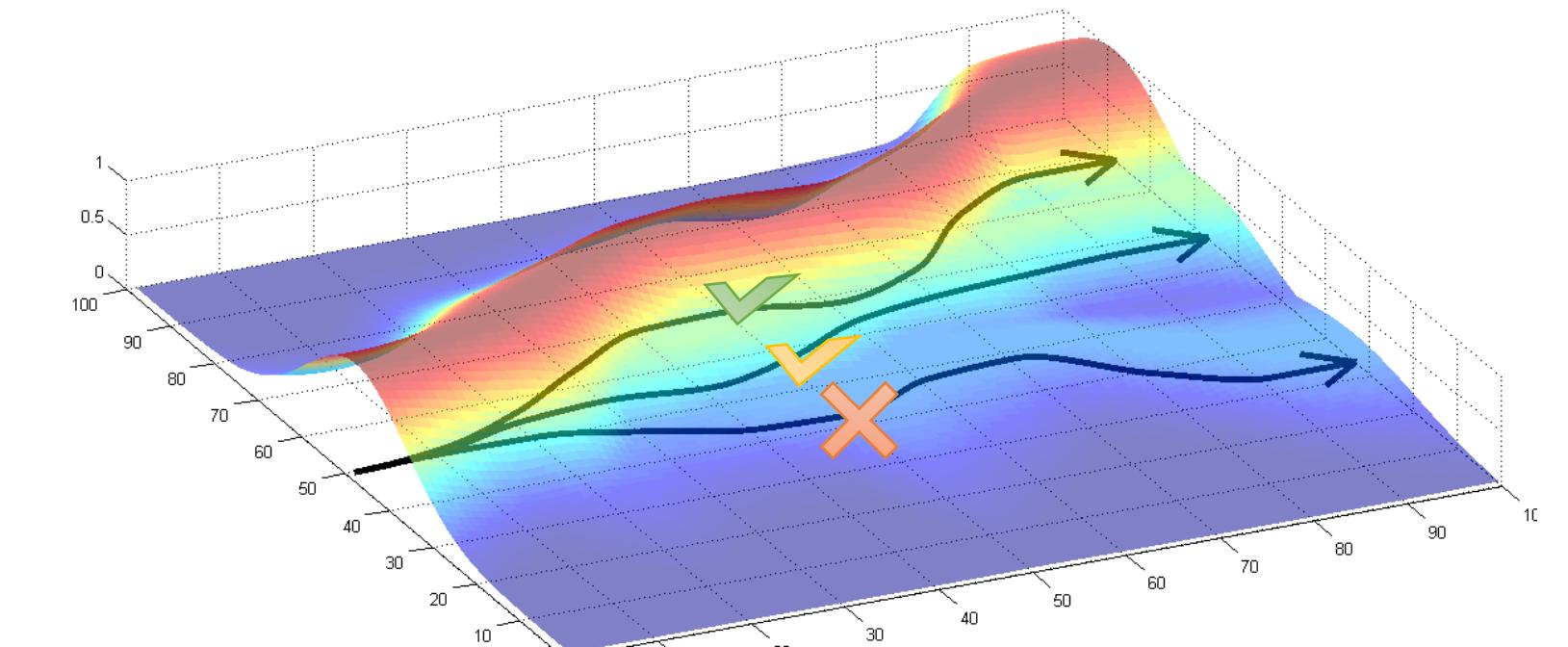
good stuff is made more likely

bad stuff is made less likely

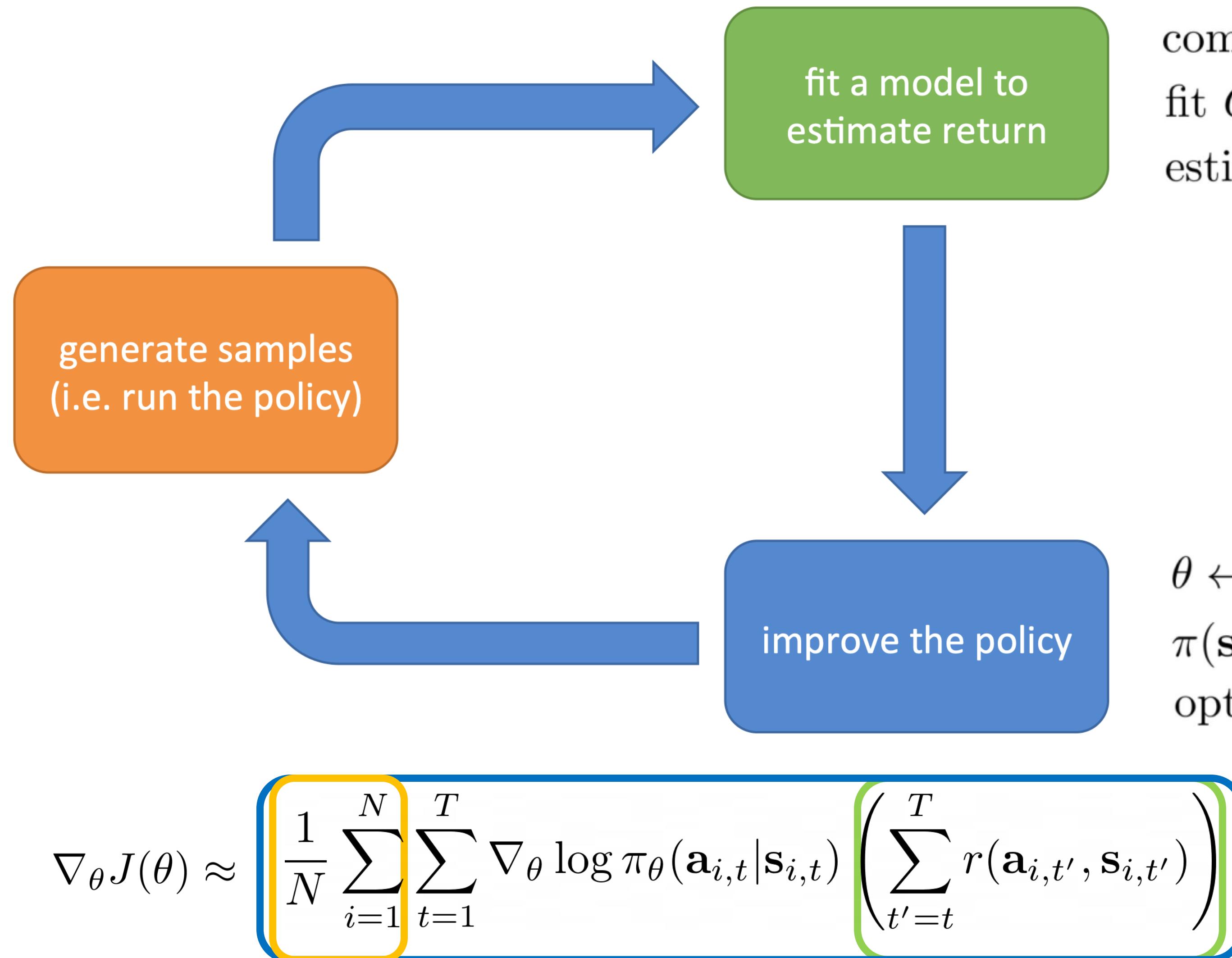
simply formalizes the notion of “trial and error”!

REINFORCE algorithm:

- 1. sample  $\{\tau^i\}$  from  $\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)$  (run it on the robot)
- 2.  $\nabla_{\theta} J(\theta) \approx \sum_i \left( \sum_t \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t^i | \mathbf{s}_t^i) \right) \left( \sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i) \right)$
- 3.  $\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$

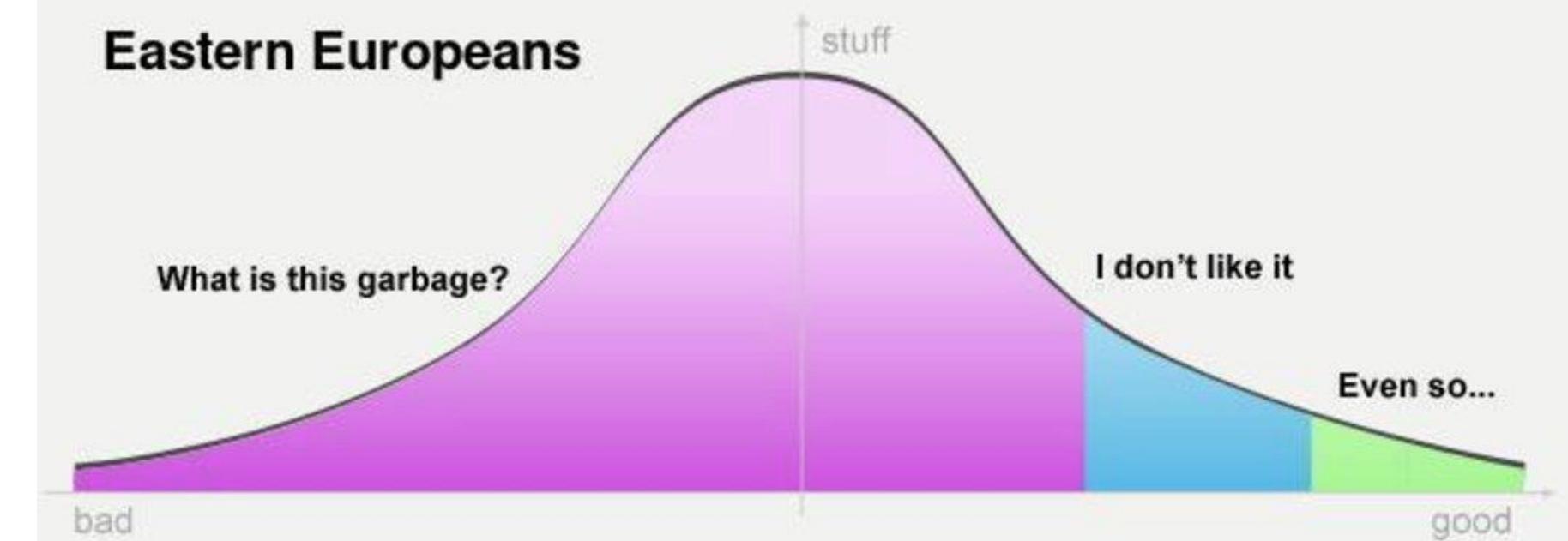
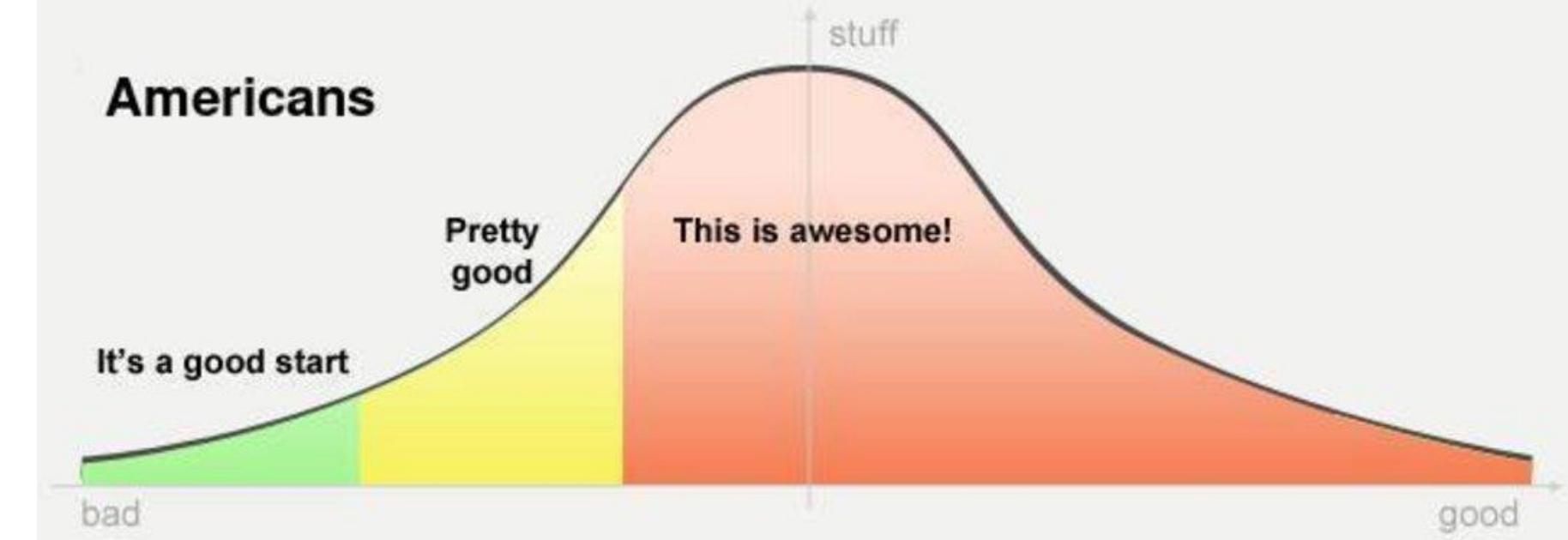
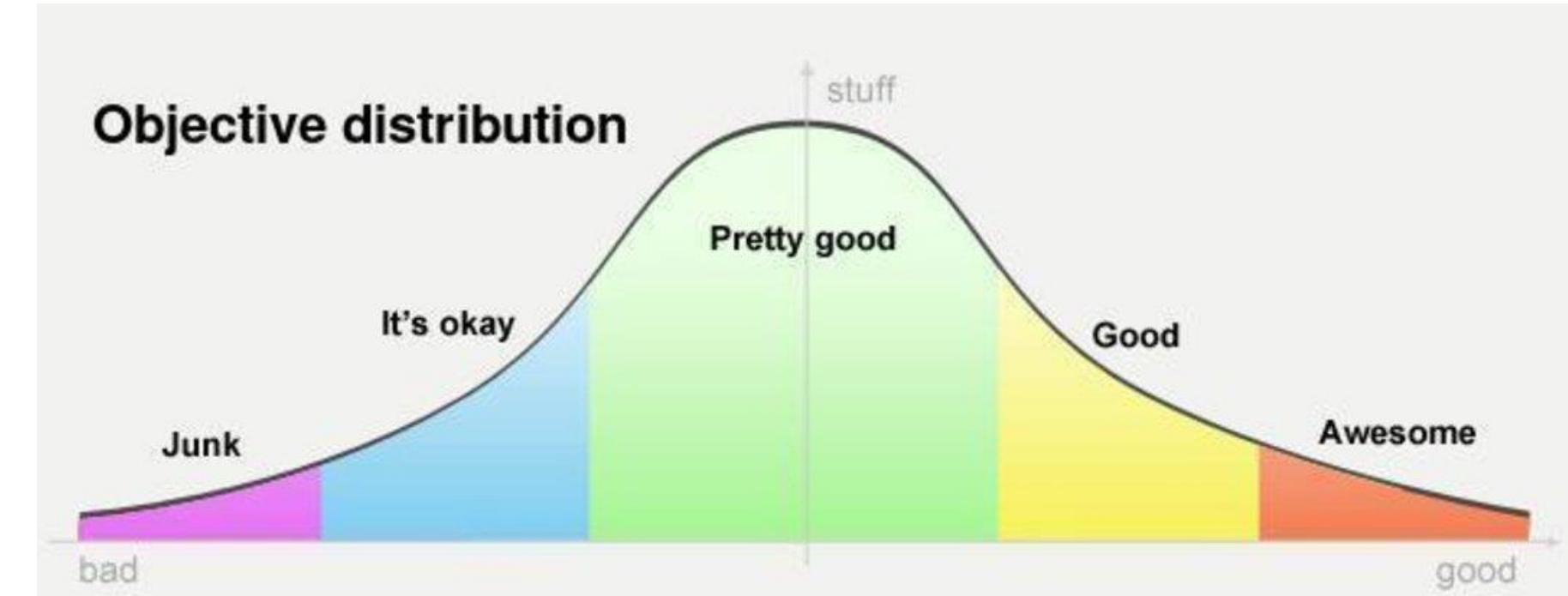


# The anatomy of a reinforcement learning algorithm



# Variance of the gradient estimator

policy gradient:  $\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \underbrace{\nabla_{\theta} \log \pi_{\theta}(\tau_i)}_{T} r(\tau_i)$   
 $\sum_{t=1}^T \nabla_{\theta} \log_{\theta} \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})$



# Small way to reduce variance

policy gradient:  $\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \left( \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \right) \left( \sum_{t=1}^T r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \right)$

