# Data Analytics
## CS301
## Modeling: Formal Basics

Week 4: 27th July

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# Modeling Basics

- What are models?
  - Data does not provide much insight unless something can be learned from it.
  - The ability to use data to extract meaning and extra value (the learning)
- Let’s talk about...
  - How to extract some meaning from your data
  - How to make predictions using your data as training

# Modeling Basics

- Topics include
  - Modeling
  - Linear regression
  - Multivariate regression
  - Interaction terms

# Types of Models (i)

- **Support Vector Machines**
  - Supervised learning models with associated learning algorithms that analyze data used for classification and regression analysis.
- **Generalized Linear Models**
  - Flexible generalization of ordinary linear regression that allows for response variables that have error distribution models other than a normal distribution
- **Generalized additive models**
  - Generalized linear model in which the linear predictor depends linearly on unknown smooth functions of some predictor variables, and interest focuses on inference about these smooth functions

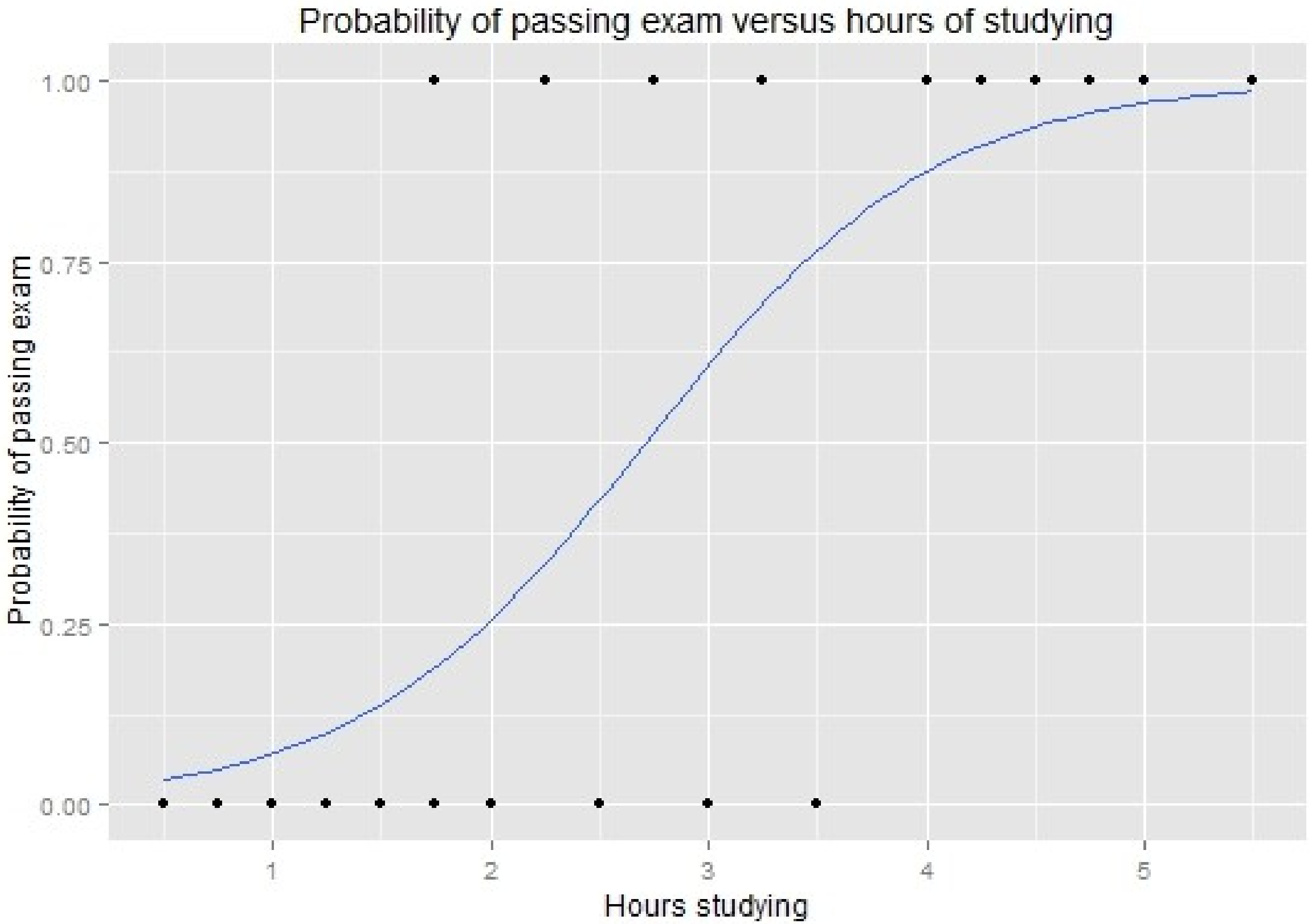
# Types of Models (ii)

