CHEME 5820: Machine Learning for Engineers **Spring 2025**

Lecture 2a: Eigendecomposition of Data and Systems

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Disclaimer: *These notes have not been subjected to the usual scrutiny reserved for formal publications.*

1 Introduction

The eigendecomposition of a matrix is a fundamental concept in linear algebra. In this lecture, we will discuss the eigendecomposition of a matrix, and how it can be used to analyze data and systems in the context of unsupervised machine learning.

What is eigendecomposition?

Eigenvalue-eigenvector problems involve finding a set of scalar values $\{\lambda_1, \dots, \lambda_m\}$ called https://mathworld.wolfram.com/Eig
and a set of linearly independent vectors $\{\mathbf{v}_1, \dots, \mathbf{v}_m\}$ called https://mathworld.wolfram.com/Eigenvector.htmleigenvectors such that:

$$\mathbf{A}\mathbf{v}_j = \lambda_j \mathbf{v}_j \qquad j = 1, 2, \dots, m$$

where $\mathbf{A} \in \mathbb{R}^{m \times m}$, $\mathbf{v} \in \mathbb{R}^{m \times 1}$, and $\lambda \in \mathbb{R}$. Eigenvalues and eigenvectors are widely used in many areas of mathematics, engineering, and physics:

- **Solution of Linear Differential Equations**: Eigenvectors form a set of linearly independent solutions, while eigenvalues determine the stability of these solutions.
- **Structural Analysis**: Eigenvalues and eigenvectors describe the structural properties of a matrix or a graph. For example, a structure's natural frequencies and vibration modes, e.g., of a building or a bridge.
- **Singular Value Decomposition (SVD)**: SVD is commonly used in data analysis, computer vision, image processing, etc, to find the most important features of the dataset.

2 Computing the Eigendecomposition of a Matrix

Let A be a square matrix of size $n \times n$. The eigendecomposition of A is given by:

$$A = Q\Lambda Q^{-1} \tag{1}$$

where Q is a matrix whose columns are the eigenvectors of A, and Λ is a diagonal matrix whose diagonal elements are the eigenvalues of A.

References