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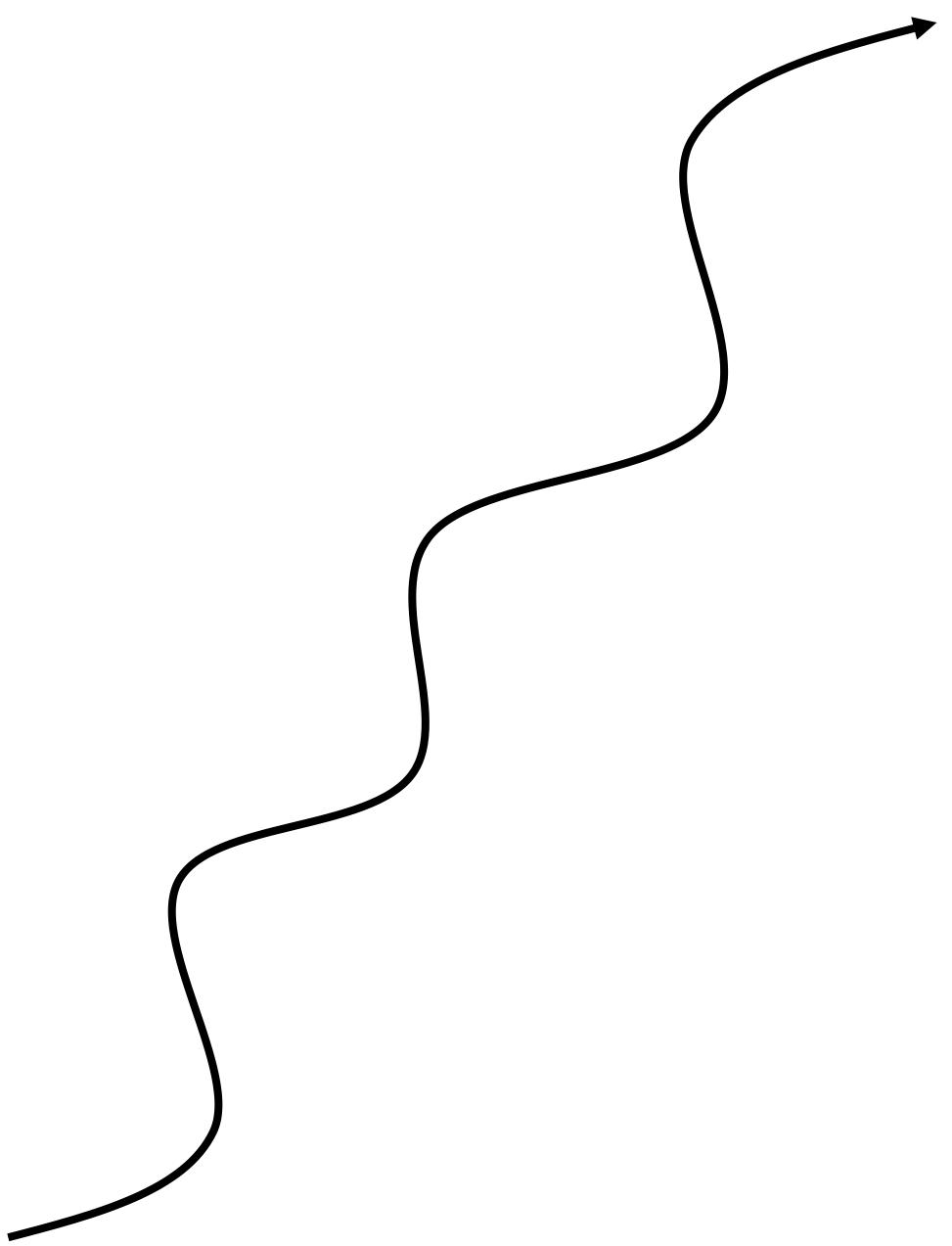
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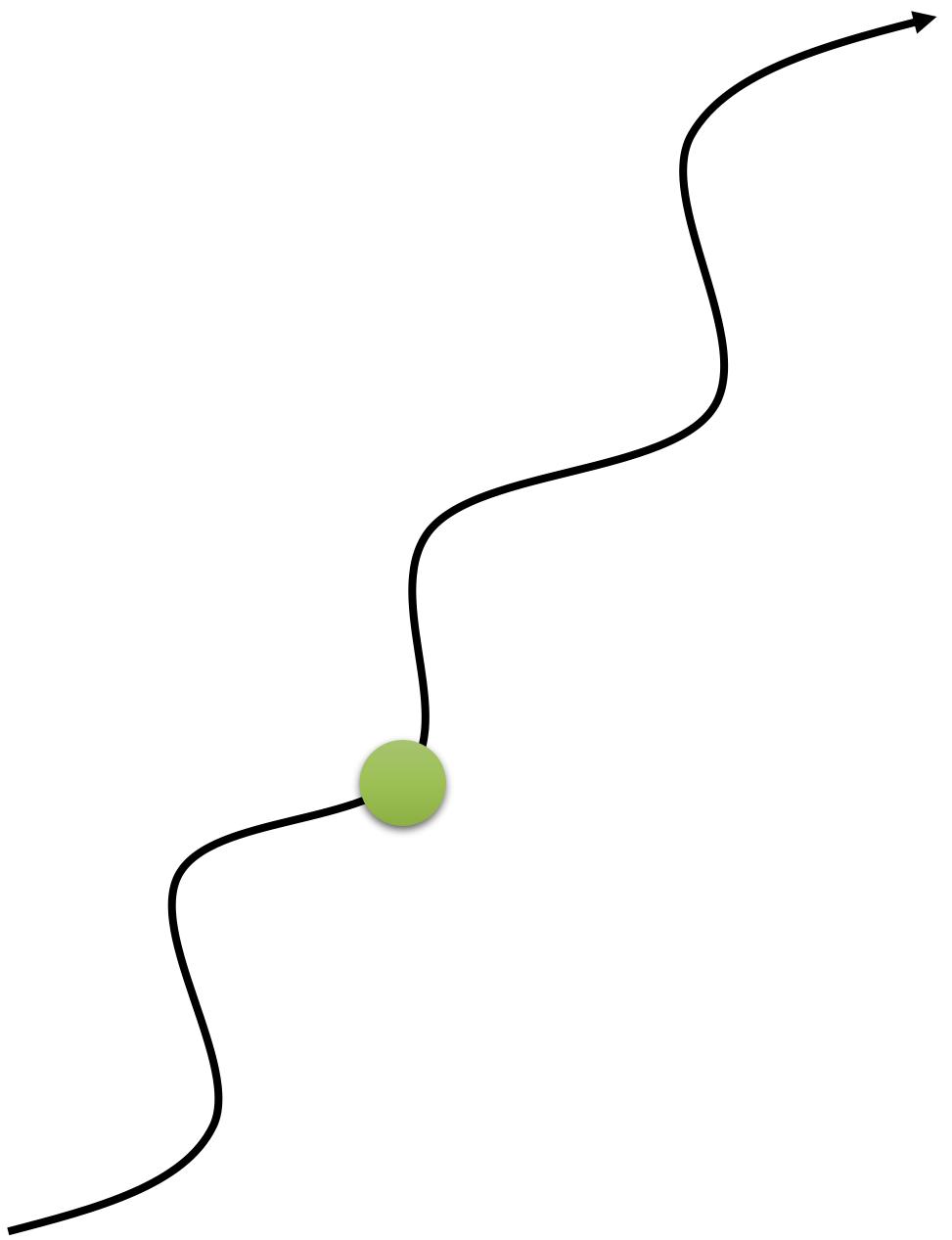
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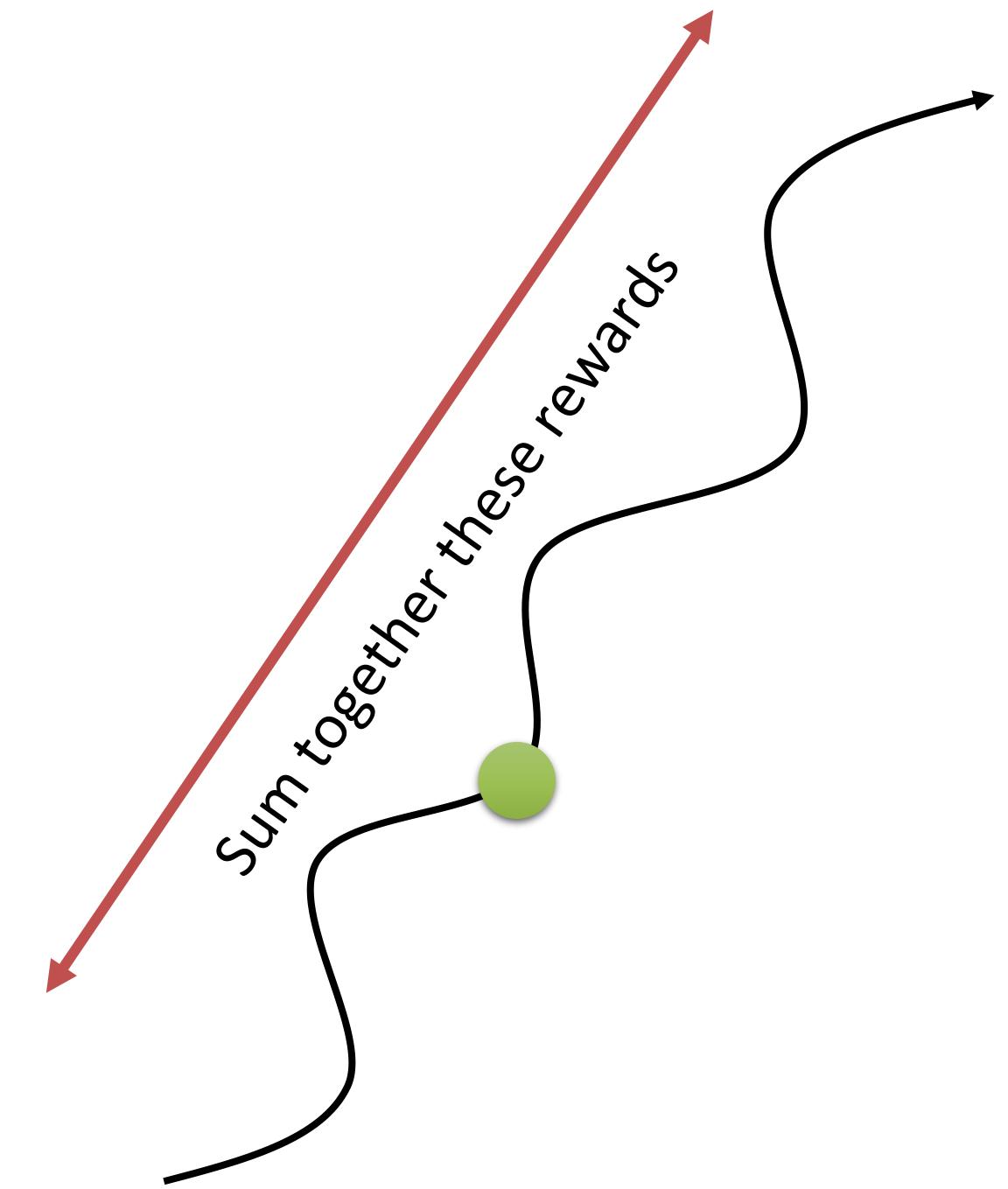
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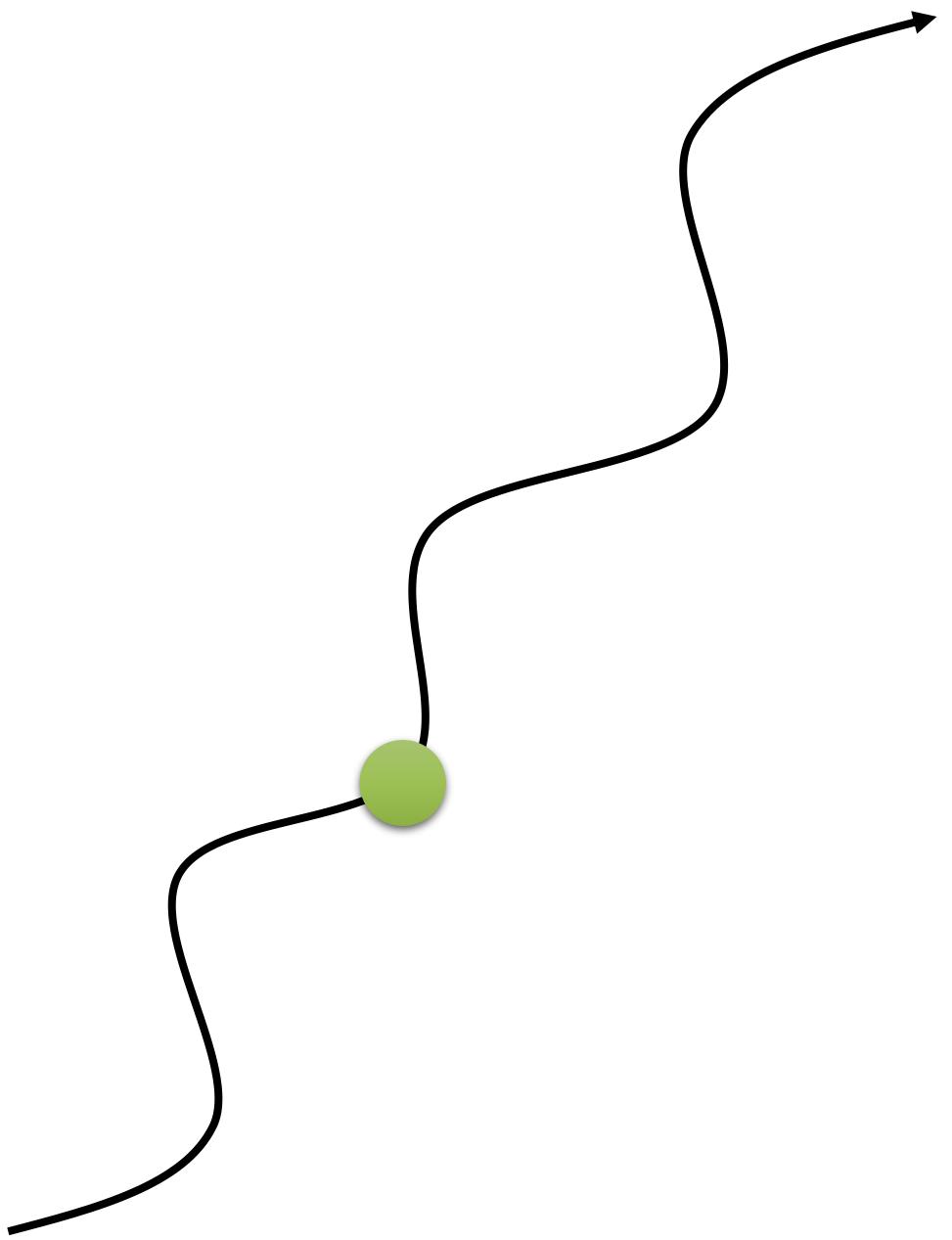
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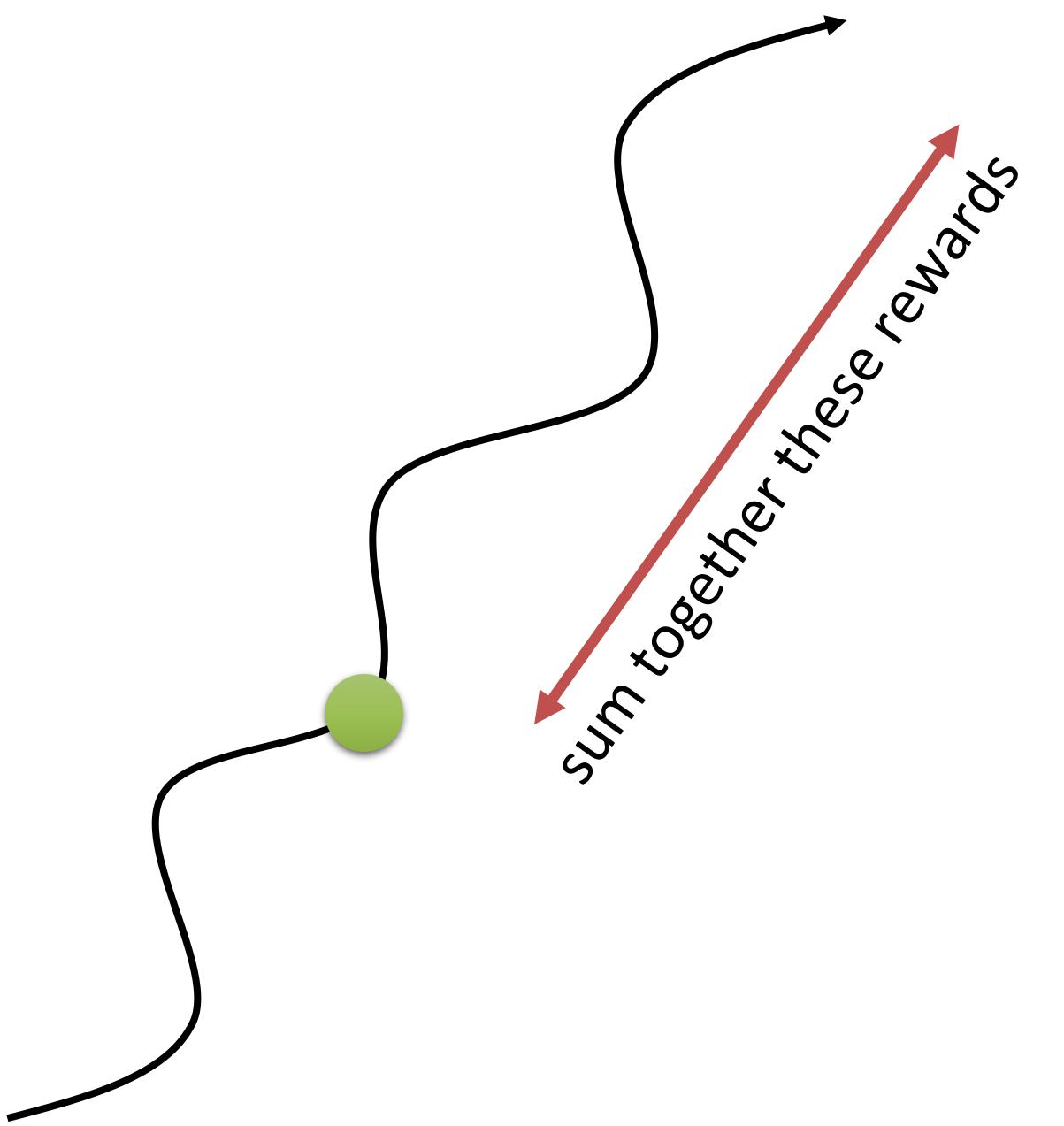
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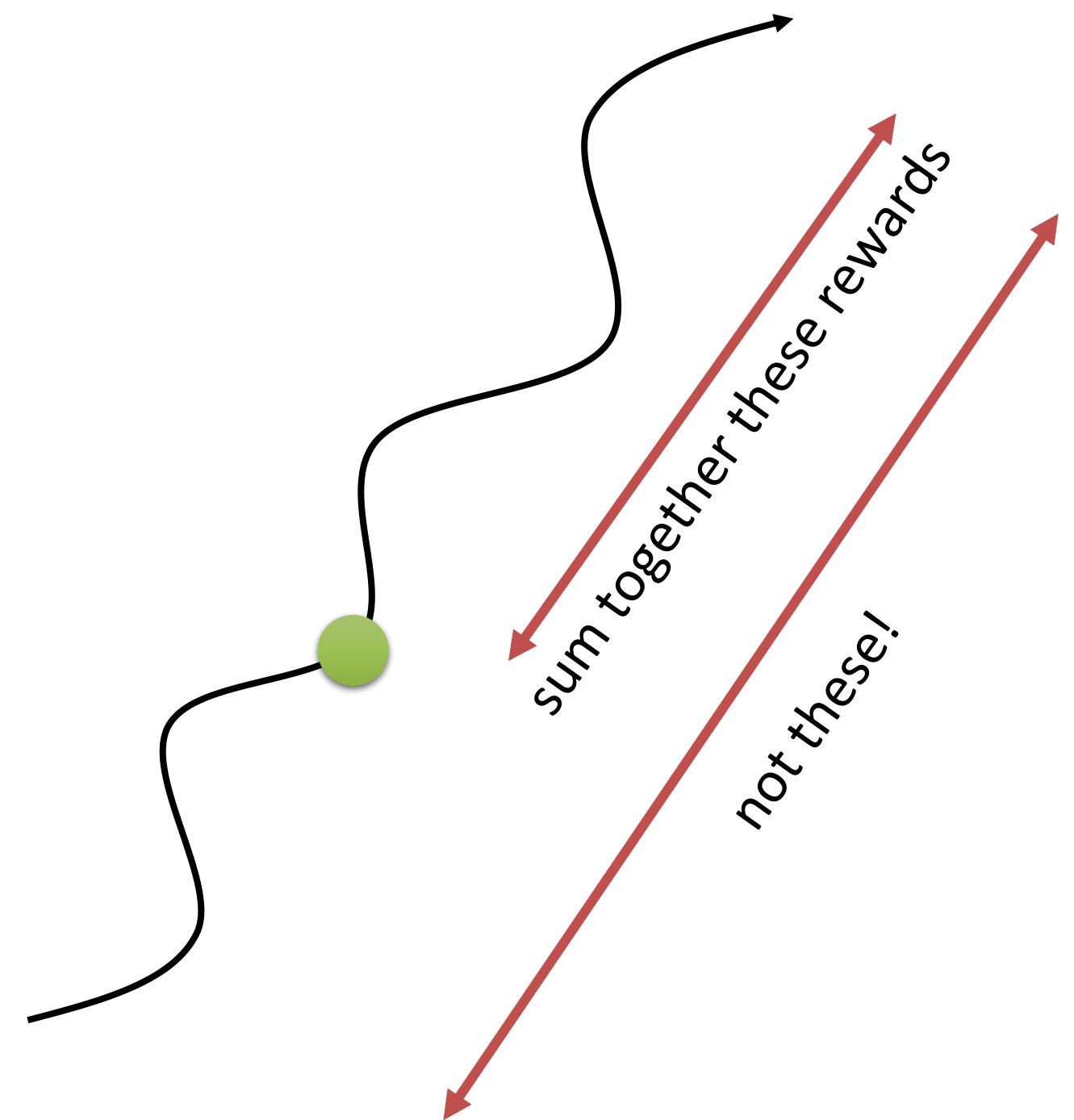
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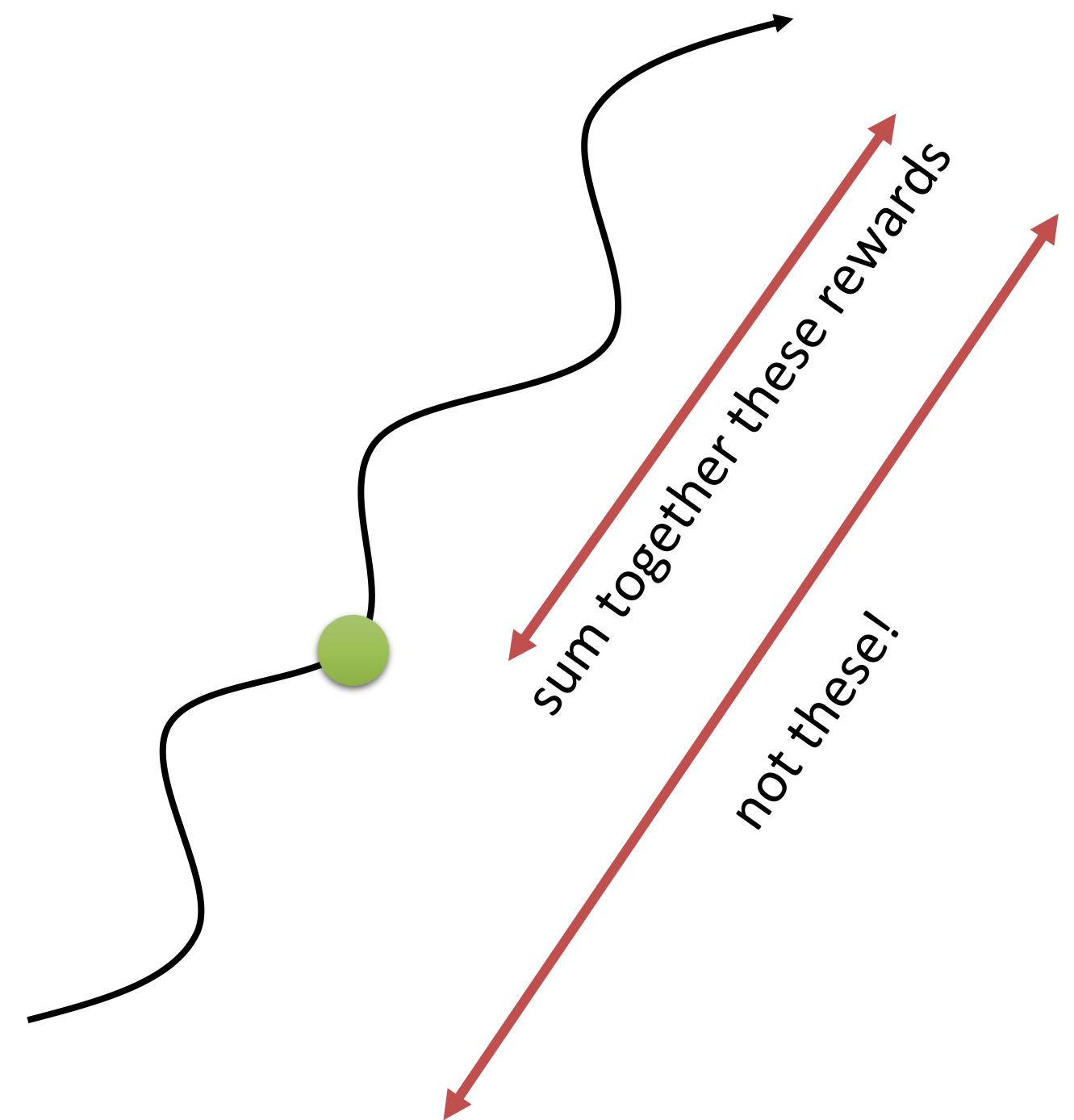


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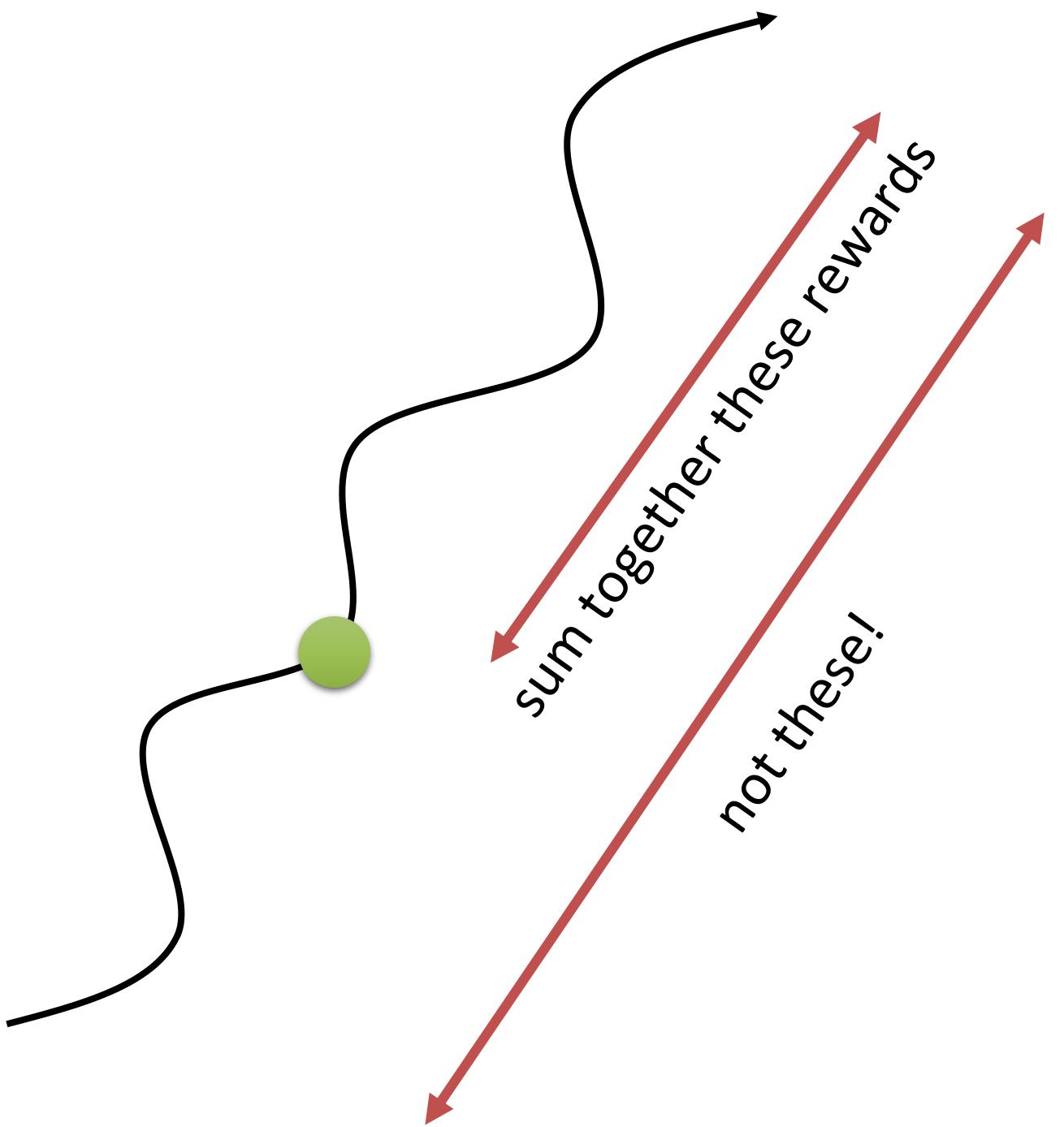


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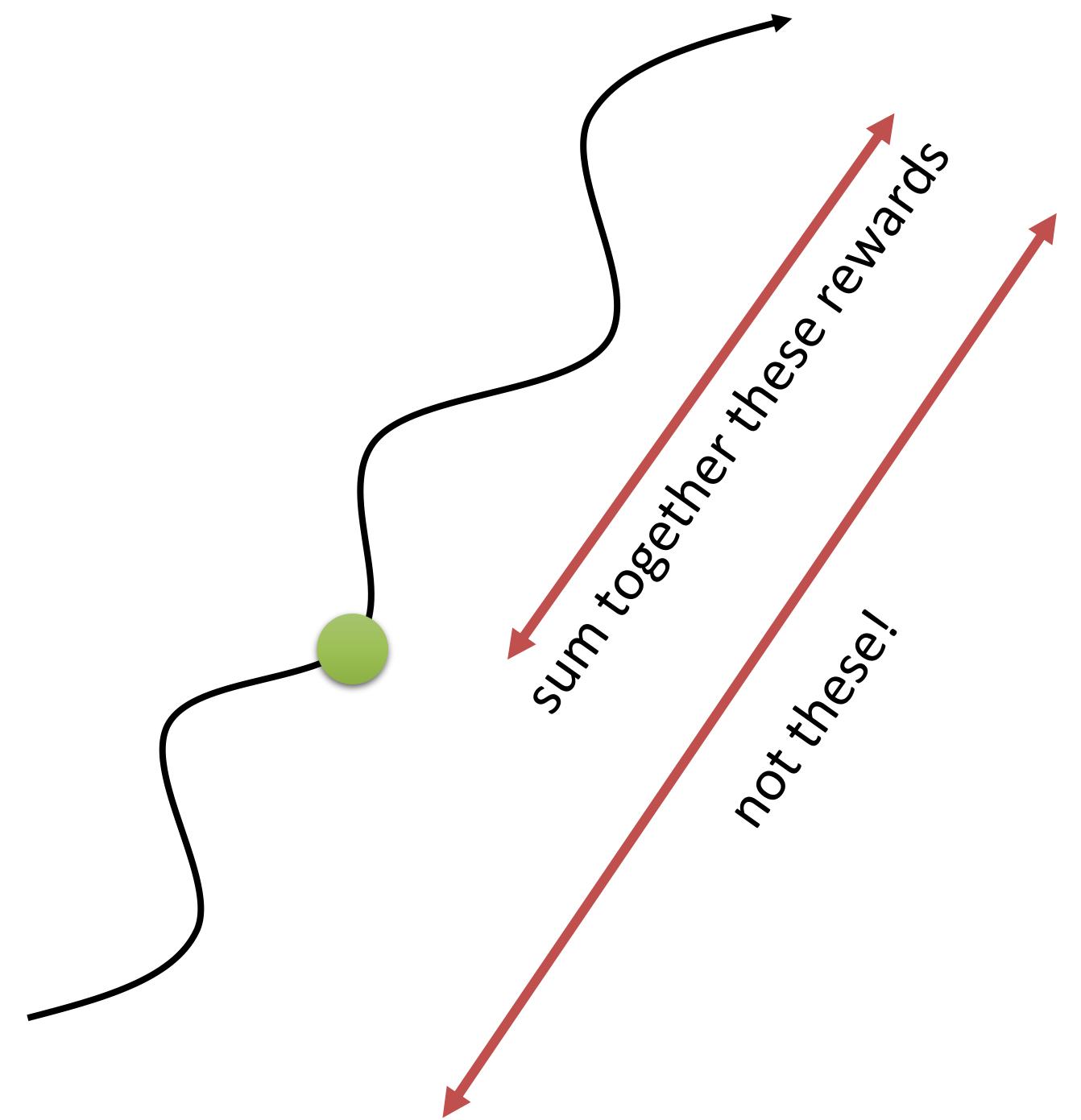