- **Linear Regression**
  - Linear approach for modeling the relationship between a scalar dependent variable y and one or more explanatory variables (or independent variables) denoted X
  - *(we have begun this study)*
- **LOESS Regression**
  - Combining much of the simplicity of linear least squares regression, but building with the flexibility of nonlinear regression.

# Types of Models (iii)

**Logistic Regression**

- Models where the dependent variable is categorical (i.e., 0's or 1's as factors)



# Let’s Begin Our Discussion...

- Working with models begins with a basic question to answer from the analysis of data.
- We will walk through each of these with a formal discussion

Q1: Do taller people make more money?

Q2: Do warmer US states have more crime?

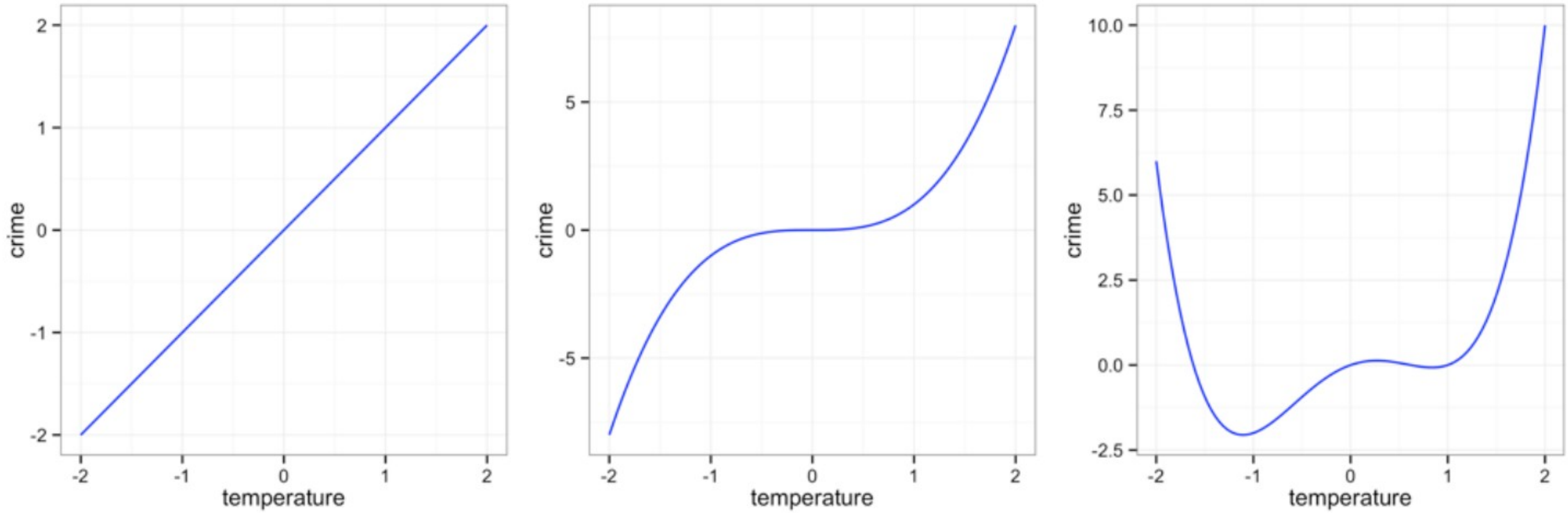
# How Do We Answer Questions?

- **Modeling**: We employ a computational framework which we use data to build (for training).
- Then, we play with the framework to see what happens when we change part(s) of the data to see what happens.