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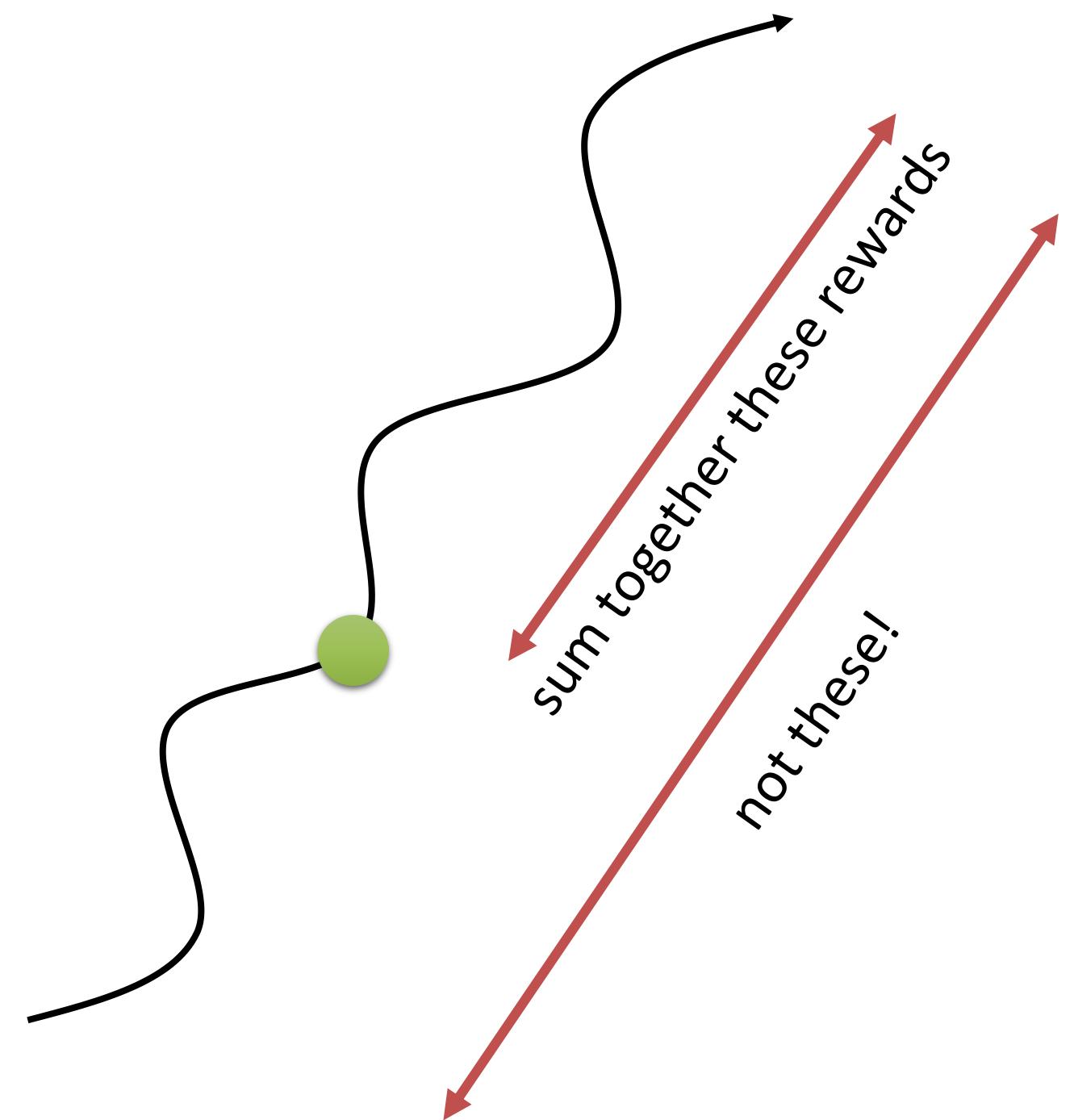
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Reward “to go”



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# Improving the policy gradient

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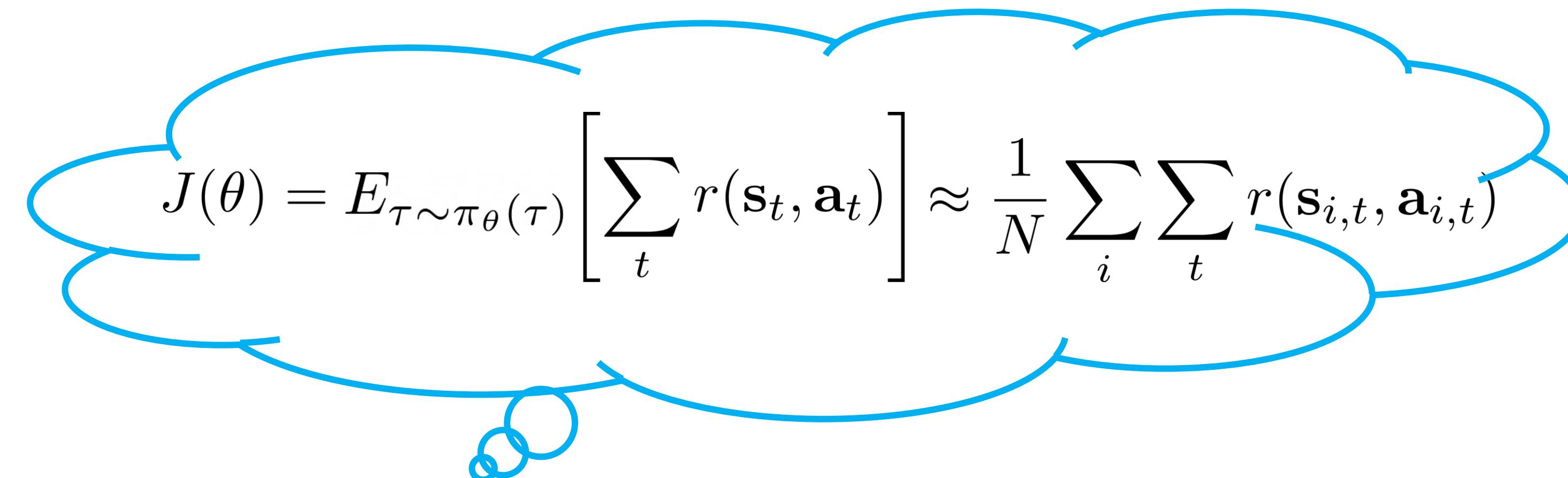
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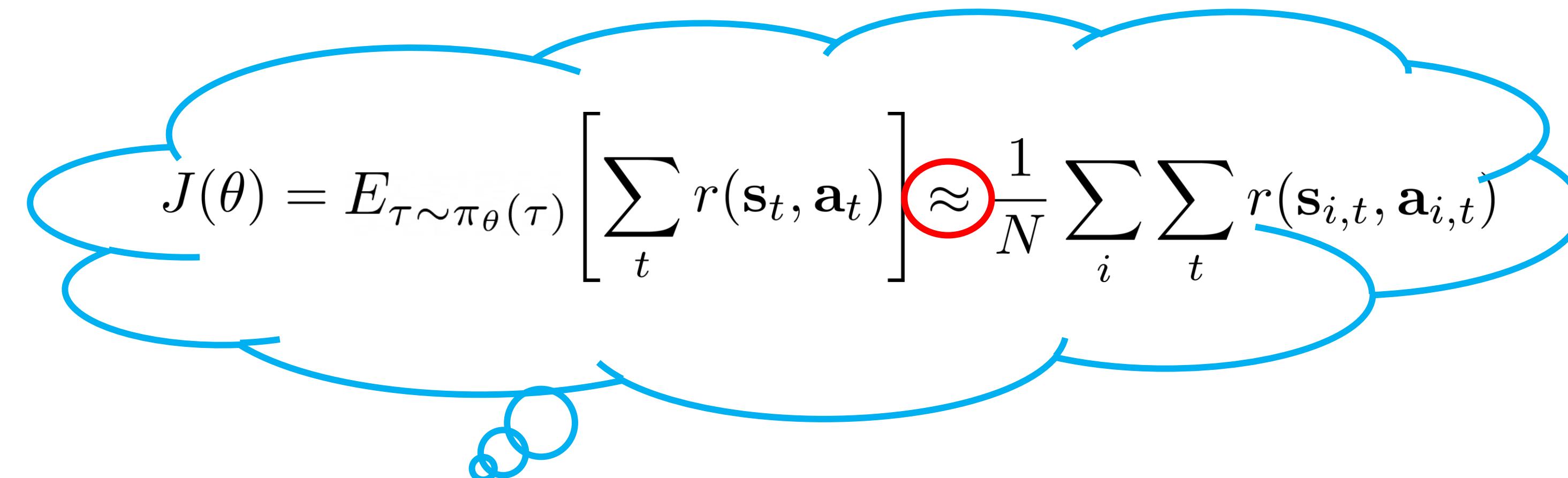


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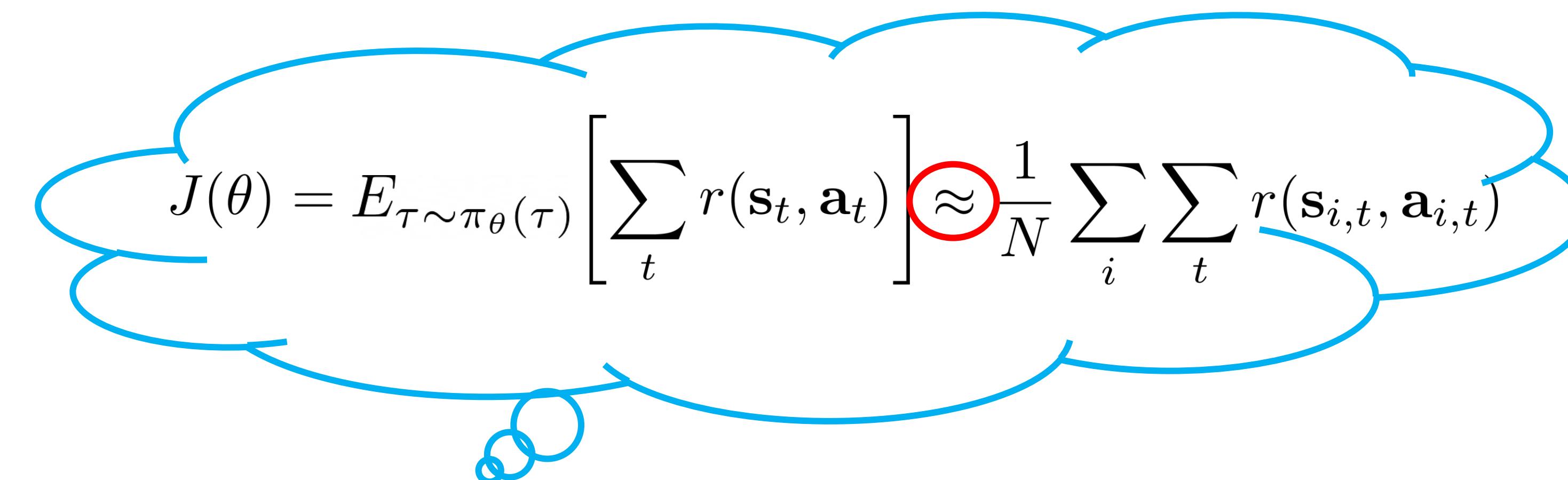
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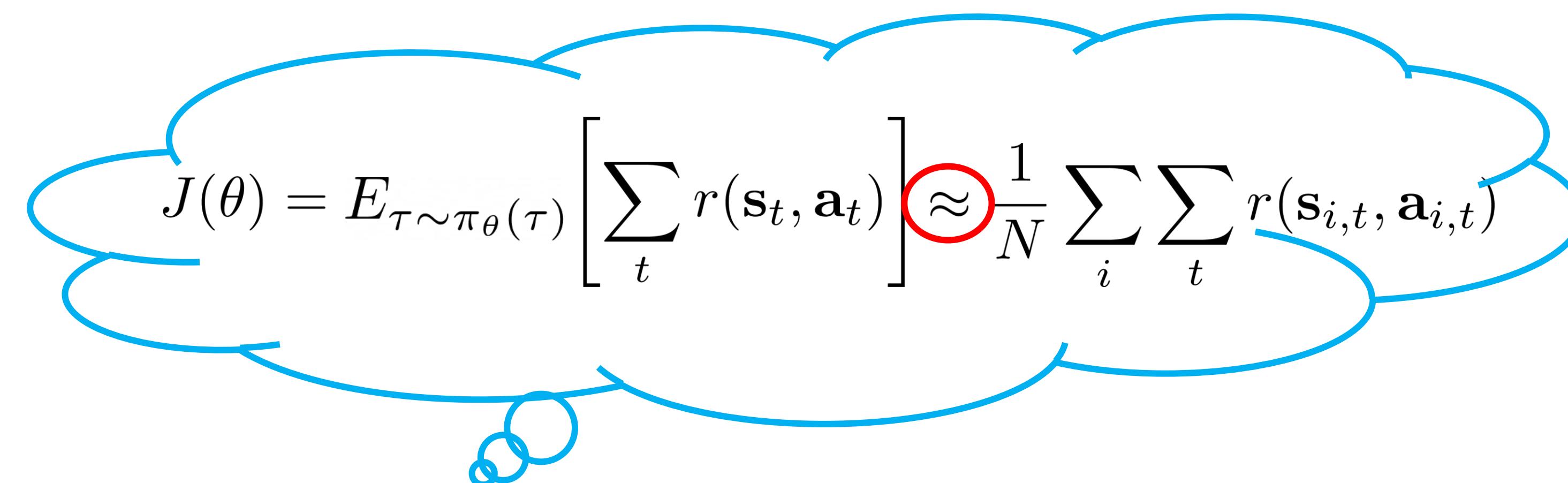
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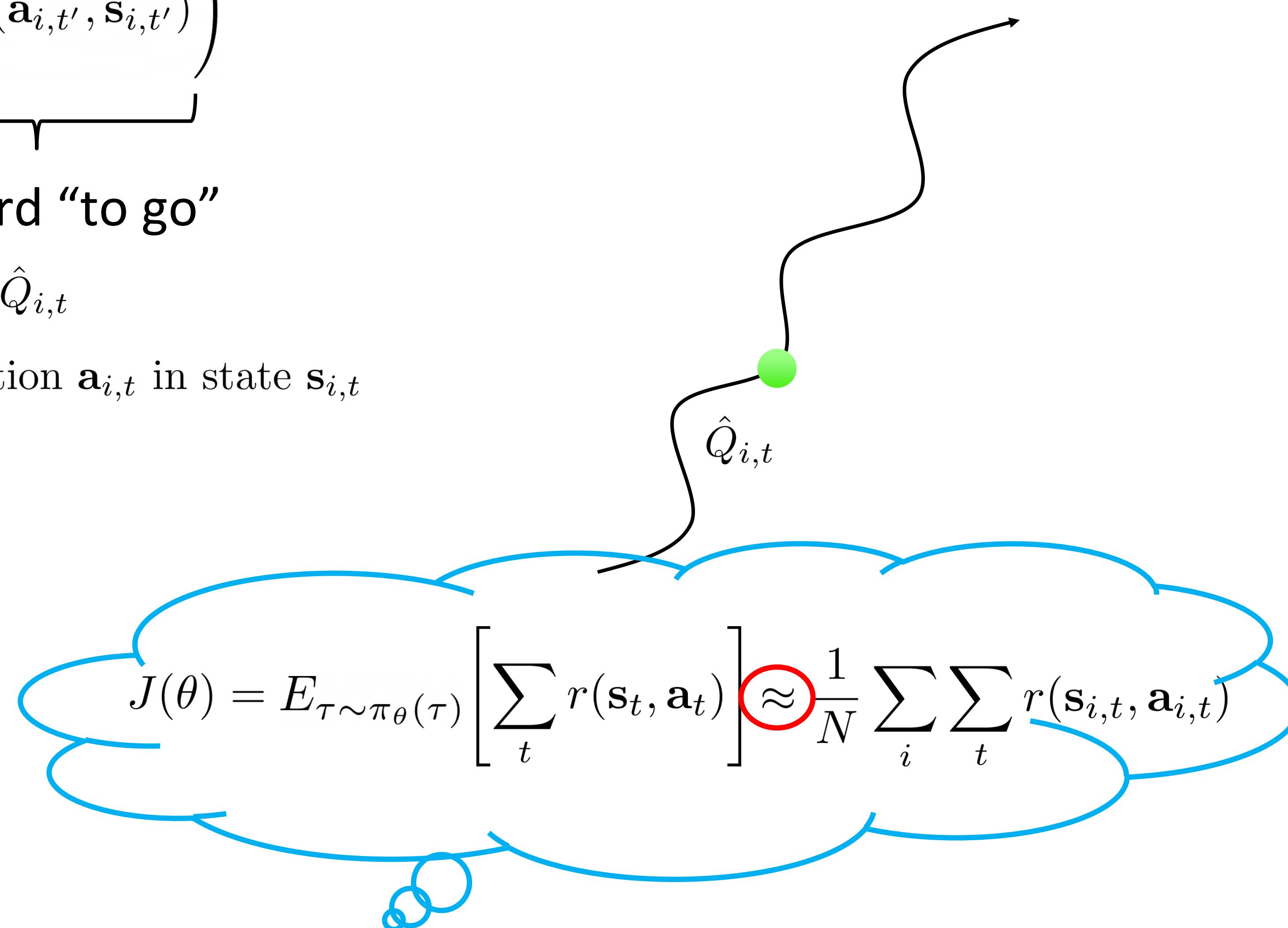
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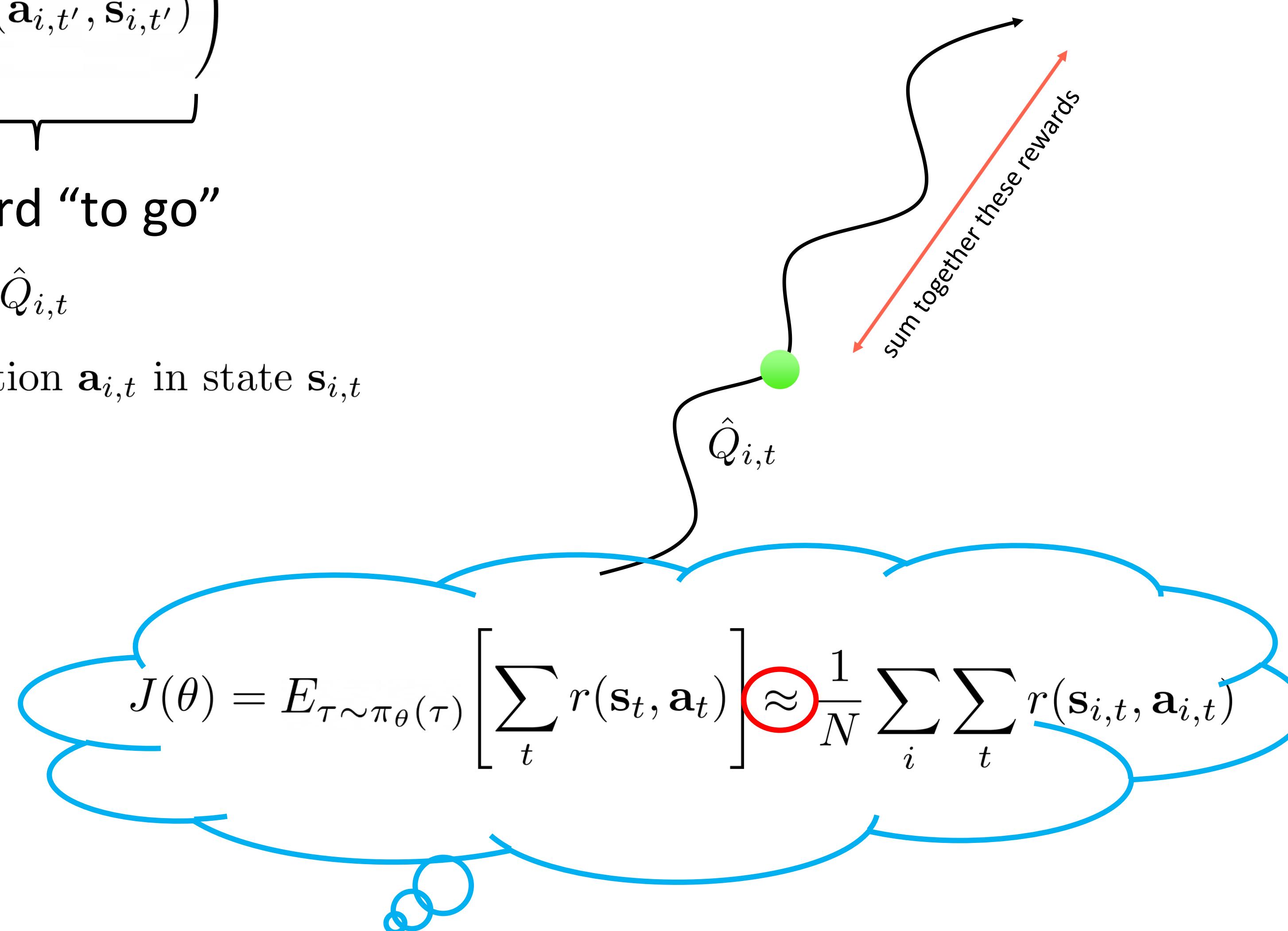
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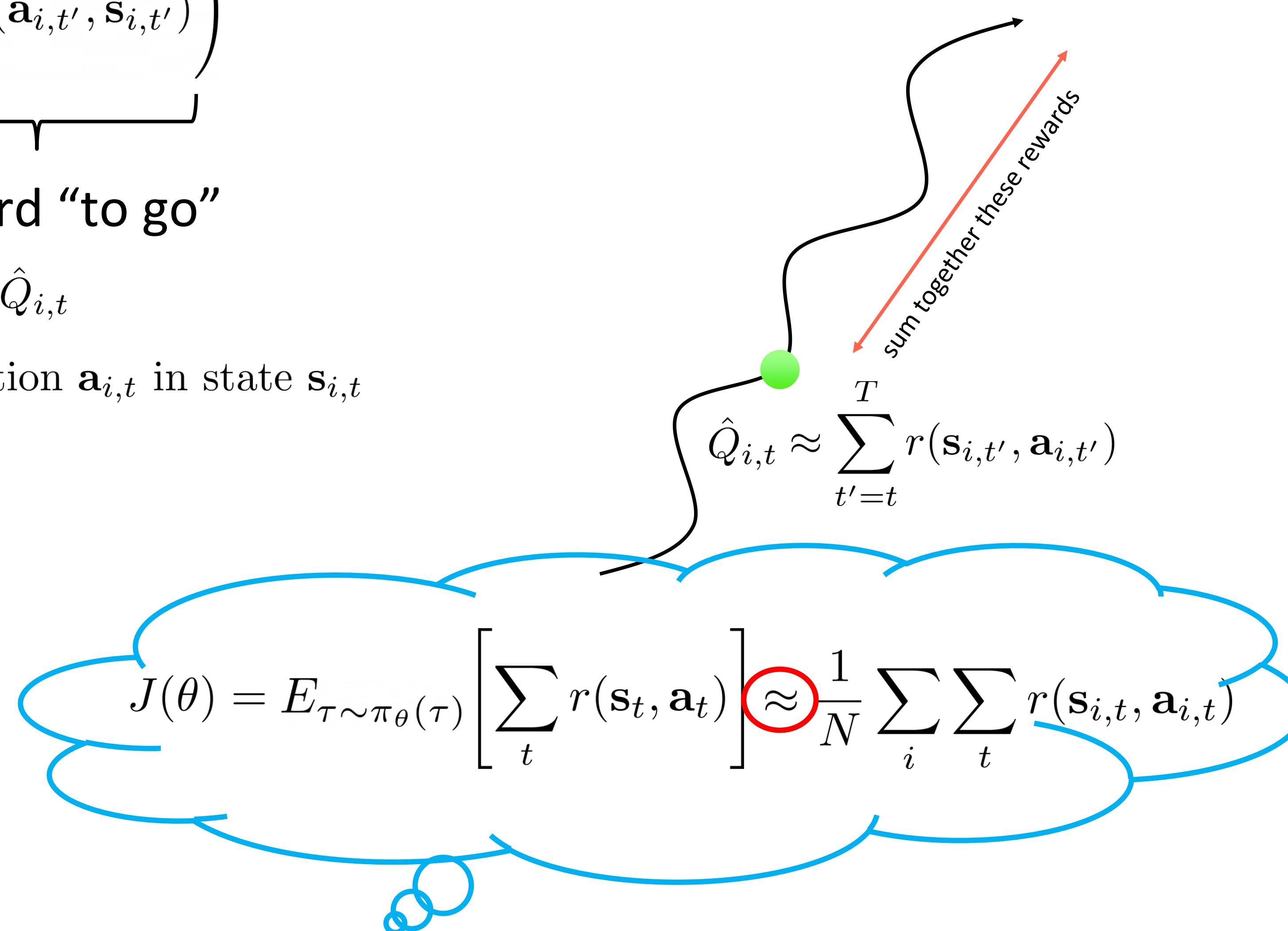
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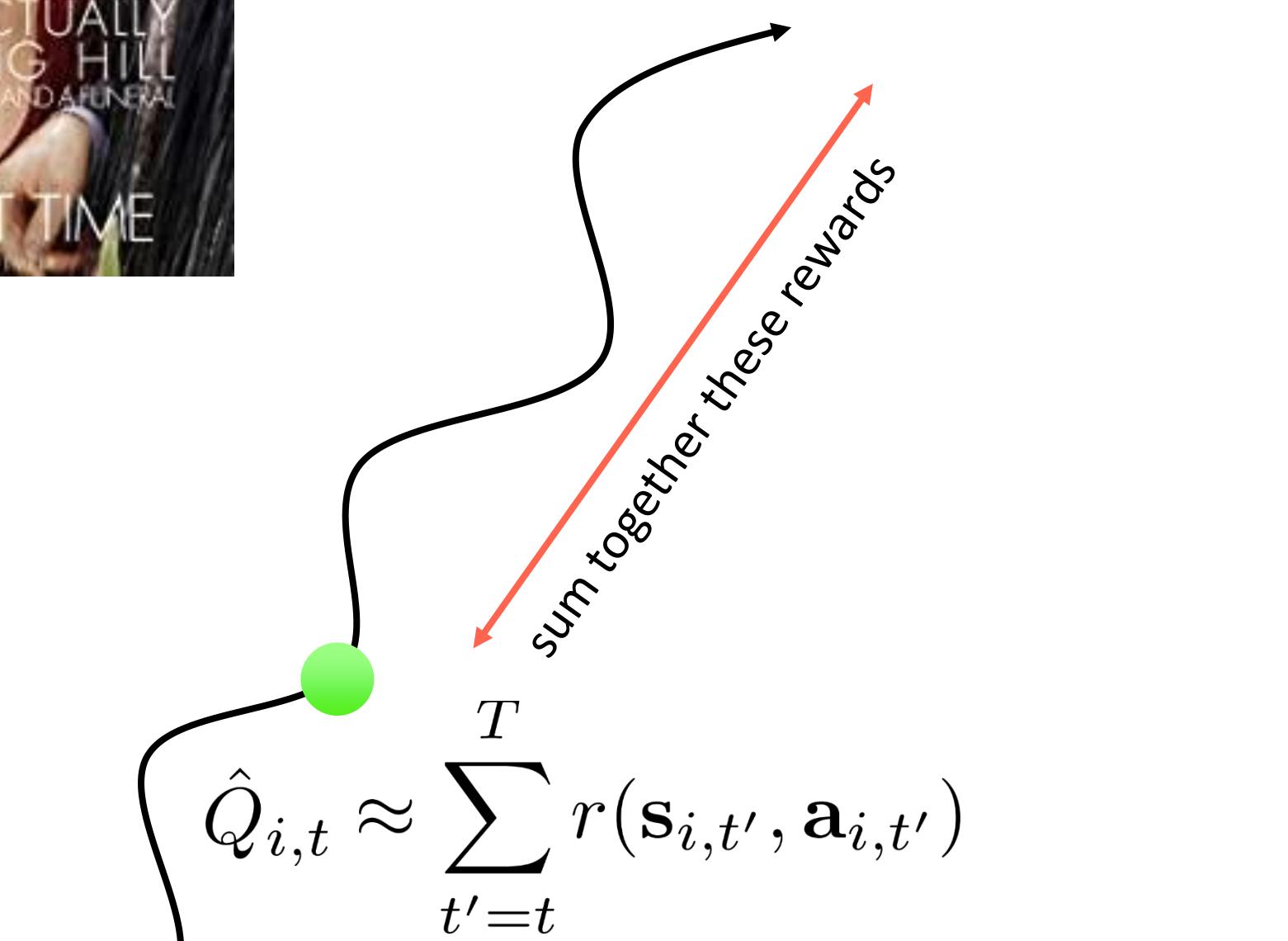
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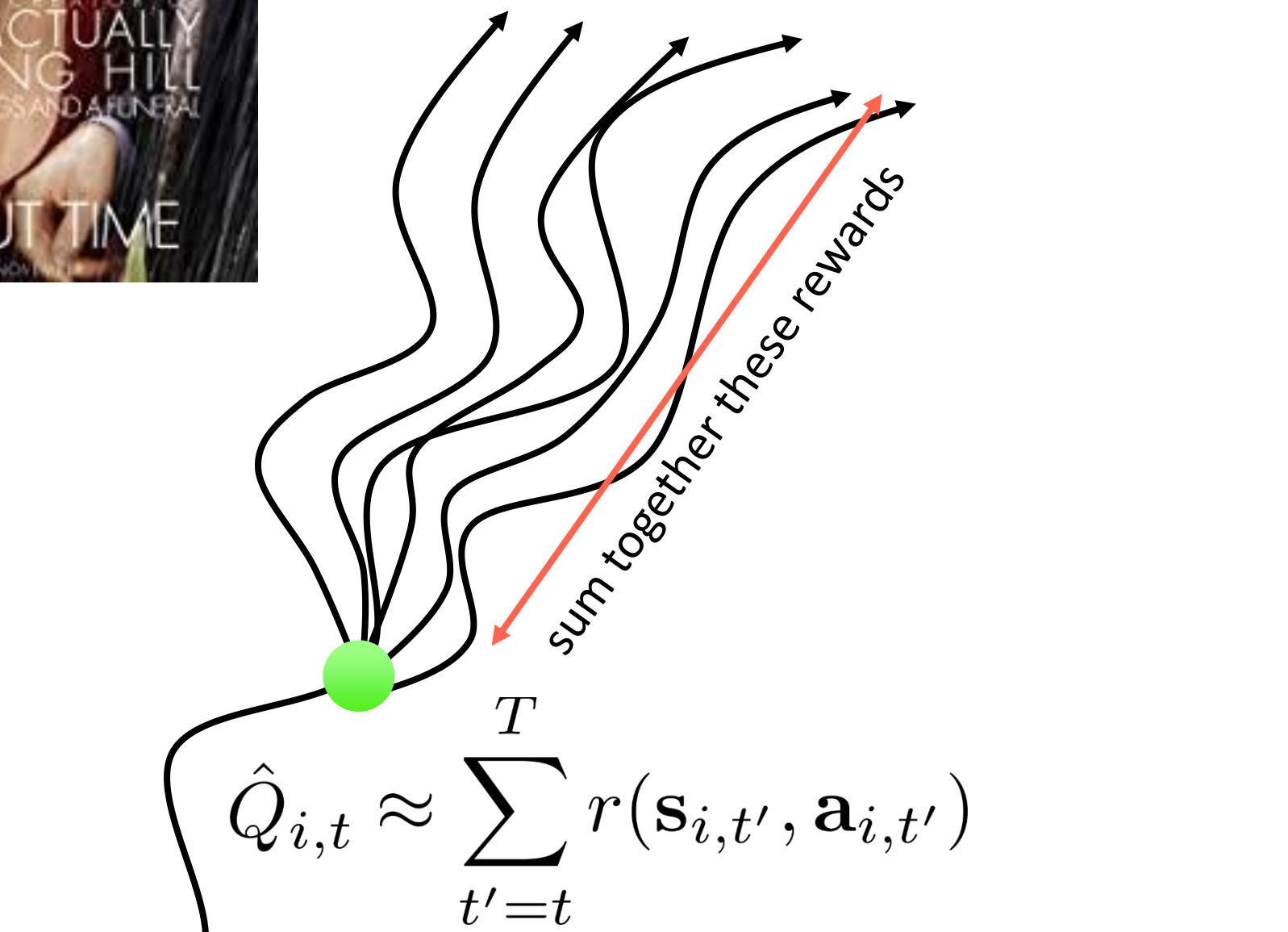
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Reward “to go”

$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$ : estimate of expected reward if we take action  $\mathbf{a}_{i,t}$  in state  $\mathbf{s}_{i,t}$

can we get a better estimate?



$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right] \approx \frac{1}{N} \sum_i \sum_t r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

# Improving the policy gradient

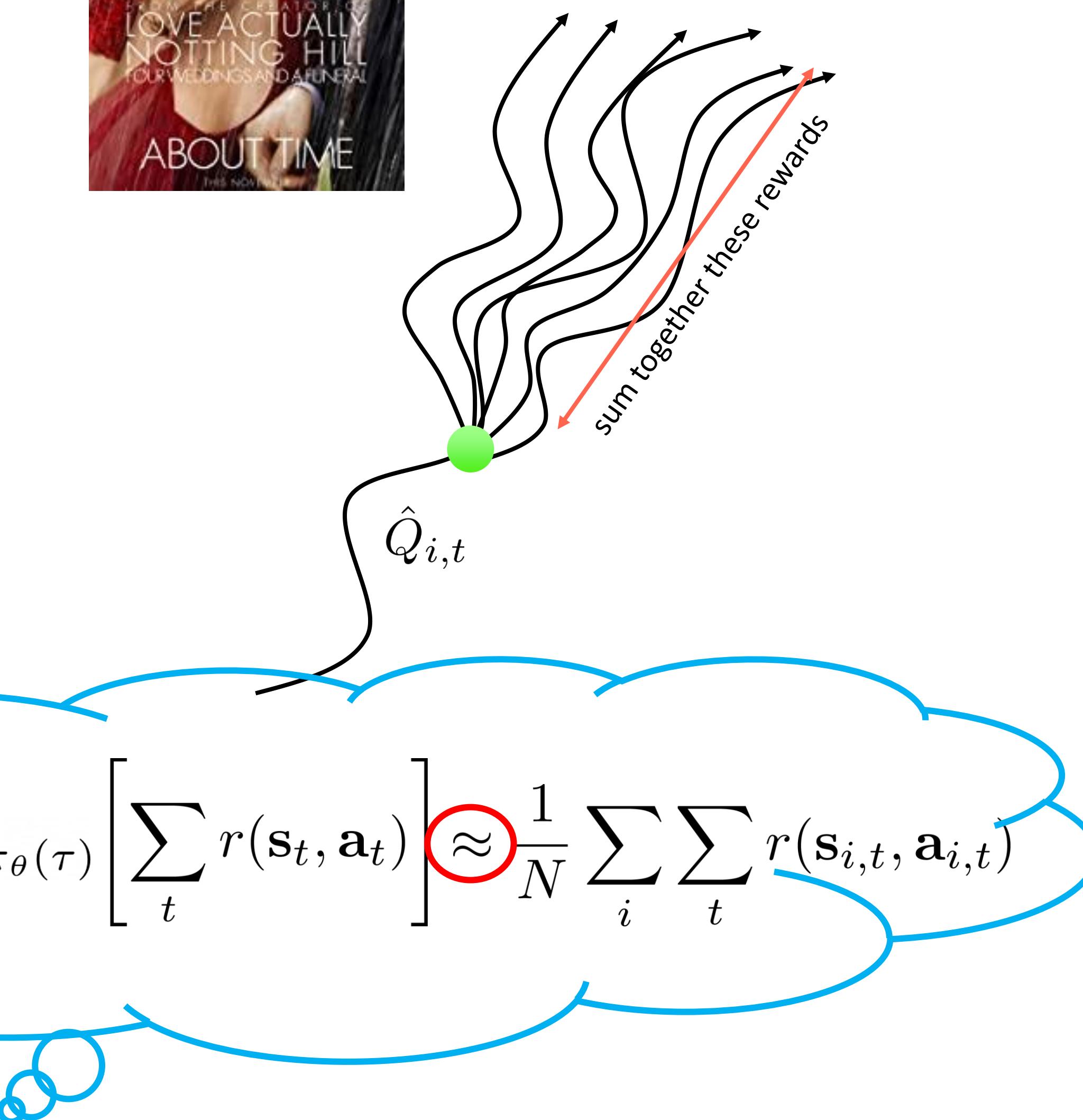
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \sum_{t'=t}^T r(\mathbf{a}_{i,t'}, \mathbf{s}_{i,t'}) \right)$$

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# Improving the policy gradient

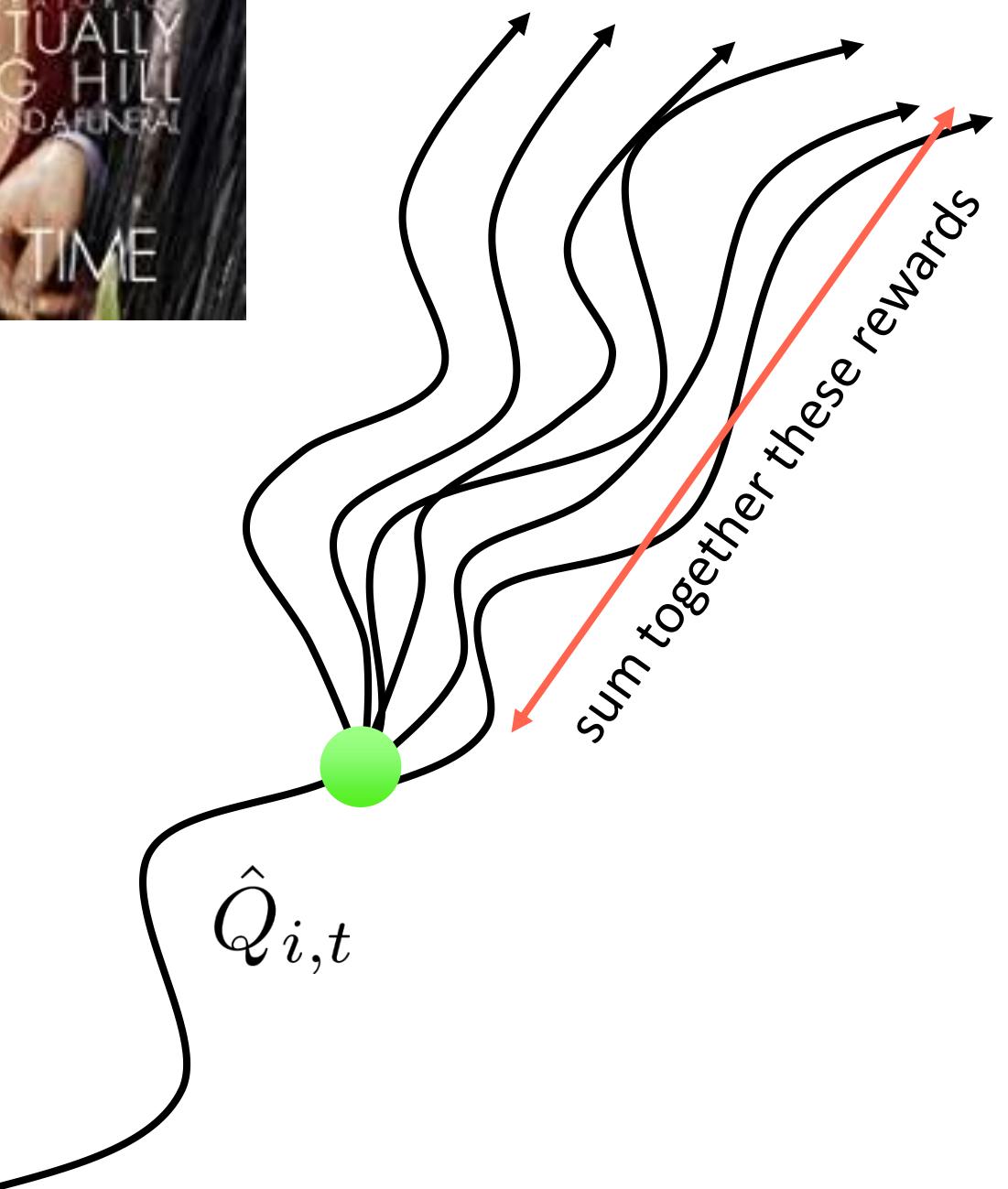
$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \sum_{t'=t}^T r(\mathbf{a}_{i,t'}, \mathbf{s}_{i,t'}) \right)$$

Reward “to go”

$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$ : estimate of expected reward if we take action  $\mathbf{a}_{i,t}$  in state  $\mathbf{s}_{i,t}$

can we get a better estimate?



# Improving the policy gradient

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \sum_{t'=t}^T r(\mathbf{a}_{i,t'}, \mathbf{s}_{i,t'}) \right)$$

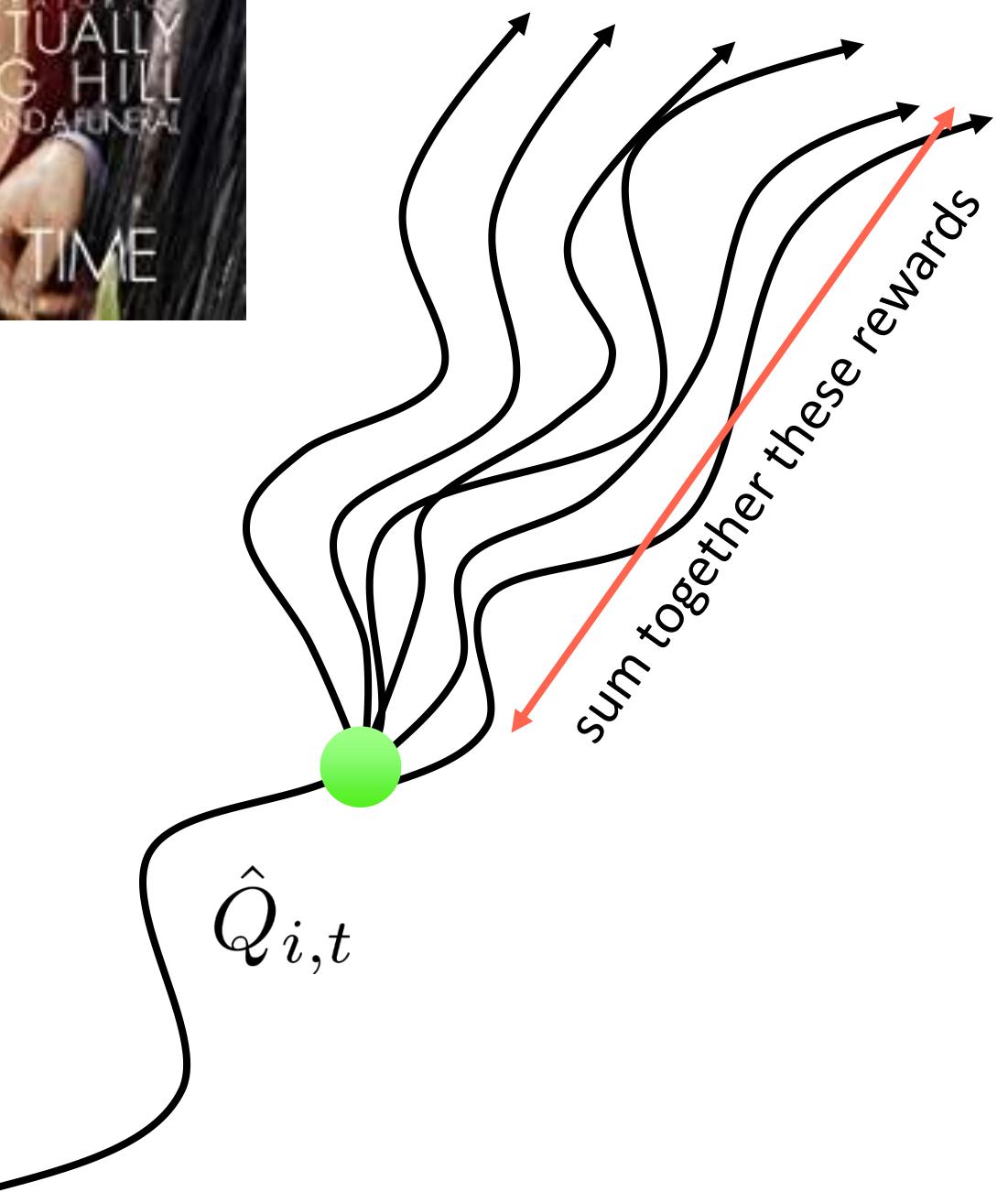
Reward “to go”

$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$ : estimate of expected reward if we take action  $\mathbf{a}_{i,t}$  in state  $\mathbf{s}_{i,t}$

can we get a better estimate?

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_{\theta}} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : true *expected* reward-to-go



# Improving the policy gradient

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \left( \sum_{t'=t}^T r(\mathbf{a}_{i,t'}, \mathbf{s}_{i,t'}) \right)$$

Reward “to go”

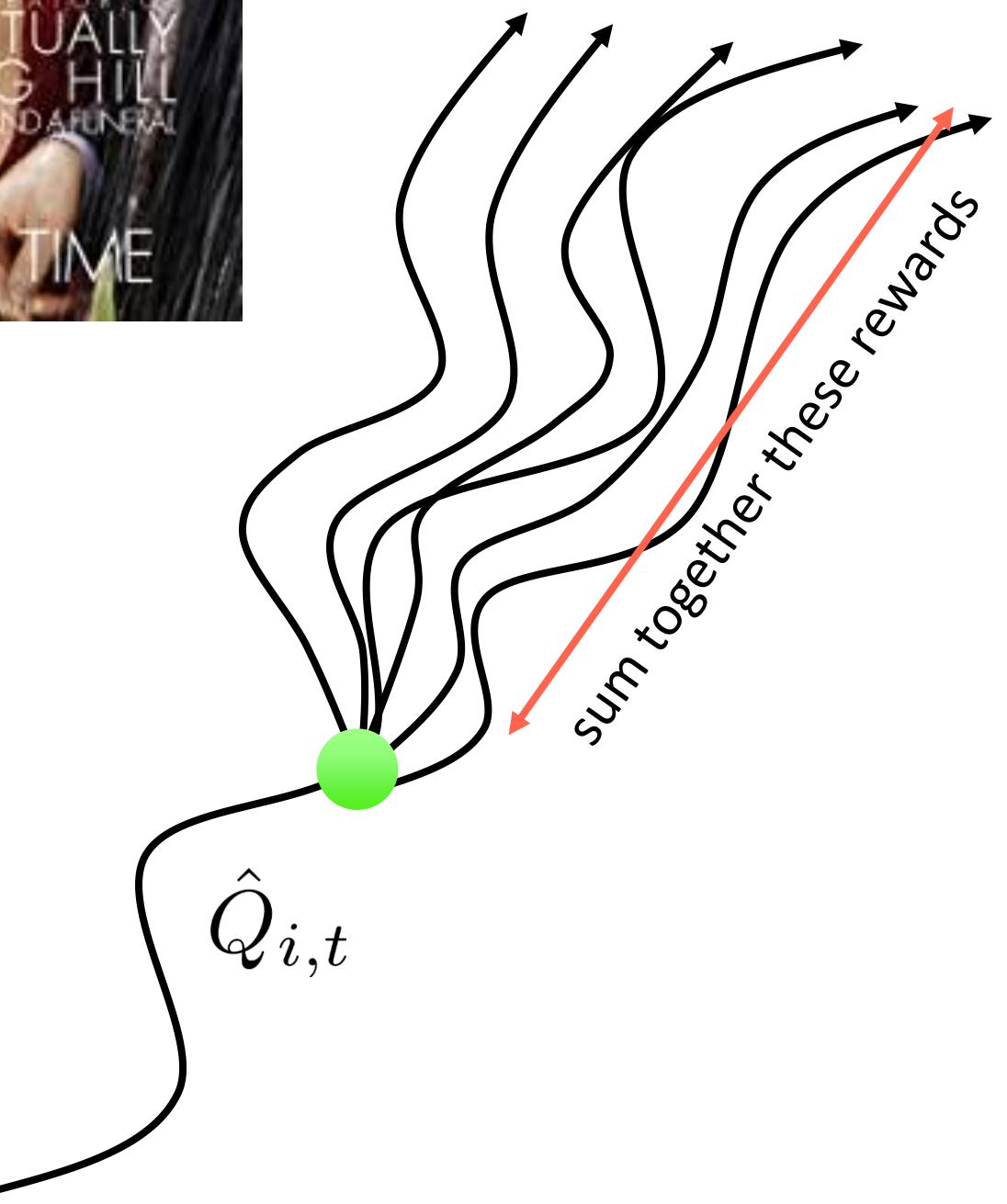
$$\hat{Q}_{i,t}$$

$\hat{Q}_{i,t}$ : estimate of expected reward if we take action  $\mathbf{a}_{i,t}$  in state  $\mathbf{s}_{i,t}$

can we get a better estimate?