***What if…***

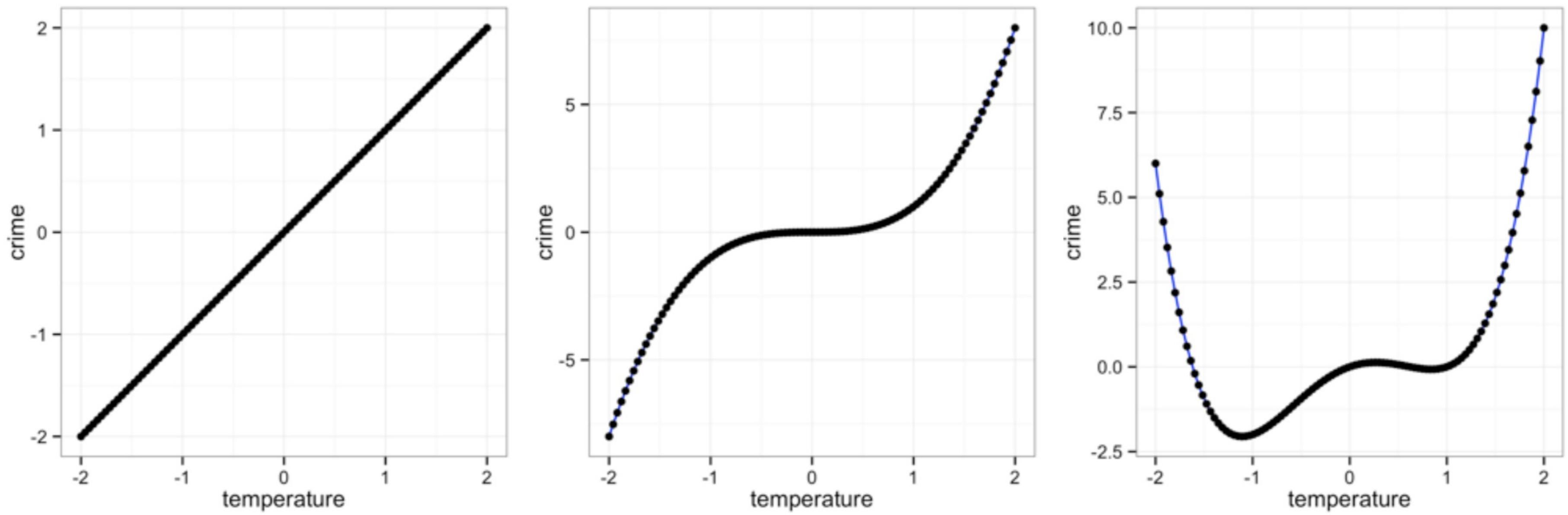
# Functions:
# the *stuff* behind the models

- Ideally, A function is a mathematical description of a relationship.



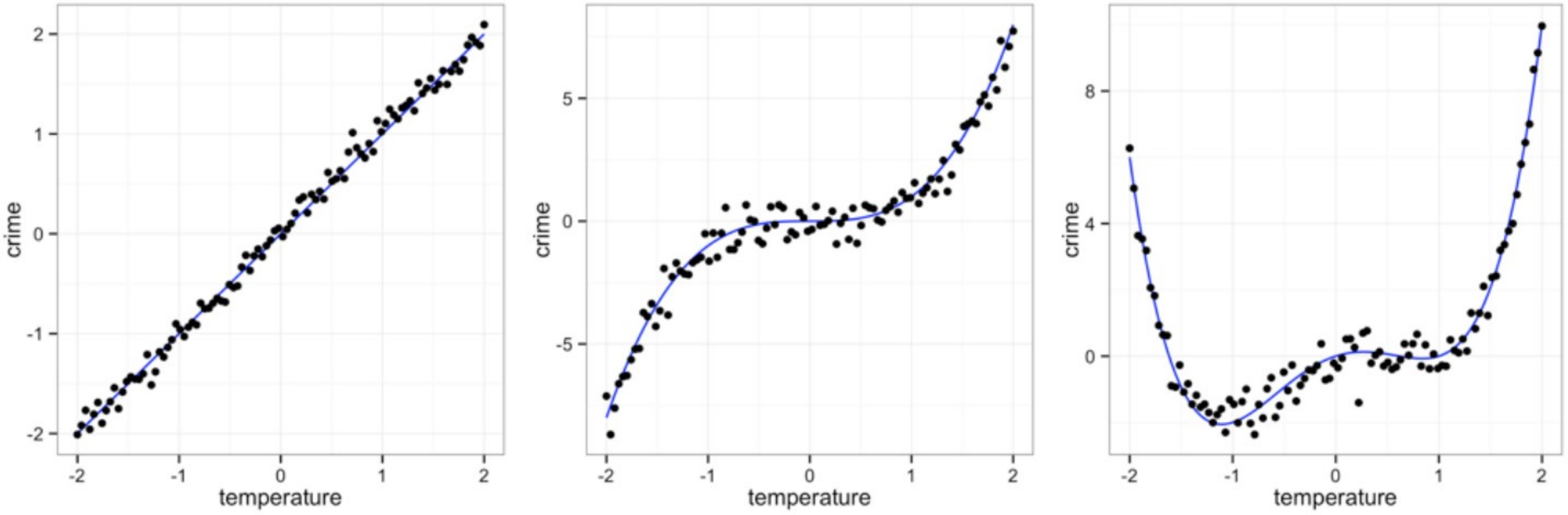
# Functions:
# the *stuff* behind the models