$Q(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_{\theta}} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : true *expected* reward-to-go

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) Q(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$ : total reward from  $\mathbf{s}_t$

$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$ : how much better  $\mathbf{a}_t$  is



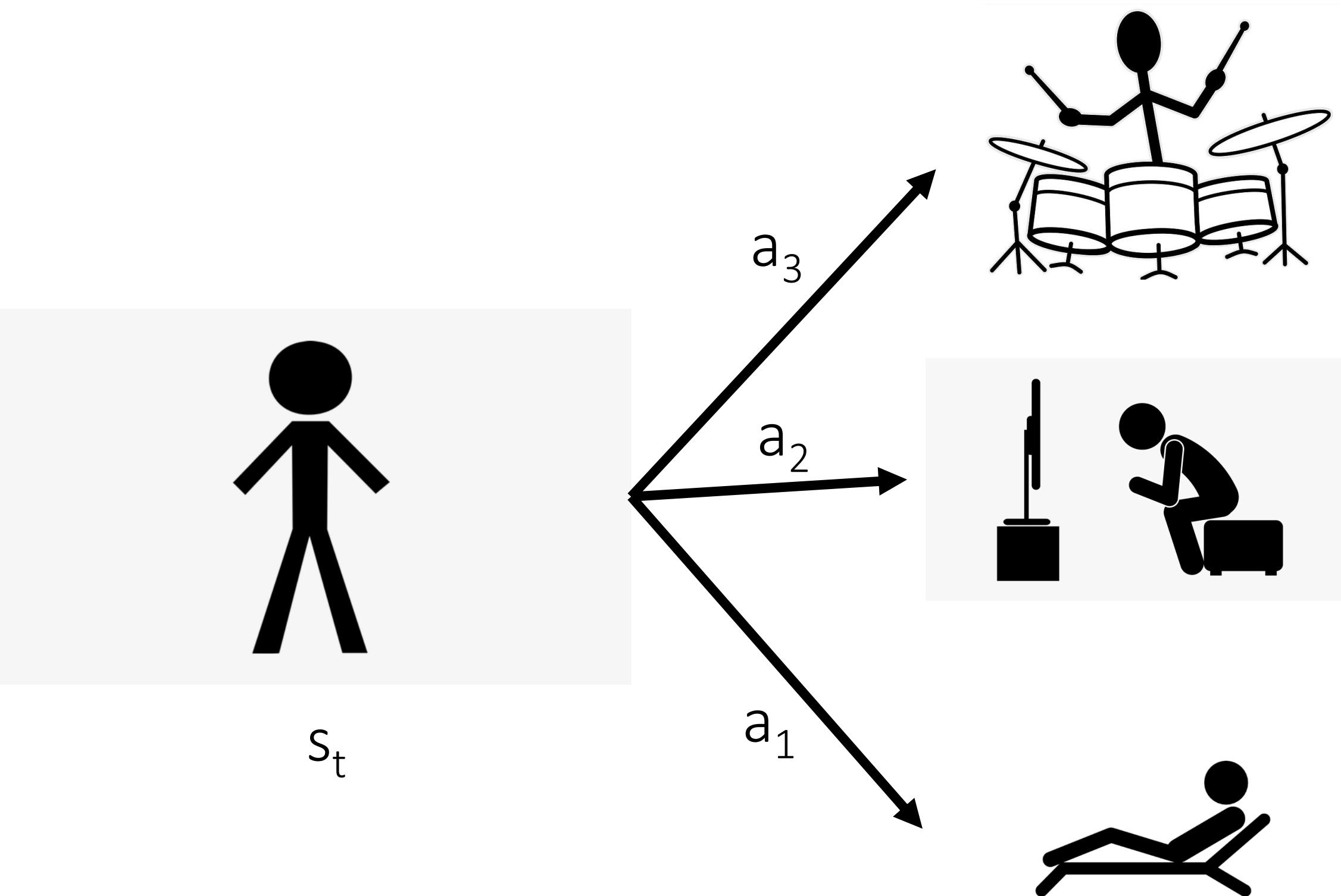
# Value-Based RL

Value function:  $V^\pi(\mathbf{s}_t) = ?$

Q function:  $Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = ?$

Advantage function:  $A^\pi(\mathbf{s}_t, \mathbf{a}_t) = ?$

Reward = 1 if I can play it  
in a month, 0 otherwise



Current  $\pi(\mathbf{a}_1|\mathbf{s}) = 1$

IMPROVISATION TEST EXAMPLES AND IDEAS FOR ROCKSCHOOL GRADE 1 DRUMS EXAM  
Written by Theo Lawrence / TL Music Lessons

$J = 70$

Exercise 1 - Rock

Exercise 2 - Rock

Exercise 3 - Rock

Exercise 4 - Rock

Exercise 5 - Funk Rock

Exercise 6 - Rock

Exercise 7 - Blues

Exercise 8 - Blues

# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$ : total reward from  $\mathbf{s}_t$

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# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

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$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$ : how much better  $\mathbf{a}_t$  is

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) Q^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

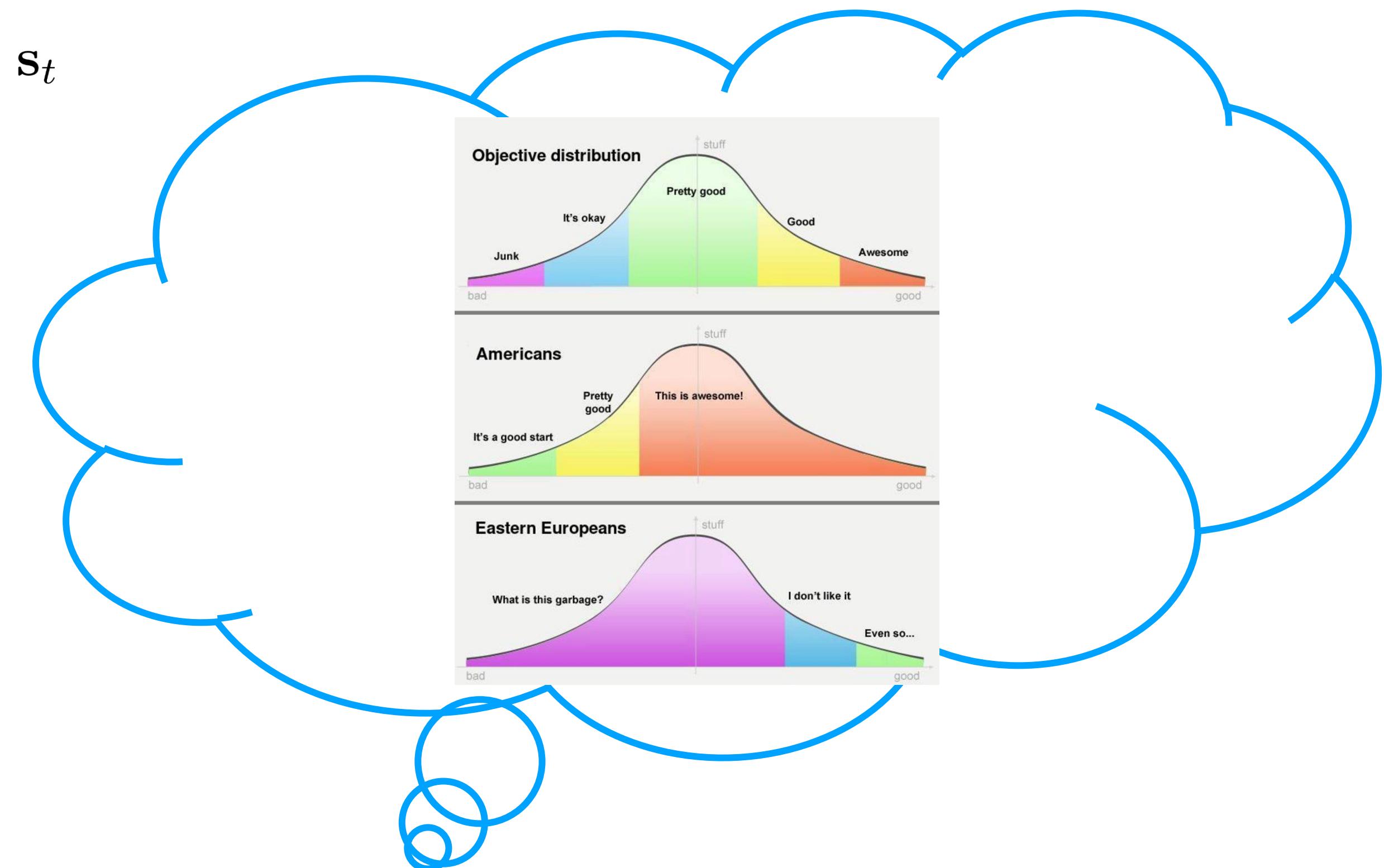
# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

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# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

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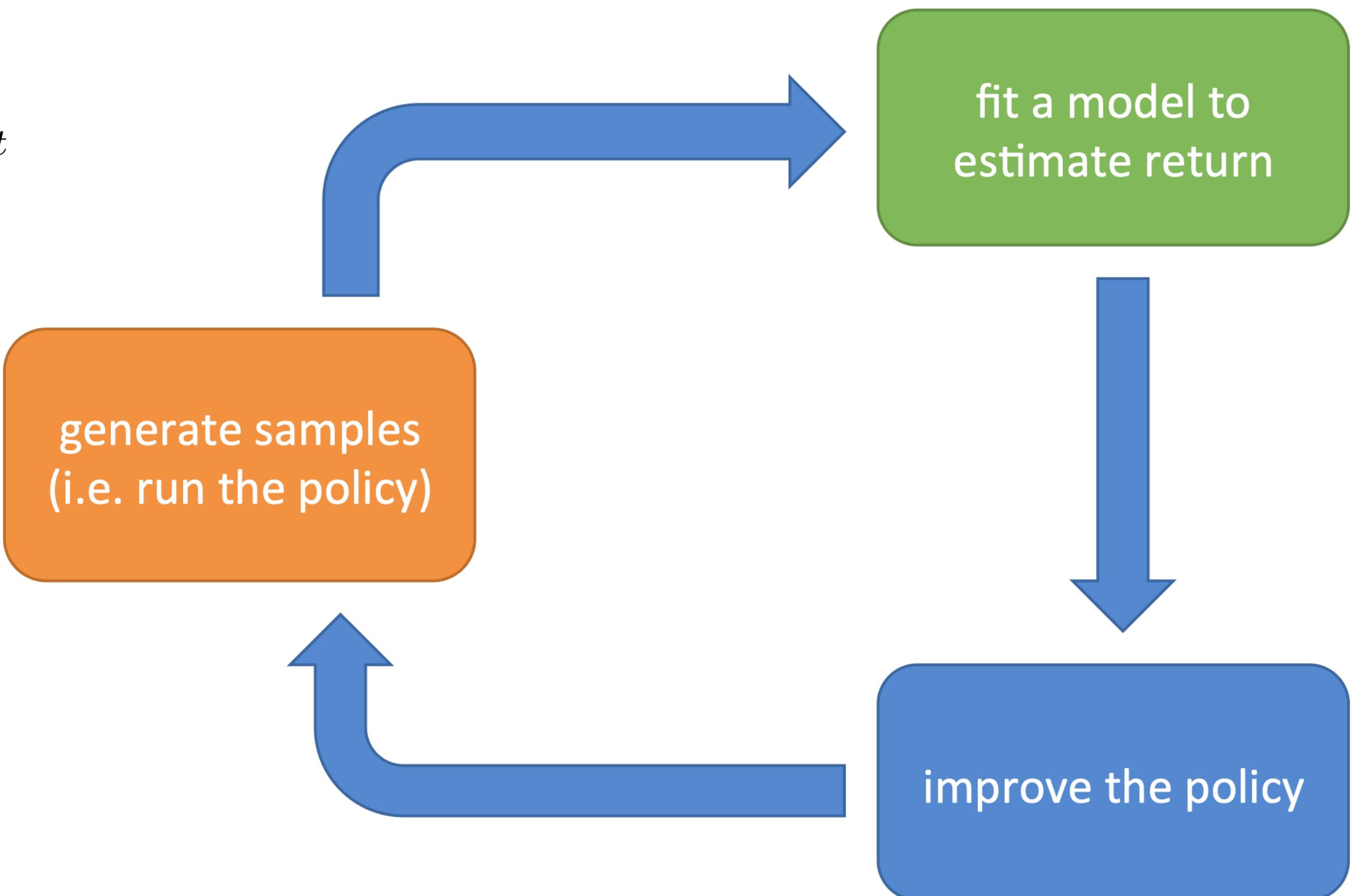
# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

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# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

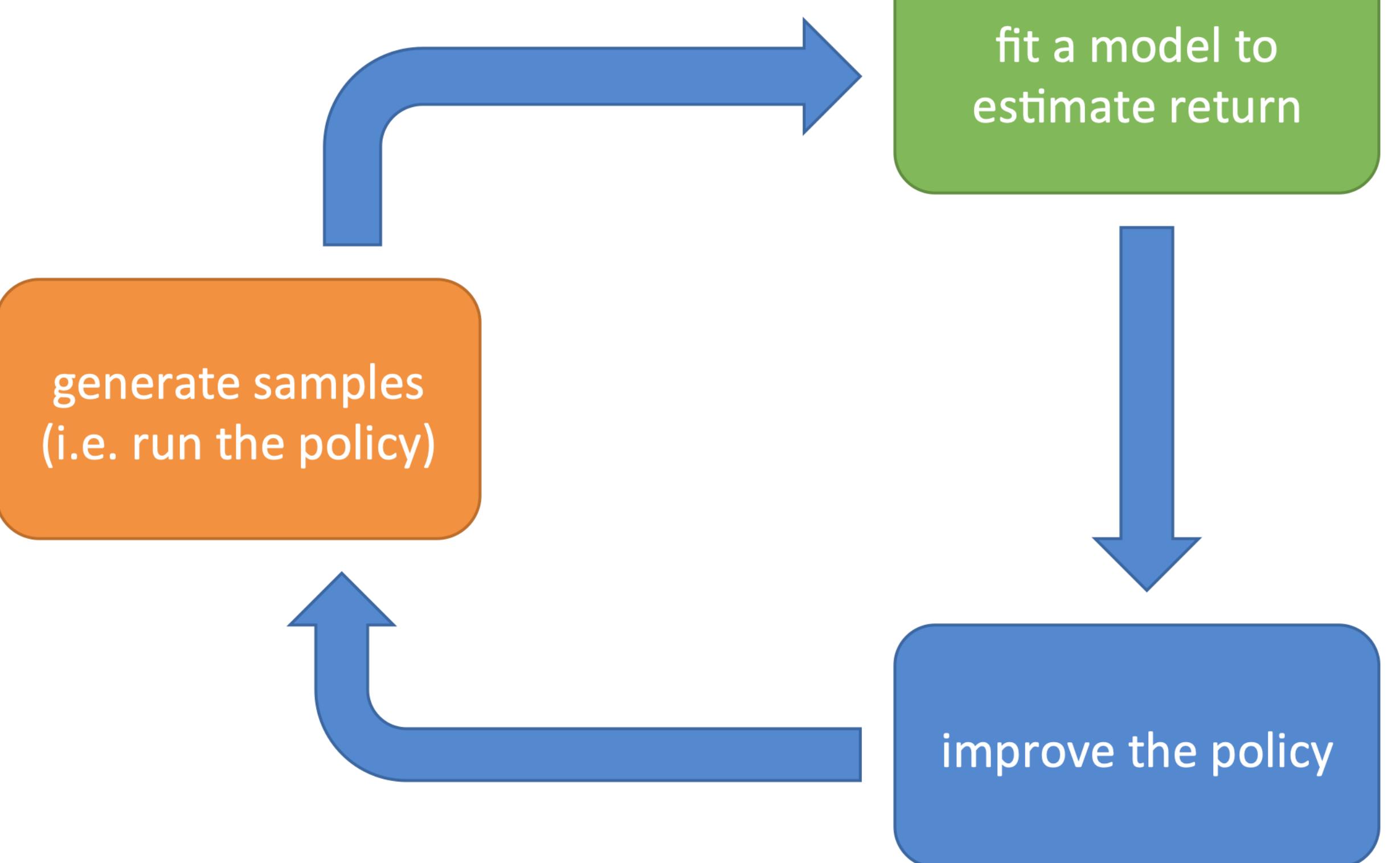
fit  $Q^\pi$ ,  $V^\pi$ , or  $A^\pi$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$ : total reward from  $\mathbf{s}_t$

fit a model to estimate return

$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$ : how much better  $\mathbf{a}_t$  is

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



# State & state-action value functions

$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$ : total reward from taking  $\mathbf{a}_t$  in  $\mathbf{s}_t$

fit  $Q^\pi$ ,  $V^\pi$ , or  $A^\pi$

$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$ : total reward from  $\mathbf{s}_t$

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$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

generate samples  
(i.e. run the policy)

improve the policy

$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

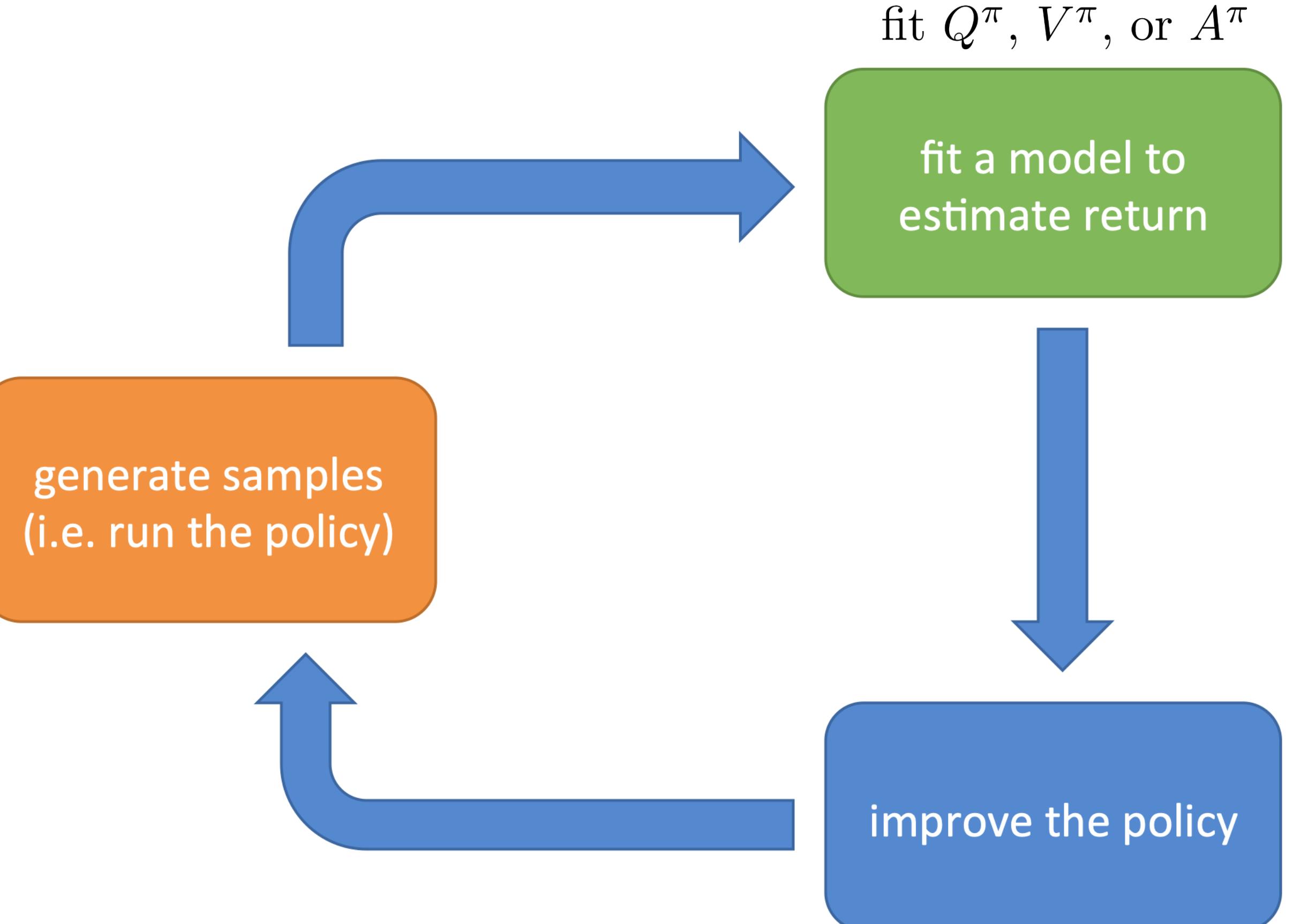
# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$



# Value function fitting

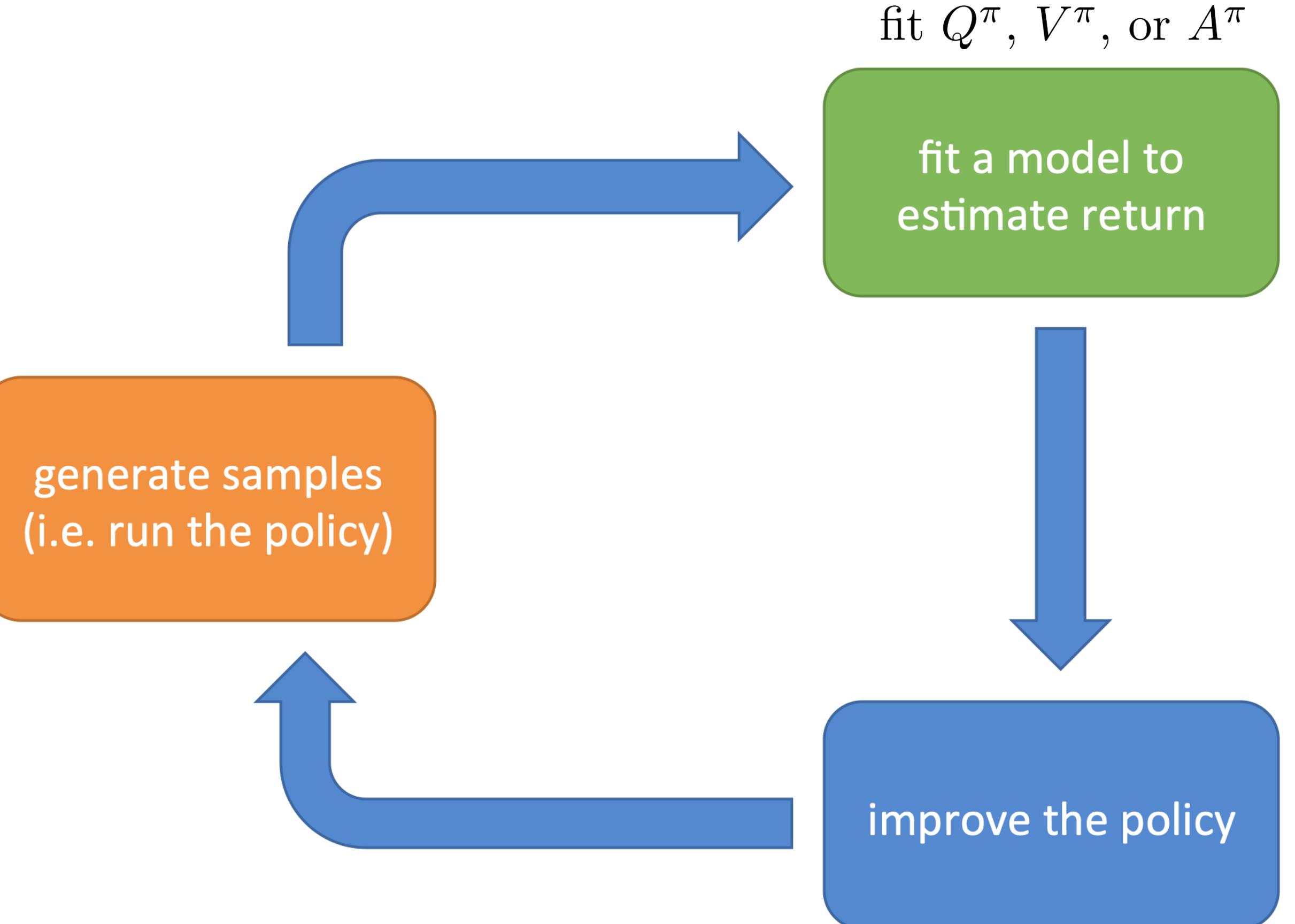
$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what*?



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$



# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

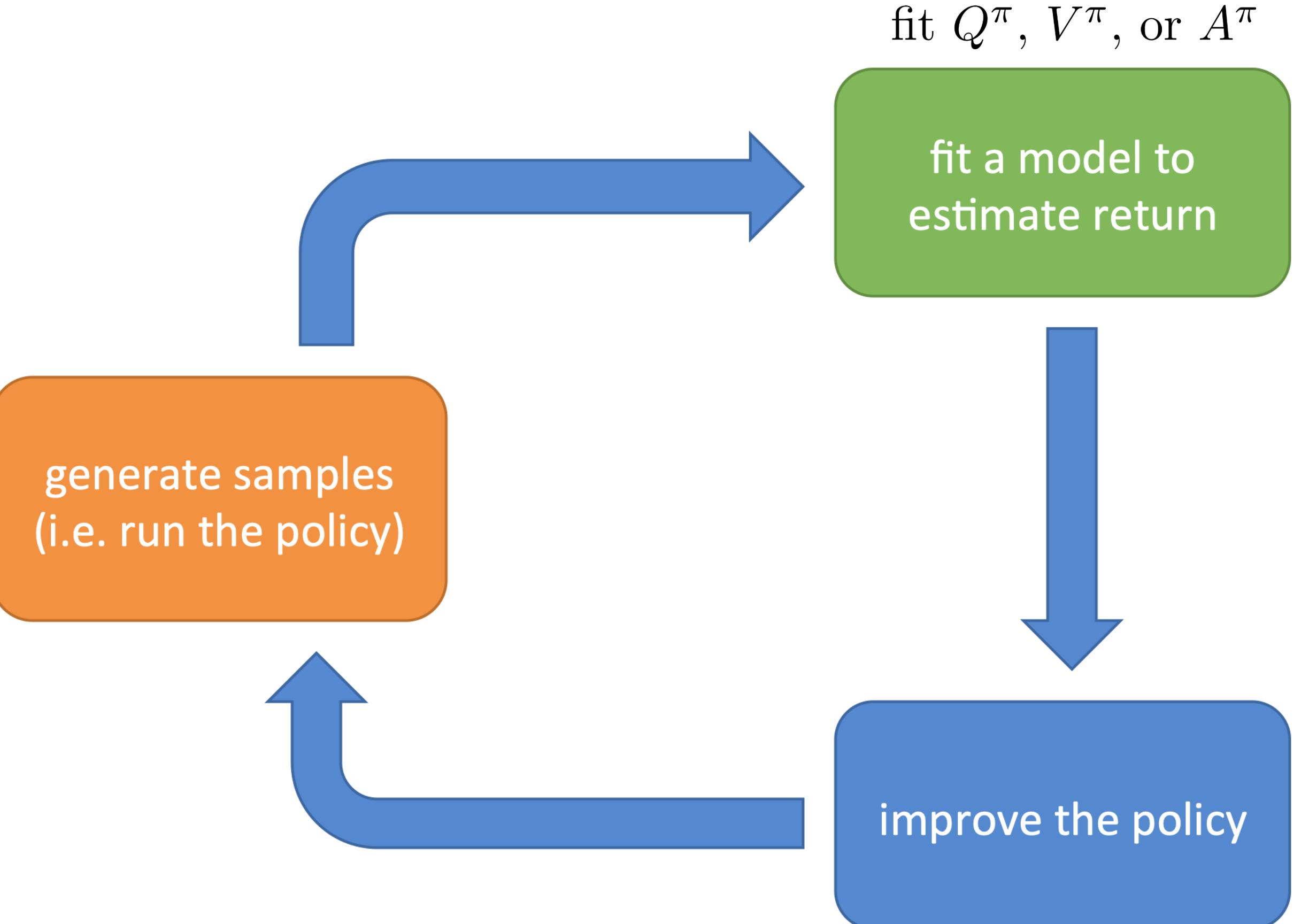
$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

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fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

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$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \sum_{t'=t+1}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

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fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + \frac{\sum_{t'=t+1}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]}{V^\pi(\mathbf{s}_{t+1})}$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

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fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = r(\mathbf{s}_t, \mathbf{a}_t) + E_{\mathbf{s}_{t+1} \sim p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)} [V^\pi(\mathbf{s}_{t+1})]$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

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fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

fit  $Q^\pi, V^\pi$ , or  $A^\pi$

fit a model to estimate return

generate samples  
(i.e. run the policy)

improve the policy

$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

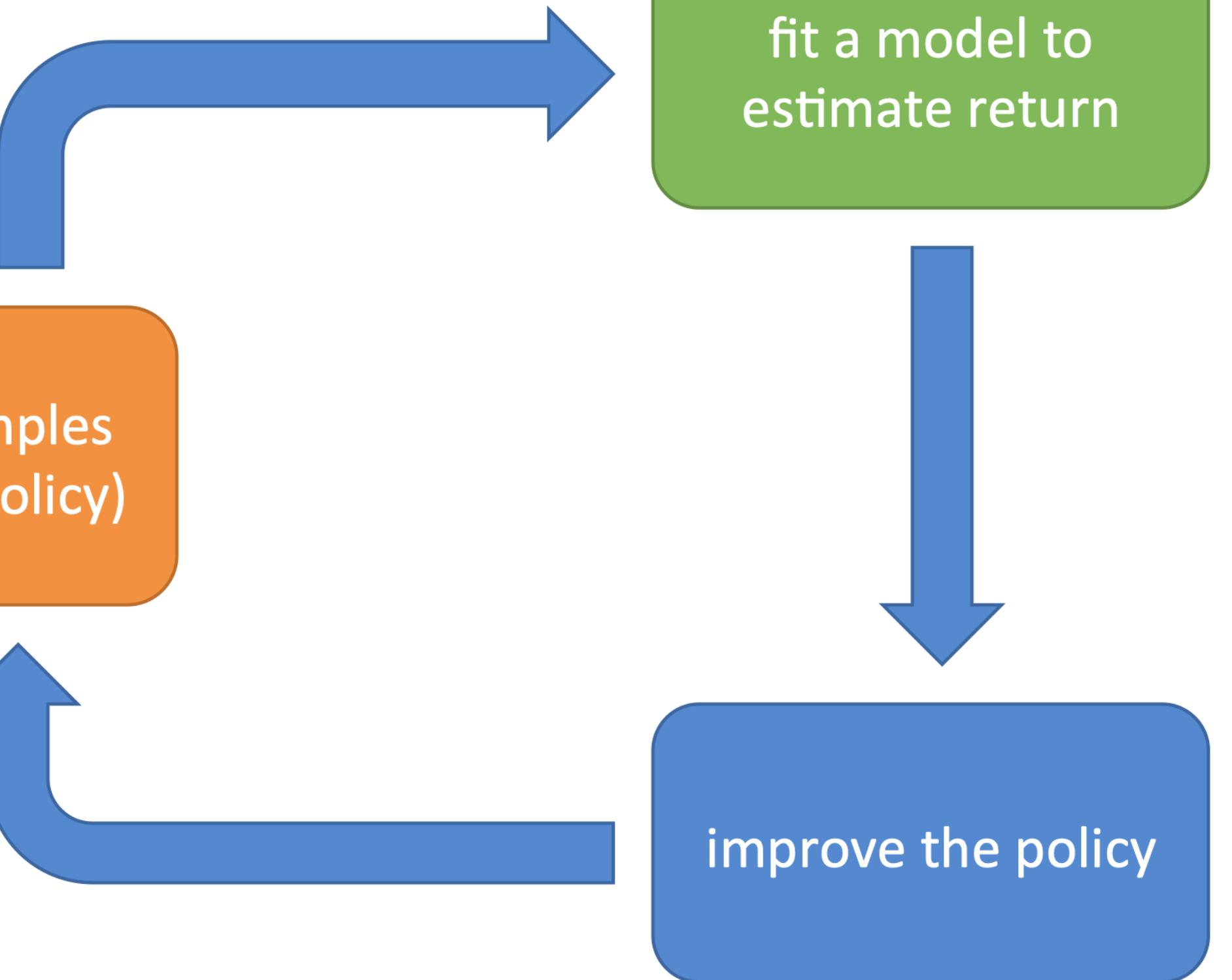
fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

let's just fit  $V^\pi(\mathbf{s})$ !



$$\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$$

# Value function fitting

$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_t, \mathbf{a}_t]$$

$$V^\pi(\mathbf{s}_t) = E_{\mathbf{a}_t \sim \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} [Q^\pi(\mathbf{s}_t, \mathbf{a}_t)]$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) = Q^\pi(\mathbf{s}_t, \mathbf{a}_t) - V^\pi(\mathbf{s}_t)$$

$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_\theta \log \pi_\theta(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) A^\pi(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})$$

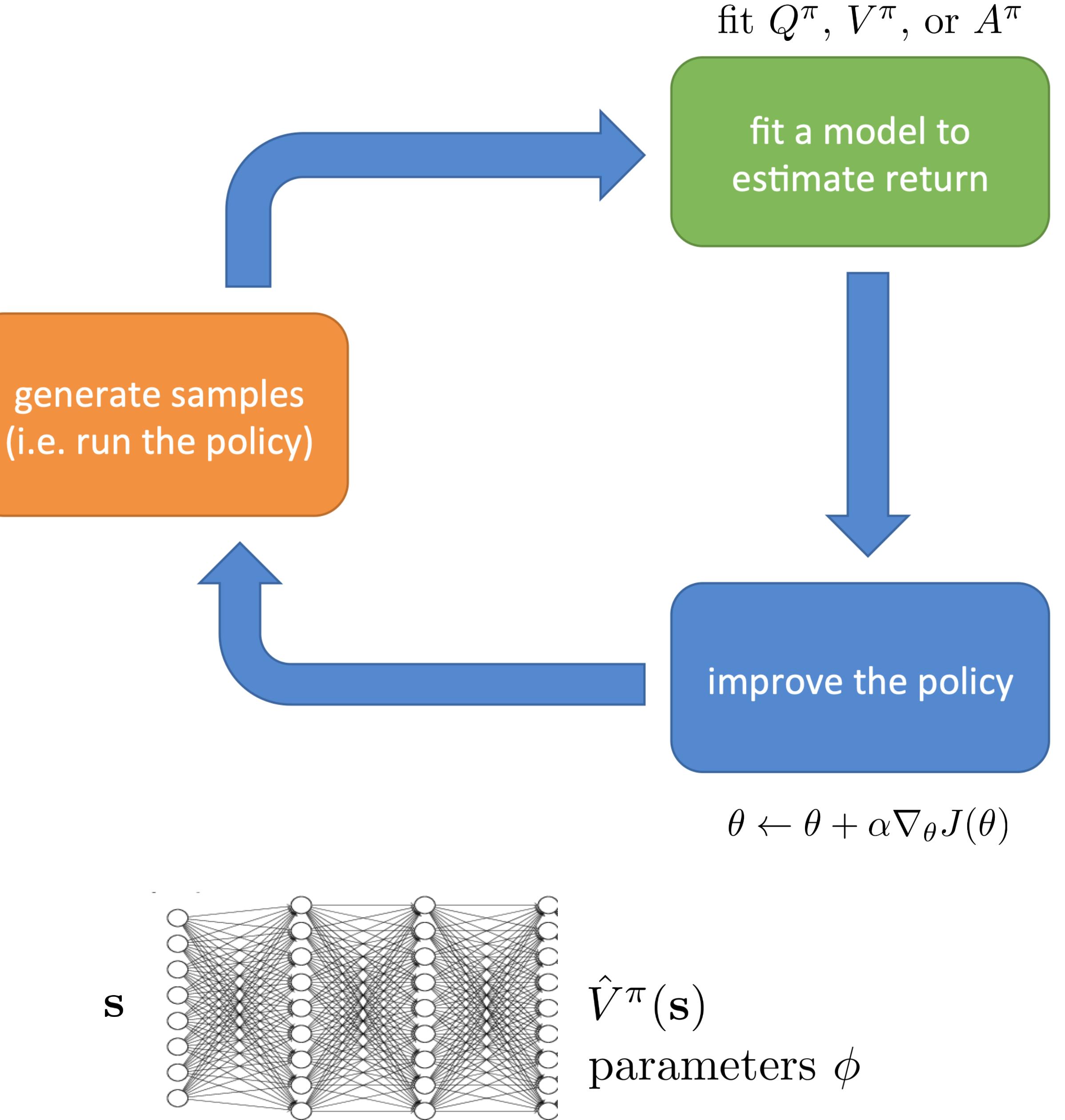
fit *what* to *what*?

$Q^\pi, V^\pi, A^\pi$ ?

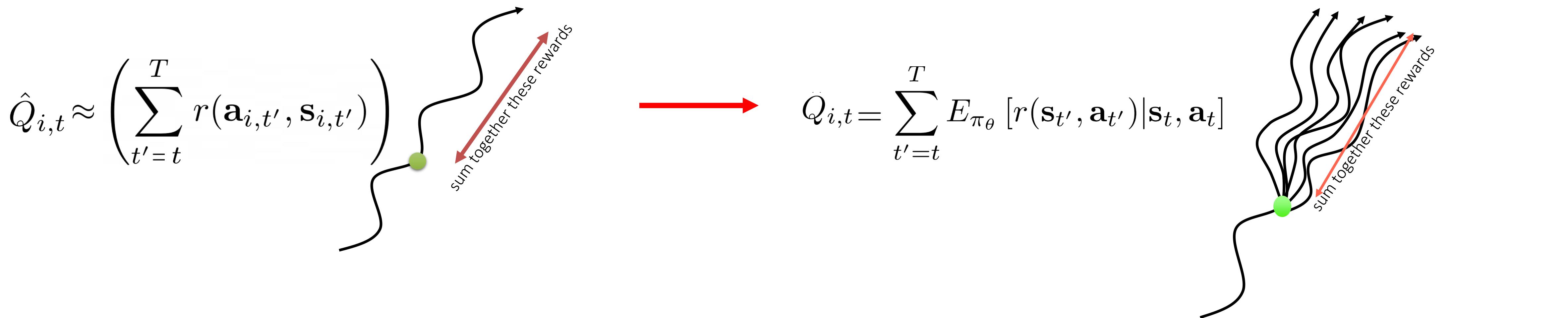
$$Q^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1})$$

$$A^\pi(\mathbf{s}_t, \mathbf{a}_t) \approx r(\mathbf{s}_t, \mathbf{a}_t) + V^\pi(\mathbf{s}_{t+1}) - V^\pi(\mathbf{s}_t)$$

let's just fit  $V^\pi(\mathbf{s})$ !

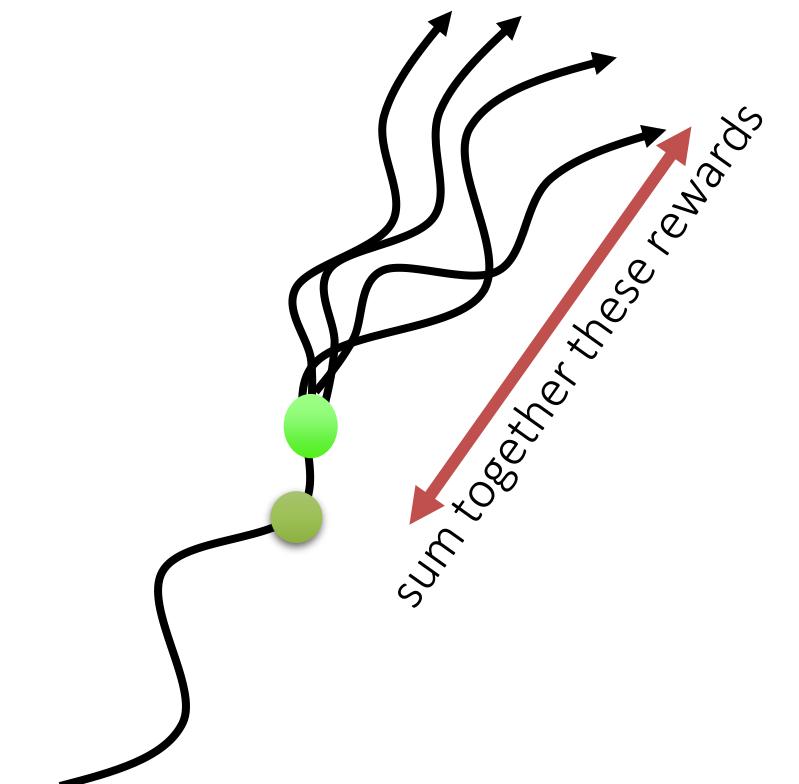


# Multi-Step Prediction



- How do you update your predictions about winning the game?
- What happens if you don't finish the game?
- Do you always wait till the end?

$$\hat{Q}_{i,t} \approx r(\mathbf{a}_{i,t}, \mathbf{s}_{i,t}) + V^\pi(\mathbf{s}_{t+1})$$



# How can we use all of this to fit a better estimator?

**Goal:** fit  $V^\pi$

ideal target:  $y_{i,t} = \sum_{t'=t}^T E_{\pi_\theta} [r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) | \mathbf{s}_{i,t}] \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + V^\pi(\mathbf{s}_{i,t+1}) \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$

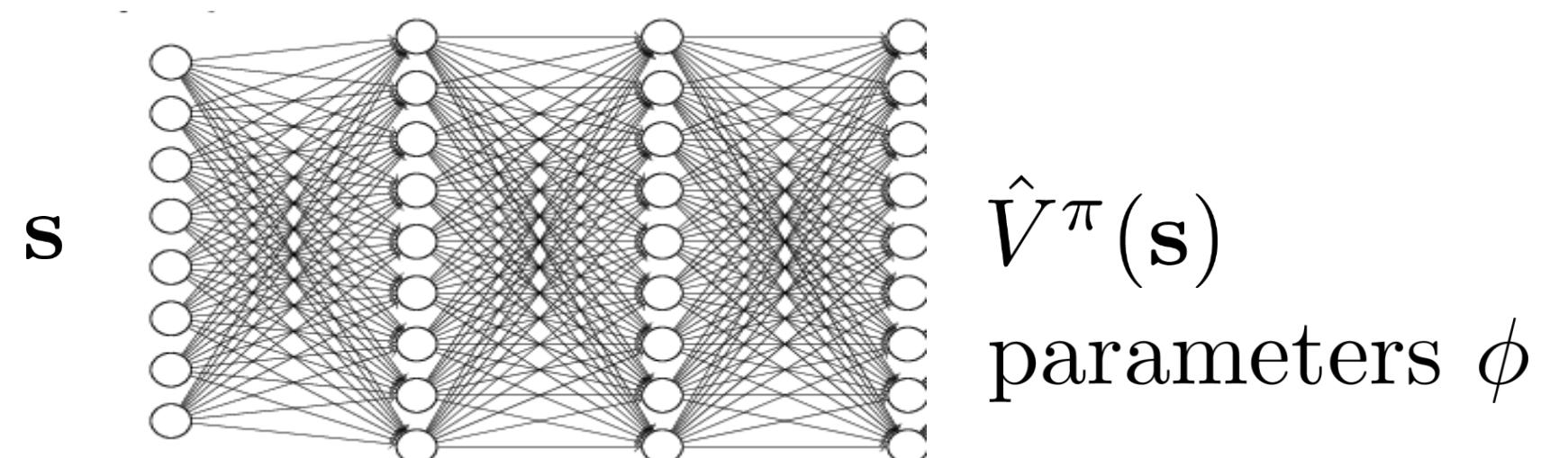
Monte Carlo target:  $y_{i,t} = \sum_{t'=t}^T r(\mathbf{s}_{i,t'}, \mathbf{a}_{i,t'})$

directly use previous fitted value function!

training data:  $\left\{ \left( \mathbf{s}_{i,t}, \underbrace{r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})}_{y_{i,t}} \right) \right\}$

supervised regression:  $\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$

sometimes referred to as a “bootstrapped” estimate



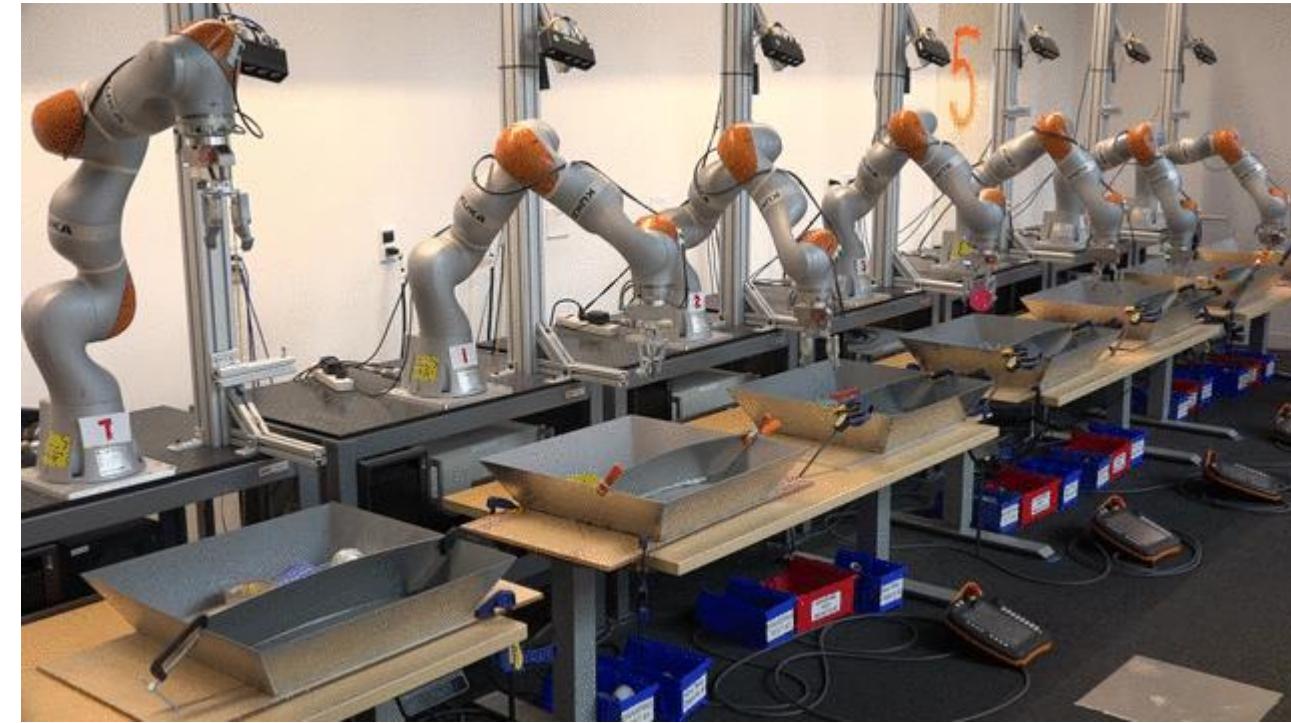
# Aside: discount factors

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

$$\mathcal{L}(\phi) = \frac{1}{2} \sum_i \left\| \hat{V}_\phi^\pi(\mathbf{s}_i) - y_i \right\|^2$$

what if  $T$  (episode length) is  $\infty$ ?

$\hat{V}_\phi^\pi$  can get infinitely large in many cases



episodic tasks



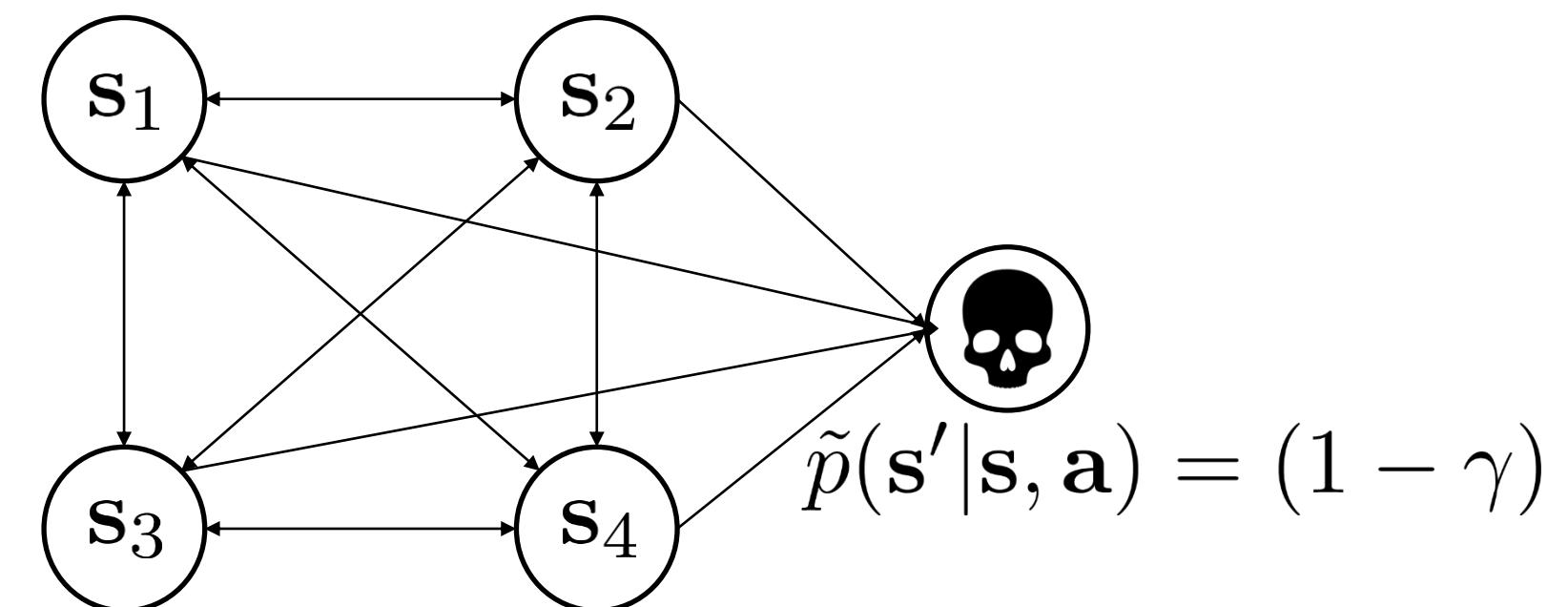
continuous/cyclical tasks

simple trick: better to get rewards sooner than later

$$y_{i,t} \approx r(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}_{i,t+1})$$

↑  
discount factor  $\gamma \in [0, 1]$  (0.99 works well)

$\gamma$  changes the MDP:



# N-step returns

$$\hat{A}_C^\pi(s_t, a_t) = r(s_t, a_t) + \gamma \hat{V}_\phi^\pi(s_{t+1}) - \hat{V}_\phi^\pi(s_t)$$

$$\hat{A}_{MC}^\pi(s_t, a_t) = \sum_{t'=t}^{\infty} \gamma^{t'-t} r(s_{t'}, a_{t'}) - \hat{V}_\phi^\pi(s_t)$$

- + lower variance
- higher bias if value is wrong (it always is)
- + no bias
- higher variance (because single-sample estimate)

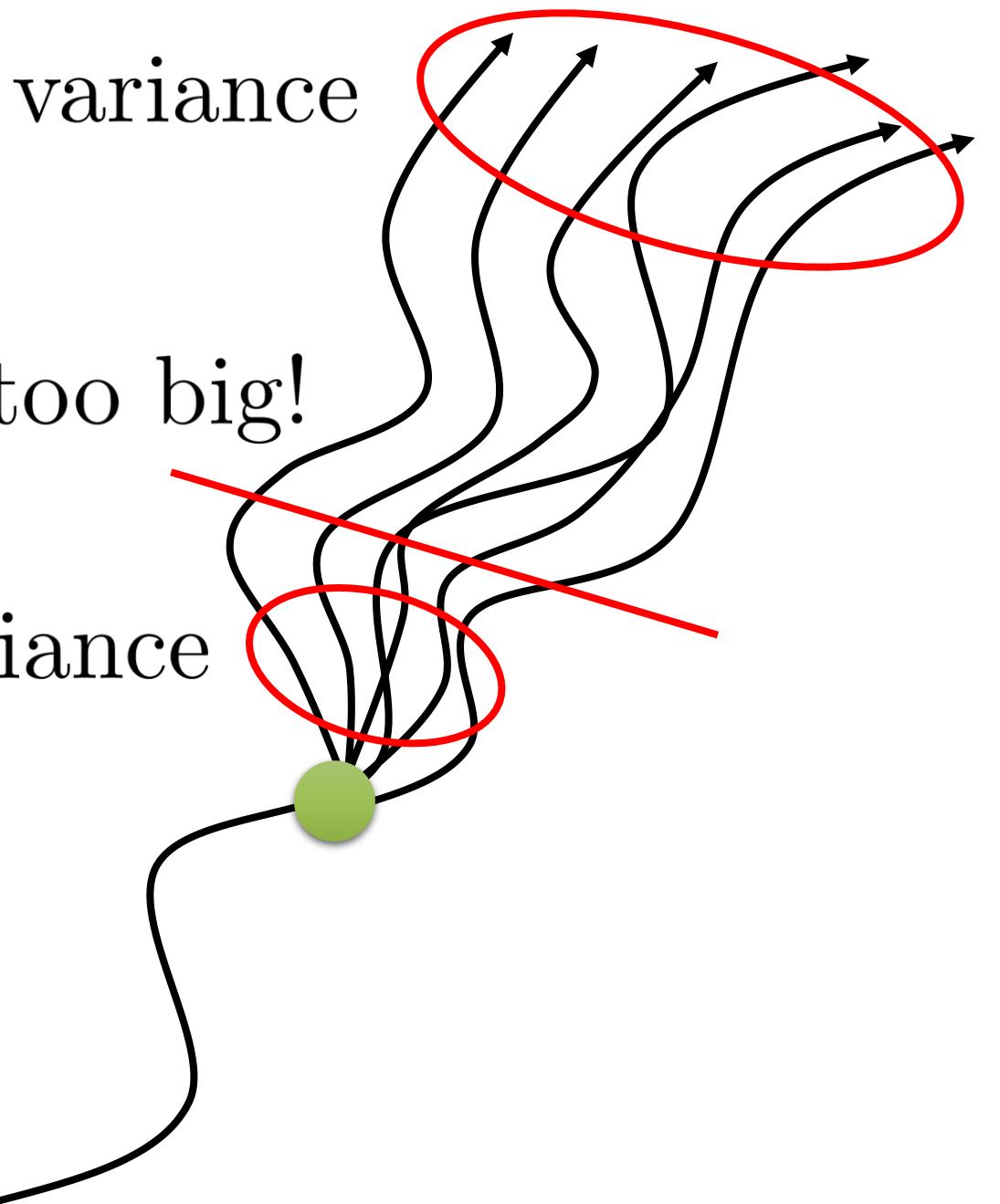
Can we combine these two, to control bias/variance tradeoff?

$$\hat{A}_n^\pi(s_t, a_t) = \sum_{t'=t}^{t+n} \gamma^{t'-t} r(s_{t'}, a_{t'}) - \hat{V}_\phi^\pi(s_t) + \gamma^n \hat{V}_\phi^\pi(s_{t+n})$$

choosing  $n > 1$  often works better!

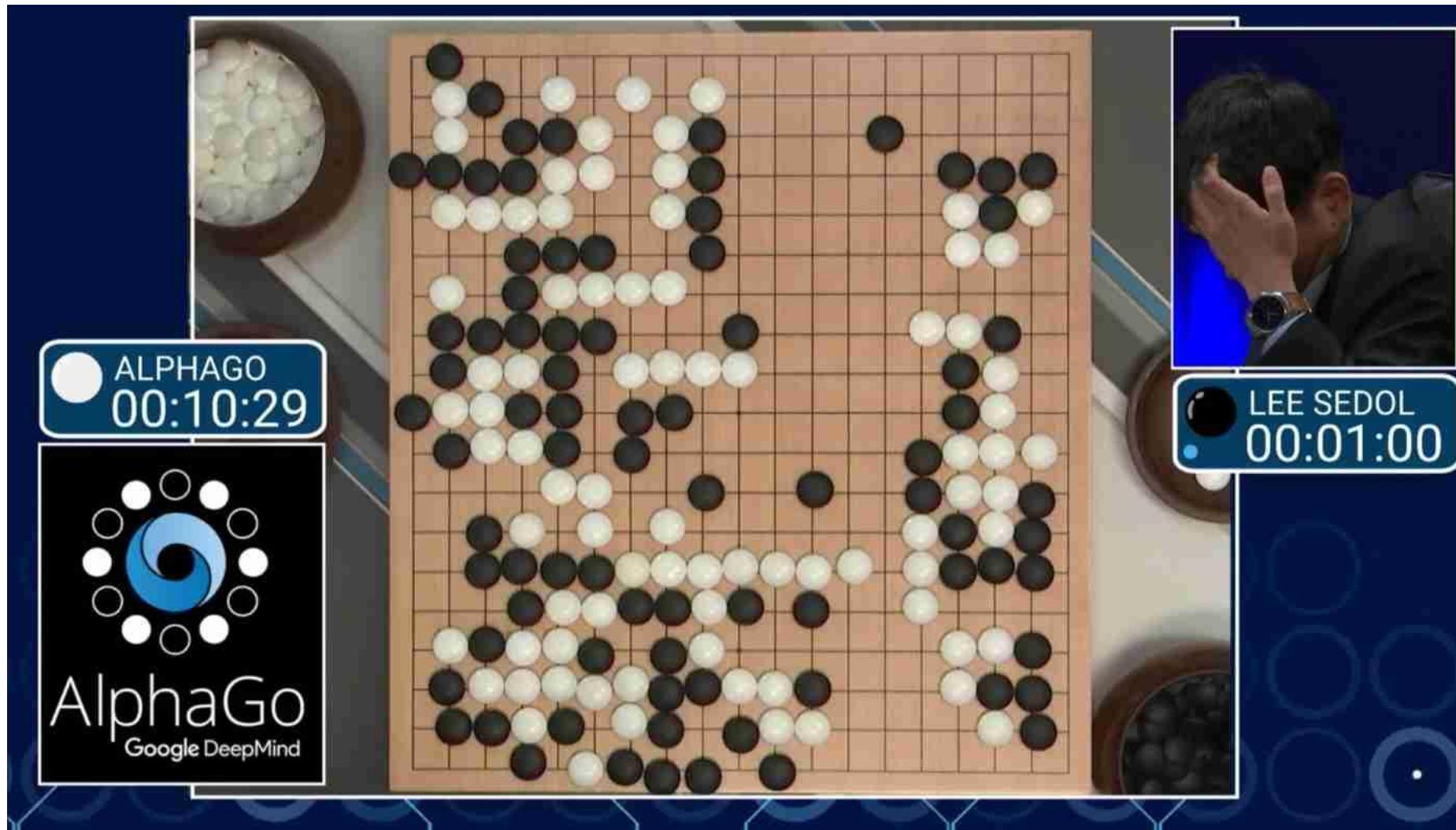
cut here before variance gets too big!

smaller variance



# Policy evaluation example

AlphaGo, Silver et al. 2016



reward: game outcome

value function  $\hat{V}_\phi^\pi(\mathbf{s}_t)$ :

expected outcome given board state

# The Plan

Policy gradients recap

Variance reduction continued

Policy gradients tricks

Actor-critic

Case studies: robotics & RLHF

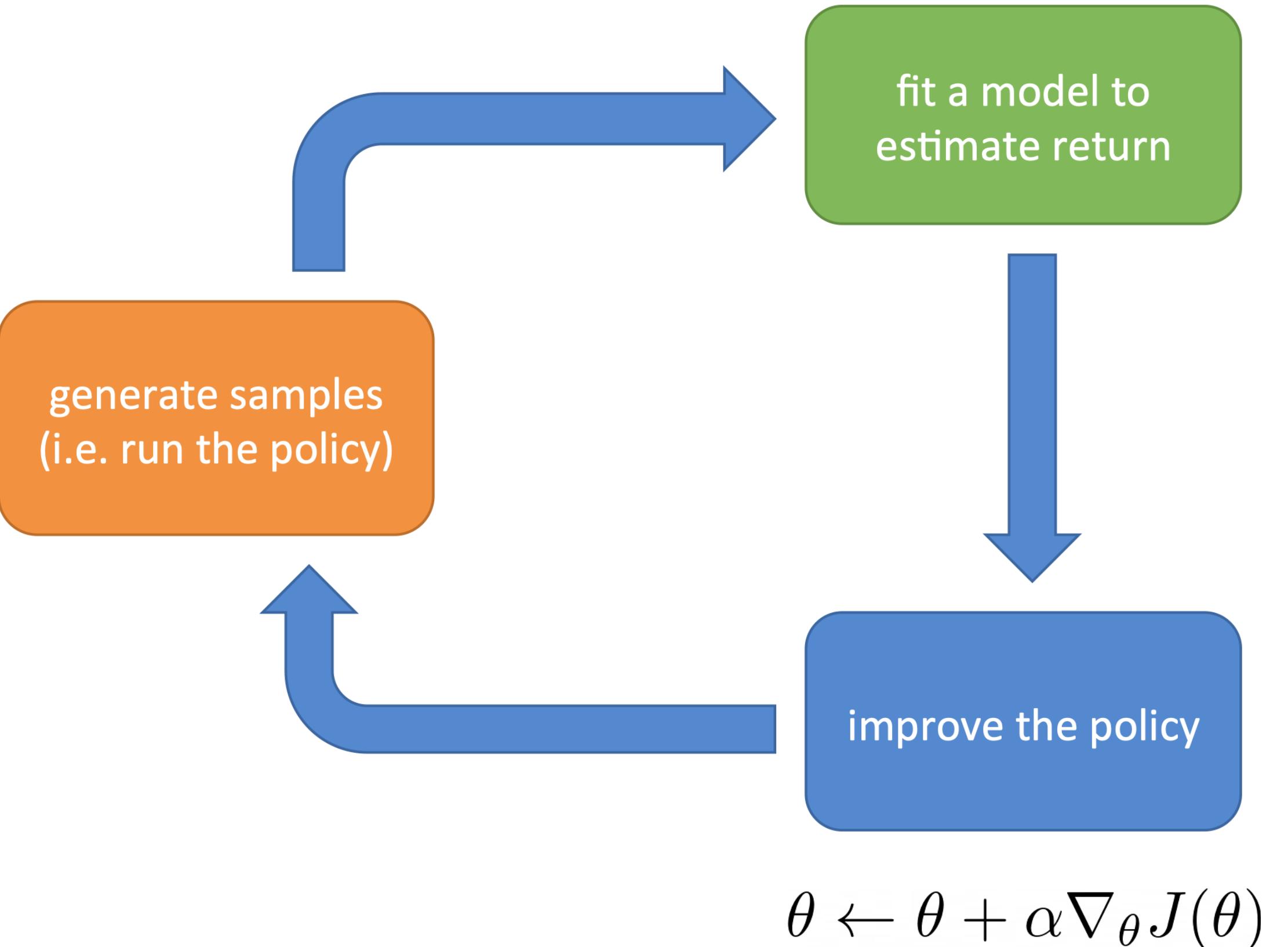
# Why is there so many RL algorithms?

Different tradeoffs:

- Continuous vs discrete actions
- Is it easier to learn the environment or the policy?
- Sample complexity

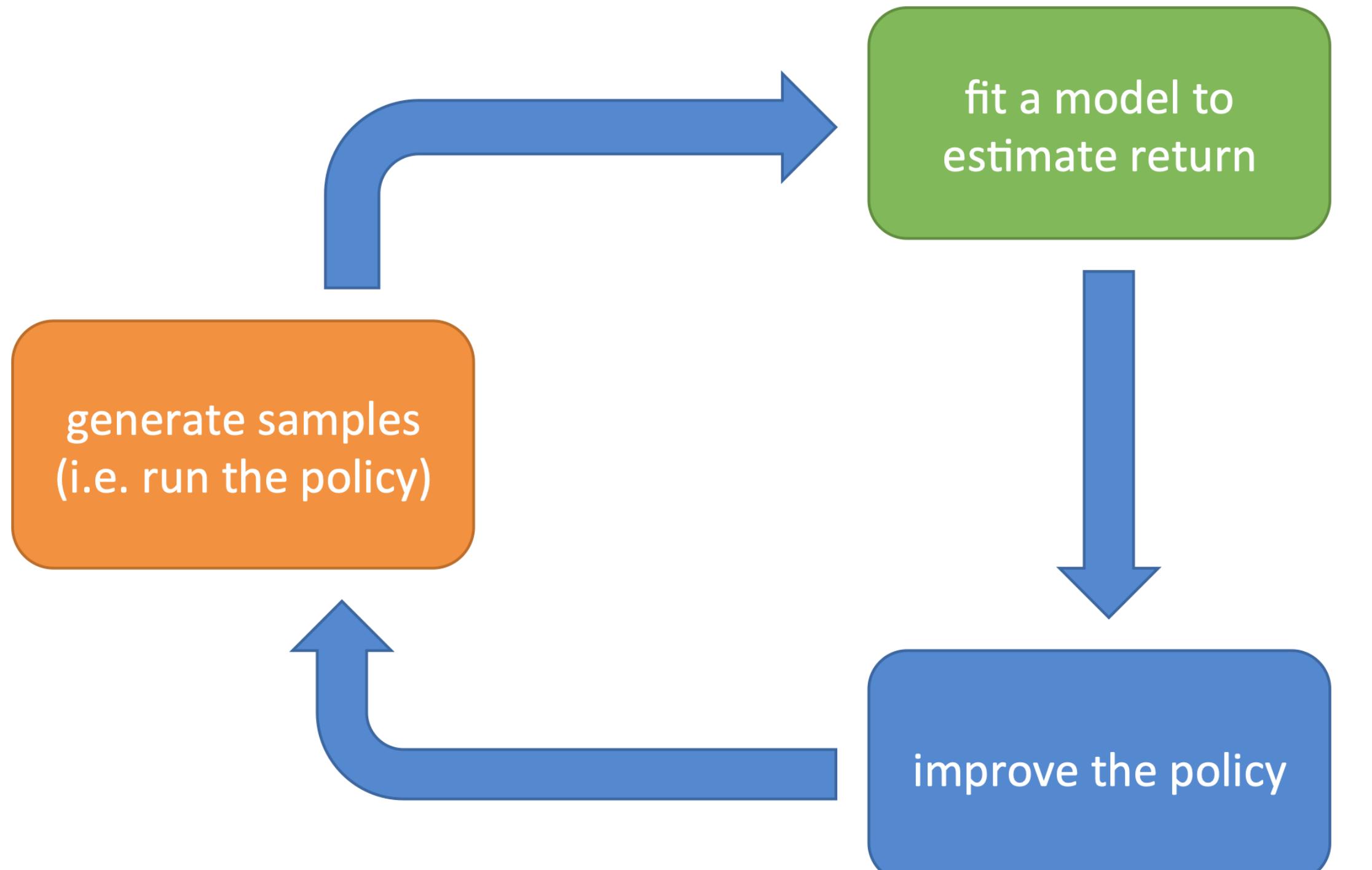
Off or on policy algorithms:

- **Off policy:** able to improve the policy without generating new samples from that policy
- **On policy:** each time the policy is changed, even a little bit, we need to generate new samples



# Can policy gradients reuse old data?

policy gradient:  $\nabla_{\theta} J(\theta) = \underline{E}_{\tau \sim \pi_{\theta}(\tau)} \left[ \left( \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right) \left( \sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$



$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

# Importance sampling

$$\theta^* = \arg \max_{\theta} E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

Importance sampling

$$E_{x \sim p(x)} [f(x)] = \int p(x) f(x) dx$$

$$\int p(x) \frac{q(x)}{q(x)} f(x) dx = \int q(x) \frac{p(x)}{q(x)} f(x) dx$$

$$E_{x \sim q(x)} \left[ \frac{p(x)}{q(x)} f(x) \right]$$

what if we don't have samples from  $\pi_{\theta}(\tau)$ ?

we have samples from  $\bar{\pi}(\tau)$

$$J(\theta) = E_{\tau \sim \bar{\pi}(\tau)} \left[ \frac{\pi_{\theta}(\tau)}{\bar{\pi}(\tau)} r(\tau) \right]$$

# Importance sampling in policy gradient

policy gradient:  $\nabla_{\theta} J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \left( \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \right) \left( \sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$

$$\theta^* = \arg \max_{\theta} E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$J(\theta) = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \sum_t r(\mathbf{s}_t, \mathbf{a}_t) \right]$$

$$J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} r(\tau) \right]$$

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \frac{\nabla_{\theta'} \pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} r(\tau) \right] = E_{\tau \sim \pi_{\theta}(\tau)} \left[ \frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau) \right]$$

$$= E_{\tau \sim \pi_{\theta}(\tau)} \left[ \left( \frac{\prod_{t=1}^T \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} \right) \left( \sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \right) \left( \sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

a convenient identity

$$\underline{\pi_{\theta}(\tau) \nabla_{\theta} \log \pi_{\theta}(\tau)} = \pi_{\theta}(\tau) \frac{\nabla_{\theta} \pi_{\theta}(\tau)}{\pi_{\theta}(\tau)} = \underline{\nabla_{\theta} \pi_{\theta}(\tau)}$$

$$\frac{\pi_{\theta'}(\tau)}{\pi_{\theta}(\tau)} = \frac{p(\mathbf{s}_1) \prod_{t=1}^T \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}, \mathbf{a})}{p(\mathbf{s}_1) \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}, \mathbf{a})}$$

# Problem with importance sampling in (policy gradient)

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim \pi_\theta(\tau)} \left[ \left( \frac{\prod_{t=1}^T \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \pi_\theta(\mathbf{a}_t | \mathbf{s}_t)} \right) \left( \sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \right) \left( \sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right]$$

Let's try it in code!

Importance sampling

$$E_{x \sim p(x)}[f(x)] = \int p(x)f(x)dx$$

$$\int p(x) \frac{q(x)}{q(x)} f(x)dx = \int q(x) \frac{p(x)}{q(x)} f(x)dx$$

$$E_{x \sim q(x)} \left[ \frac{p(x)}{q(x)} f(x) \right]$$

# Solution?

$$\theta \leftarrow \theta + \alpha \nabla_{\theta} J(\theta)$$

Stay close to the previous policy!

$$\theta' \leftarrow \arg \max(\theta' - \theta) \nabla_{\theta} J(\theta) \quad s.t. ||\theta' - \theta||^2 \leq \epsilon$$

**Policy** not parameters

$$\theta' \leftarrow \arg \max(\theta' - \theta) \nabla_{\theta} J(\theta) \quad s.t. D_{KL}(\pi_{\theta'}, \pi_{\theta}) \leq \epsilon$$

# Trust region policy optimization (TRPO)

Apply all the tricks:

- Use advantage function to reduce the variance
- Use importance sampling to take multiple gradient steps
- Constrain the optimization objective in the policy space

---

## Trust Region Policy Optimization

---

**John Schulman**  
Sergey Levine  
**Philipp Moritz**  
Michael Jordan  
**Pieter Abbeel**

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Our optimization problem in Equation (13) is exactly equivalent to the following one, written in terms of expectations:

$$\begin{aligned} & \underset{\theta}{\text{maximize}} \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}, a \sim q} \left[ \frac{\pi_{\theta}(a|s)}{q(a|s)} Q_{\theta_{\text{old}}}(s, a) \right] \\ & \text{subject to } \mathbb{E}_{s \sim \rho_{\theta_{\text{old}}}} [D_{\text{KL}}(\pi_{\theta_{\text{old}}}(\cdot|s) \| \pi_{\theta}(\cdot|s))] \leq \delta. \end{aligned} \quad (14)$$

# Proximal policy optimization (PPO)

Apply all the tricks:

- Use advantage function to reduce the variance
- Use importance sampling to take multiple gradient steps
- Constrain the optimization objective in the policy space

## Proximal Policy Optimization Algorithms

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, Oleg Klimov  
OpenAI

{joschu, filip, prafulla, alec, oleg}@openai.com

Let  $r_t(\theta)$  denote the probability ratio  $r_t(\theta) = \frac{\pi_\theta(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)}$ , so  $r(\theta_{\text{old}}) = 1$ . TRPO maximizes a “surrogate” objective

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t \left[ \frac{\pi_\theta(a_t | s_t)}{\pi_{\theta_{\text{old}}}(a_t | s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t [r_t(\theta) \hat{A}_t]. \quad (6)$$

The superscript  $CPI$  refers to conservative policy iteration [KL02], where this objective was proposed. Without a constraint, maximization of  $L^{CPI}$  would lead to an excessively large policy update; hence, we now consider how to modify the objective, to penalize changes to the policy that move  $r_t(\theta)$  away from 1.

The main objective we propose is the following:

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[ \min(r_t(\theta) \hat{A}_t, \text{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_t) \right] \quad (7)$$

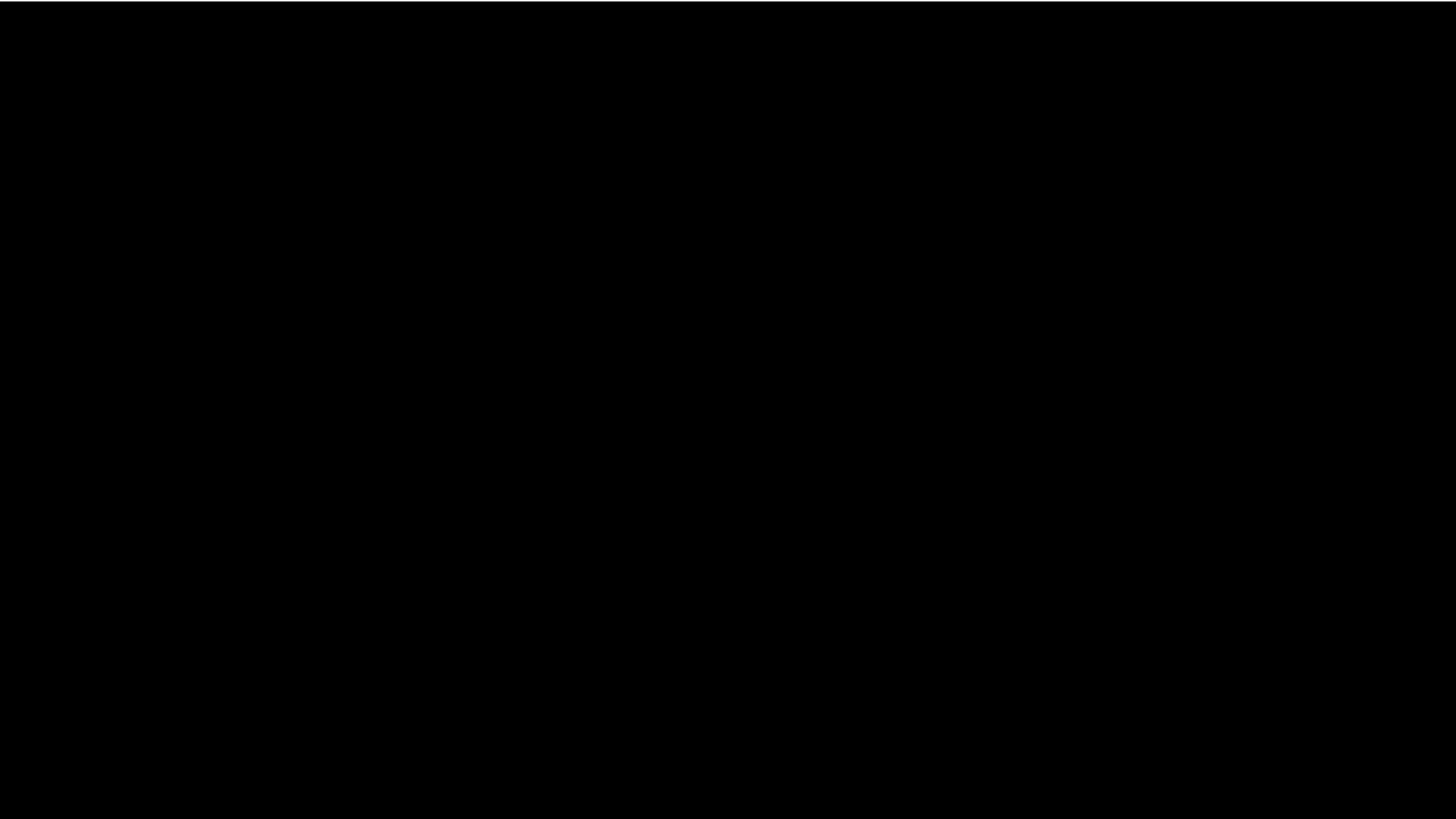
# Examples

TRPO applied to continuous control



Trust Region Policy Optimization

PPO applied to Dota



# The Plan

Policy gradients recap

Variance reduction continued

Policy gradients tricks

Actor-critic

Case studies: robotics & RLHF

REINFORCE algorithm:

- 
1. sample  $\{\tau^i\}$  from  $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$  (run the policy)
  2.  $\nabla_\theta J(\theta) \approx \sum_i (\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)) (\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i))$
  3.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

fit a model to  
estimate return

online actor-critic algorithm:

- 
1. take action  $\mathbf{a} \sim \pi_\theta(\mathbf{a}|\mathbf{s})$ , get  $(\mathbf{s}, \mathbf{a}, \mathbf{s}', r)$
  2. update  $\hat{V}_\phi^\pi$  using target  $r + \gamma \hat{V}_\phi^\pi(\mathbf{s}')$
  3. evaluate  $\hat{A}^\pi(\mathbf{s}, \mathbf{a}) = r(\mathbf{s}, \mathbf{a}) + \gamma \hat{V}_\phi^\pi(\mathbf{s}') - \hat{V}_\phi^\pi(\mathbf{s})$
  4.  $\nabla_\theta J(\theta) \approx \nabla_\theta \log \pi_\theta(\mathbf{a}|\mathbf{s}) \hat{A}^\pi(\mathbf{s}, \mathbf{a})$
  5.  $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

generate samples  
(i.e. run the policy)

improve the policy

Can we make it more off-policy friendly?

# The Plan

Policy gradients recap

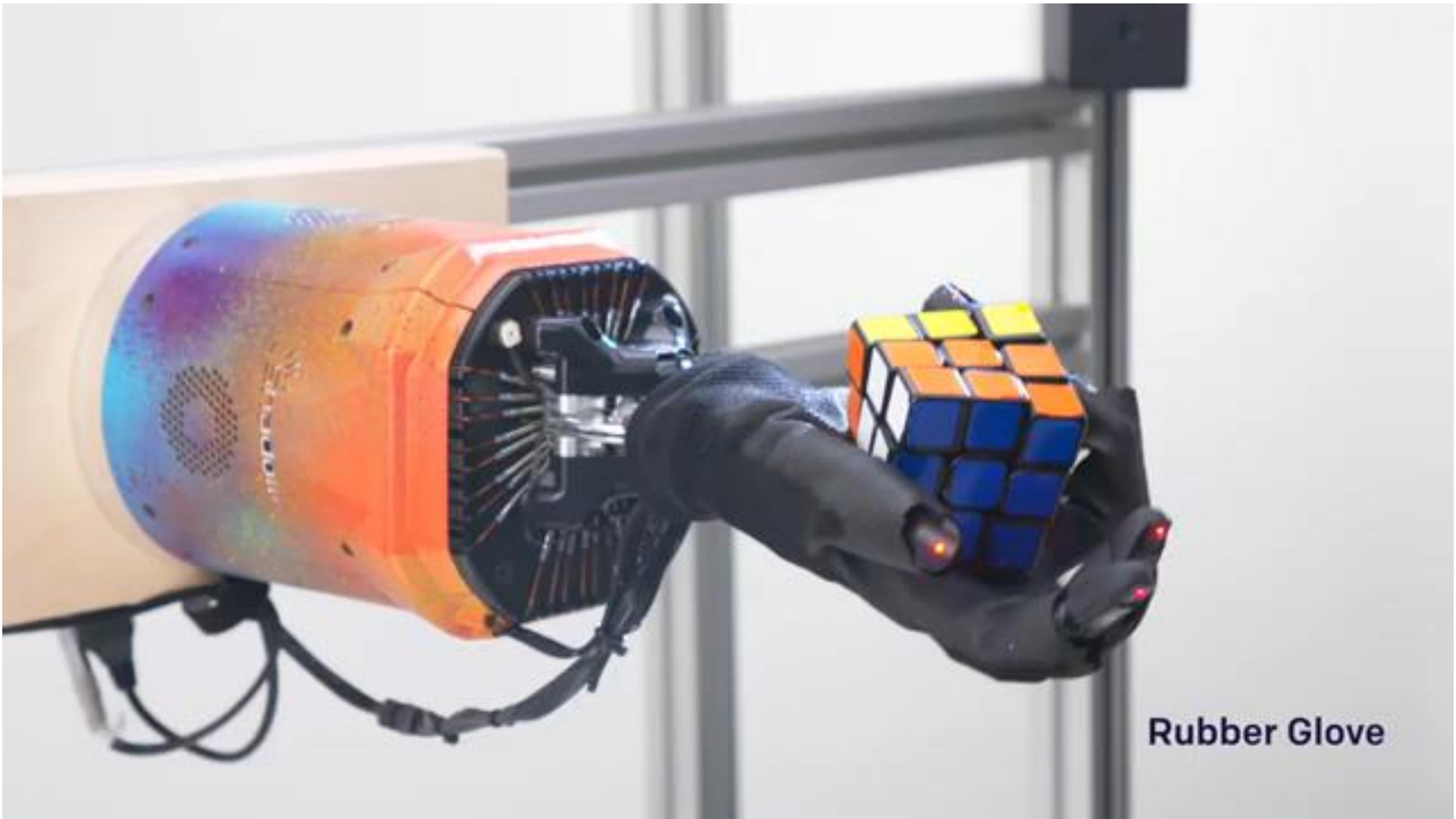
Variance reduction continued

Policy gradients tricks

Actor-critic

Case studies: robotics & RLHF

# Case study: PPO applied to robotics



# Case study: PPO applied to robotics

Solving the Rubik's Cube with a robot hand is still not easy. Our method currently solves the Rubik's Cube 20% of the time when applying a maximally difficult scramble that requires 26 face rotations. For simpler scrambles that require 15 rotations to undo, the success rate is 60%. When the Rubik's Cube is dropped or a timeout is reached, we consider the attempt failed. However, our network is capable of solving the Rubik's Cube from any initial condition. So if the cube is dropped, it is possible to put it back into the hand and continue solving.

# Case study: PPO applied to robotics

We train neural networks to solve the Rubik's Cube in simulation using reinforcement learning and Kociemba's algorithm for picking the solution steps.<sup>A</sup> Domain randomization enables networks trained solely in simulation to transfer to a real robot.



**Simulator physics.** We randomize simulator physics parameters such as geometry, friction, gravity, etc. See Section B.1 for details of their ADR parameterization.

**Custom physics.** We model additional physical robot effects that are not modelled by the simulator, for example, action latency or motor backlash. See [77, Appendix C.2] for implementation details of these models. We randomize the parameters in these models in a similar way to simulator physics randomizations.

**Adversarial.** We use an adversarial approach similar to [82, 83] to capture any remaining unmodeled physical effects in the target domain. However, we use random networks instead of a trained adversary. See Section B.3 for details on implementation and ADR parameterization.

**Observation.** We add Gaussian noise to policy observations to better approximate observation conditions in reality. We apply both correlated noise, which is sampled once at the start of an episode and uncorrelated noise, which is sampled at each time step. We randomize the parameters of the added noise. See Section B.4 for details of their ADR parameterization.

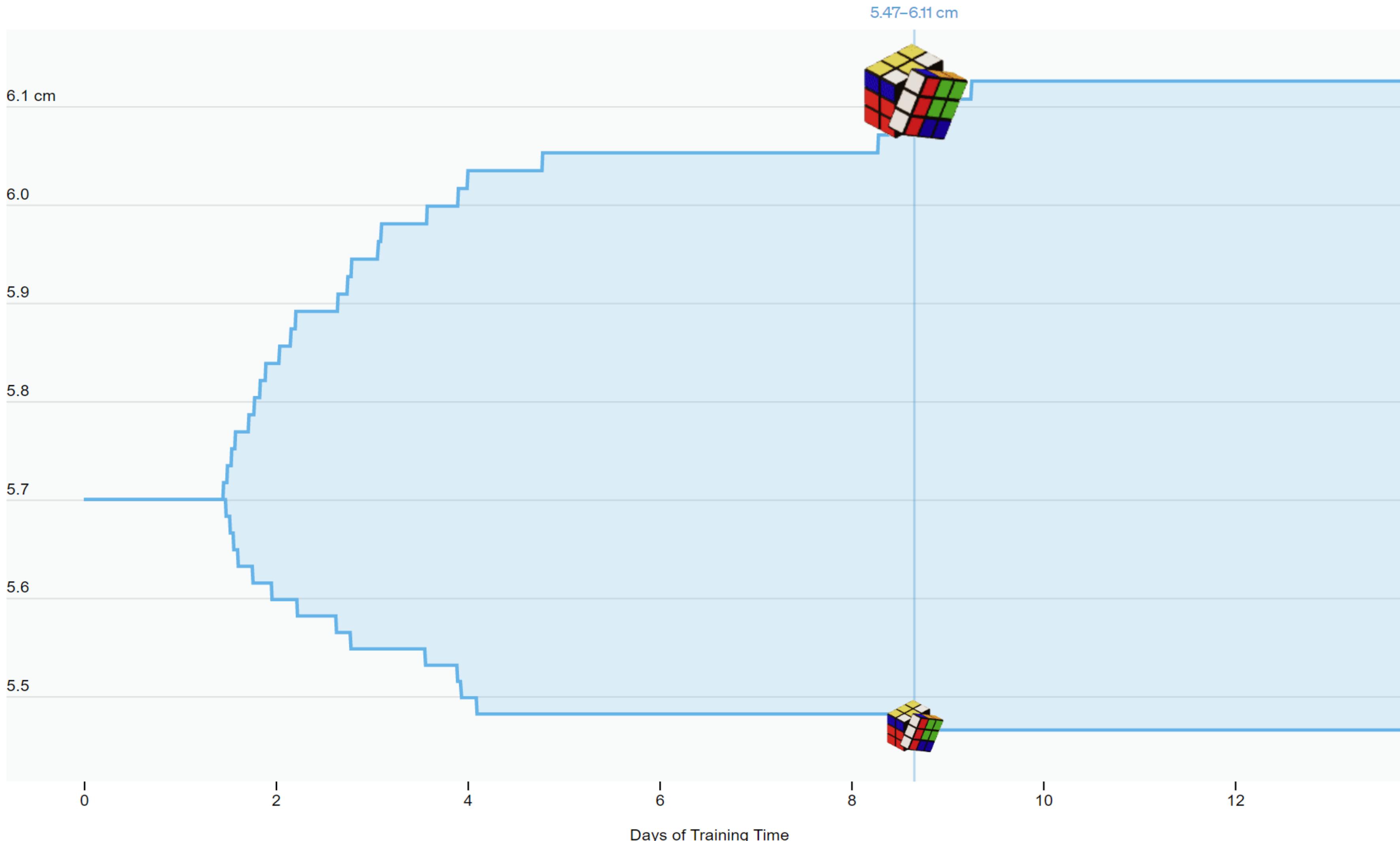
**Vision.** We randomize several aspects in ORRB [16] to control the rendered scene, including lighting conditions, camera positions and angles, materials and appearances of all the objects, the texture of the background, and the post-processing effects on the rendered images. See Section B.5 for details.

# Case study: PPO applied to robotics

ADR starts with a single, nonrandomized environment, wherein a neural network learns to solve Rubik's Cube. As the neural network gets better at the task and reaches a performance threshold, the amount of domain randomization is increased automatically. This makes the task harder, since the neural network must now learn to generalize to more randomized environments. The network keeps learning until it again exceeds the performance threshold, when more randomization kicks in, and the process is repeated.

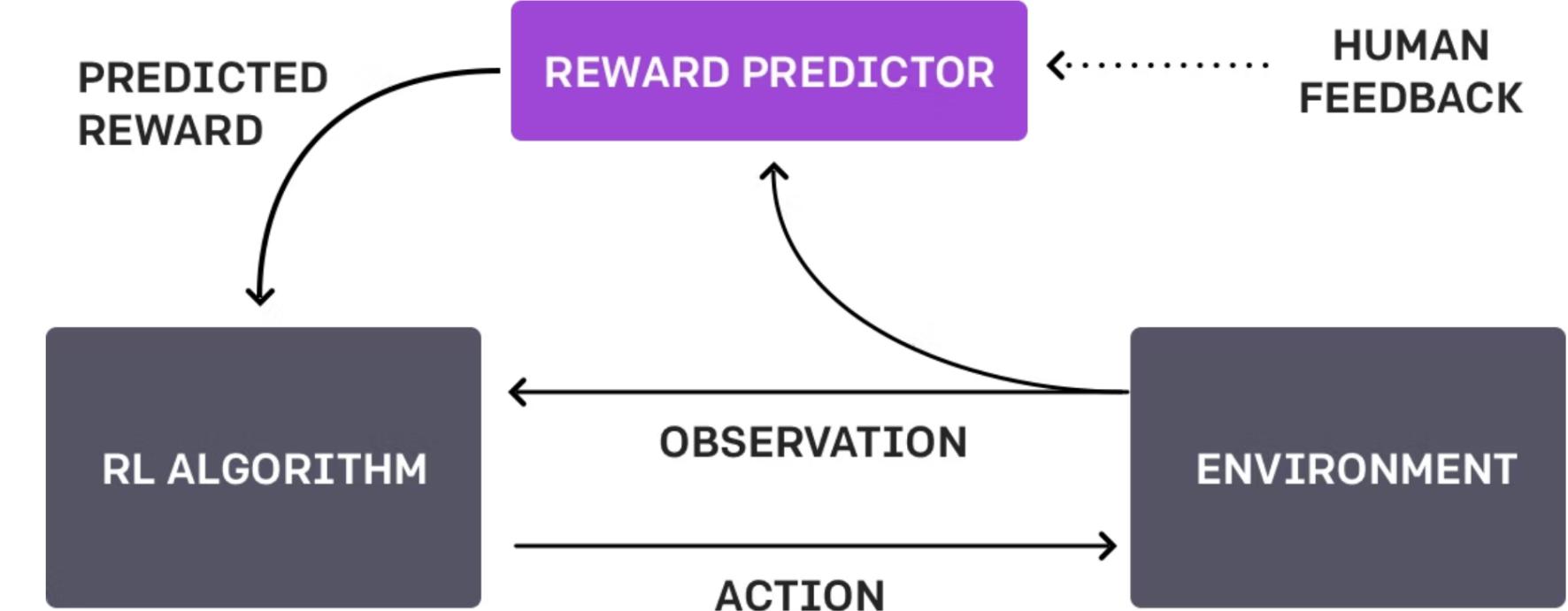
# Case study: PPO applied to robotics

**ADR applied to the size of the Rubik's Cube**



# Case study: PPO applied to LLMs (speculations)

RL from human feedback (RLHF)



# Reward model

- Imagine a reward function:  $R(s; p) \in \mathbb{R}$  for any output  $s$  to prompt  $p$
- The reward is higher when humans prefer the output

SAN FRANCISCO,  
California (CNN) --  
A magnitude 4.2  
earthquake shook the  
San Francisco  
...  
overturn unstable  
objects.

An earthquake hit  
San Francisco.  
There was minor  
property damage,  
but no injuries.

$s_1$

$$R(s_1; p) = 0.8$$

The Bay Area has  
good weather but is  
prone to  
earthquakes and  
wildfires.

$s_2$

$$R(s_2; p) = 1.2$$

# Regularizing pre-trained model

- Challenge: how do we ensure that  $R(s; p)$  prefers natural language generations?
- Since  $R(s; p)$  is trained on natural language inputs, it might fail to assign low scores to unnatural  $s$ .
- Solution: add regularization term to  $R(s; p)$  that penalizes outputs that deviate from natural language.

$$\hat{R}(s; p) := R(s; p) - \beta \log \left( \frac{p^{RL}(s)}{p^{PT}(s)} \right)$$

pay a price when  
 $p^{RL}(s) < p^{PT}(s)$

- This is a penalty which prevents us from diverging too far from the pretrained model.

# RLHF + PPO (speculations)

Prompt *Explain the moon landing to a 6 year old in a few sentences.*

Completion GPT-3

Explain the theory of gravity to a 6 year old.

Explain the theory of relativity to a 6 year old in a few sentences.

Explain the big bang theory to a 6 year old.

Explain evolution to a 6 year old.

InstructGPT

People went to the moon, and they took pictures of what they saw, and sent them back to the earth so we could all see them.

GPT-3 models aren't trained to follow user instructions. Our InstructGPT models (highlighted) generate much more helpful outputs in response to user instructions.

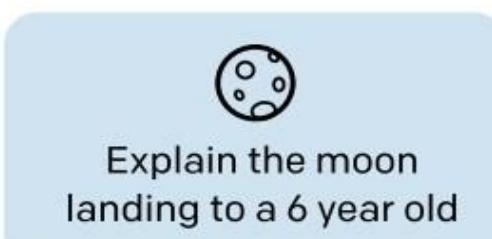
# RLHF + PPO (speculations)

Step 1

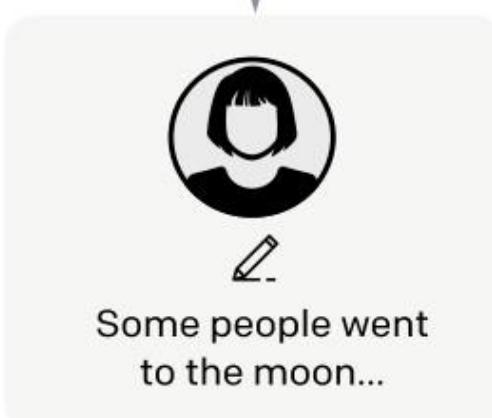
**Collect demonstration data,  
and train a supervised policy.**

30k tasks!

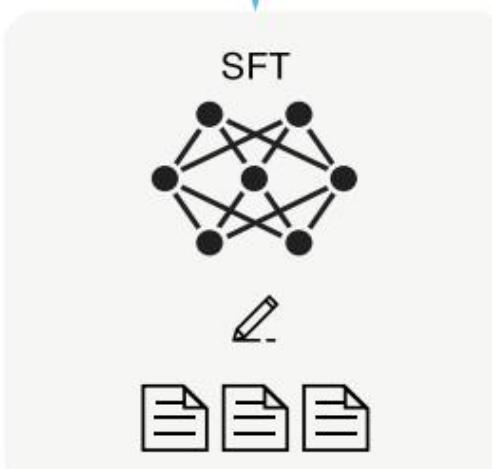
A prompt is sampled from our prompt dataset.



A labeler demonstrates the desired output behavior.



This data is used to fine-tune GPT-3 with supervised learning.



Step 2

**Collect comparison data,  
and train a reward model.**

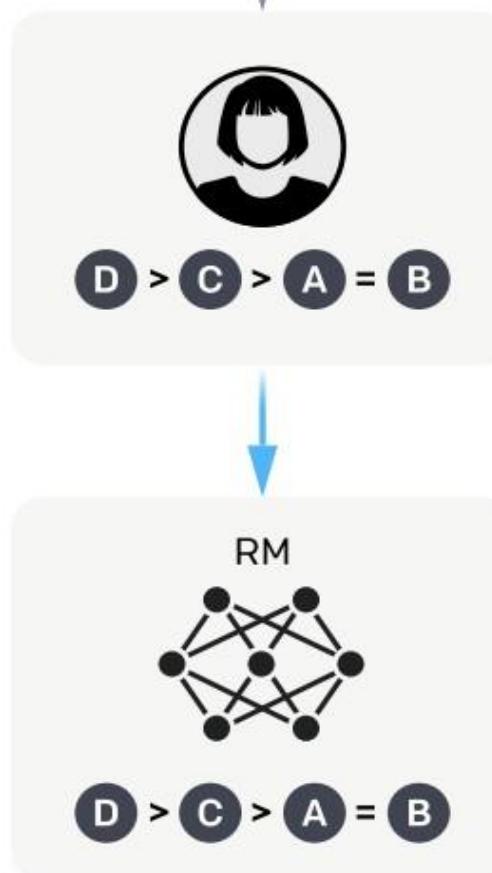
A prompt and several model outputs are sampled.



A labeler ranks the outputs from best to worst.



This data is used to train our reward model.



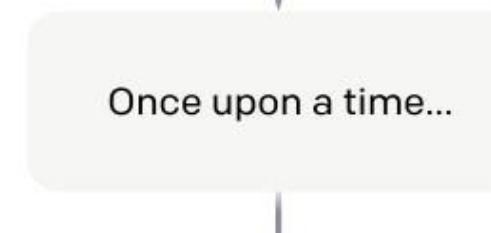
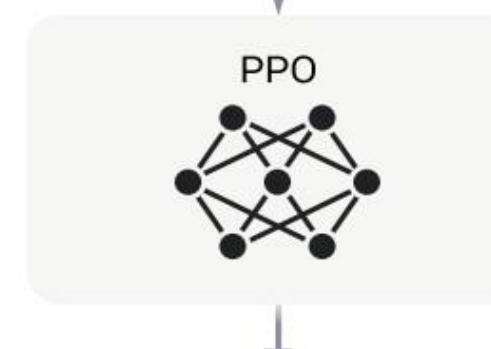
Step 3

**Optimize a policy against the reward model using reinforcement learning.**

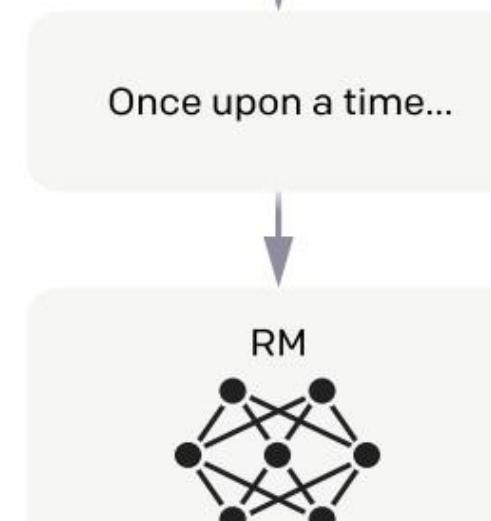
A new prompt is sampled from the dataset.



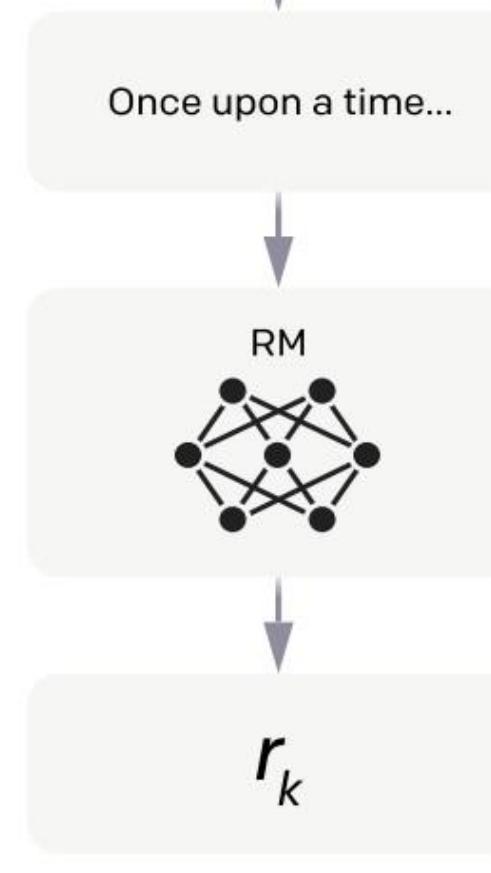
The policy generates an output.



The reward model calculates a reward for the output.



The reward is used to update the policy using PPO.



# Case study: PPO applied to LLMs (speculations)

- What would you try next?
- Is HF more important than RL?
- What do you think the challenges are?

PROMPT *Write a short poem about a wise frog.*

COMPLETION GPT-3

Write a short story in which a character has two different names.

Write a short story in which you try to get something back that you have lost.

Write a short story in which a character has a bad dream.

InstructGPT

The frog is a symbol of wisdom  
He knows all the secrets of the world  
He is a master of disguise  
And a great teacher of life  
He is a symbol of transformation  
And the bringer of change  
He is the frog who has seen it all  
And knows the meaning of it all

# Recap

## Key learning goals:

- Practical policy gradient implementation tricks & case studies
- Understanding a generic actor-critic method

## PG implementation:

- Variance reduction
- Importance sampling and trust region
- RLHF and robotics applications

## AC method:

- Uses the advantage fn

# Next

Do we even need a policy?

Can we be even more off-policy?

Q learning and its applications