- If one variable completely determines another, every (x, y) data point will fall on the **function**-line.



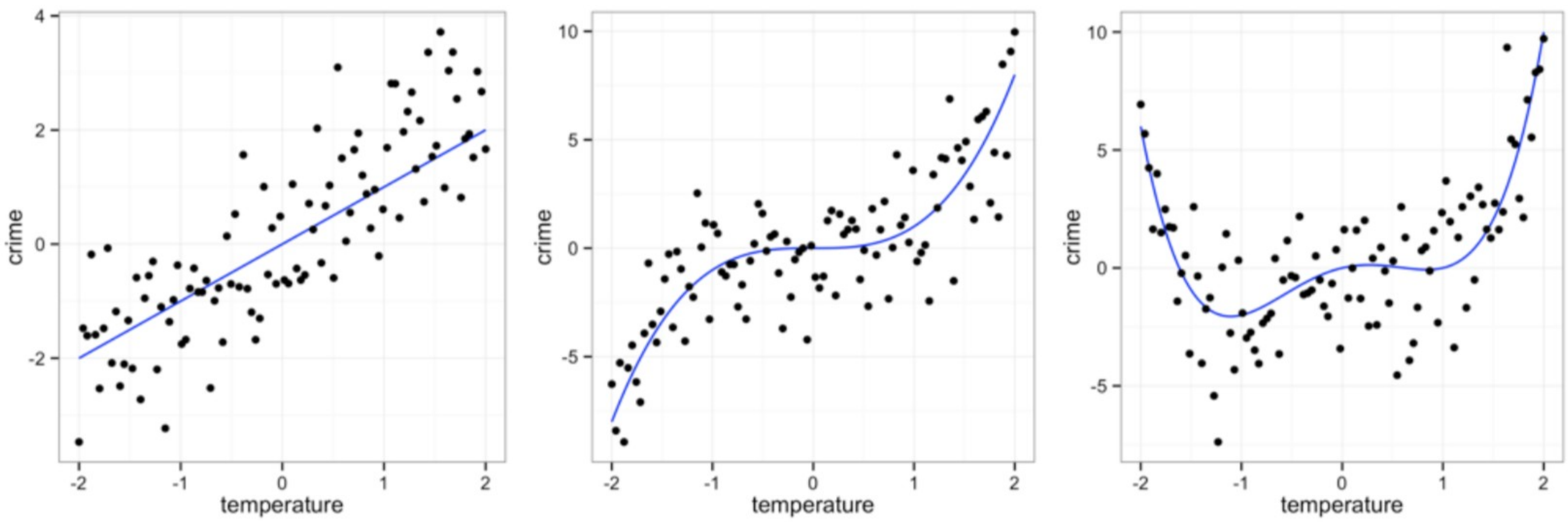
# Relationships Between Variables are Messy

- This is what real data looks like on a good day!



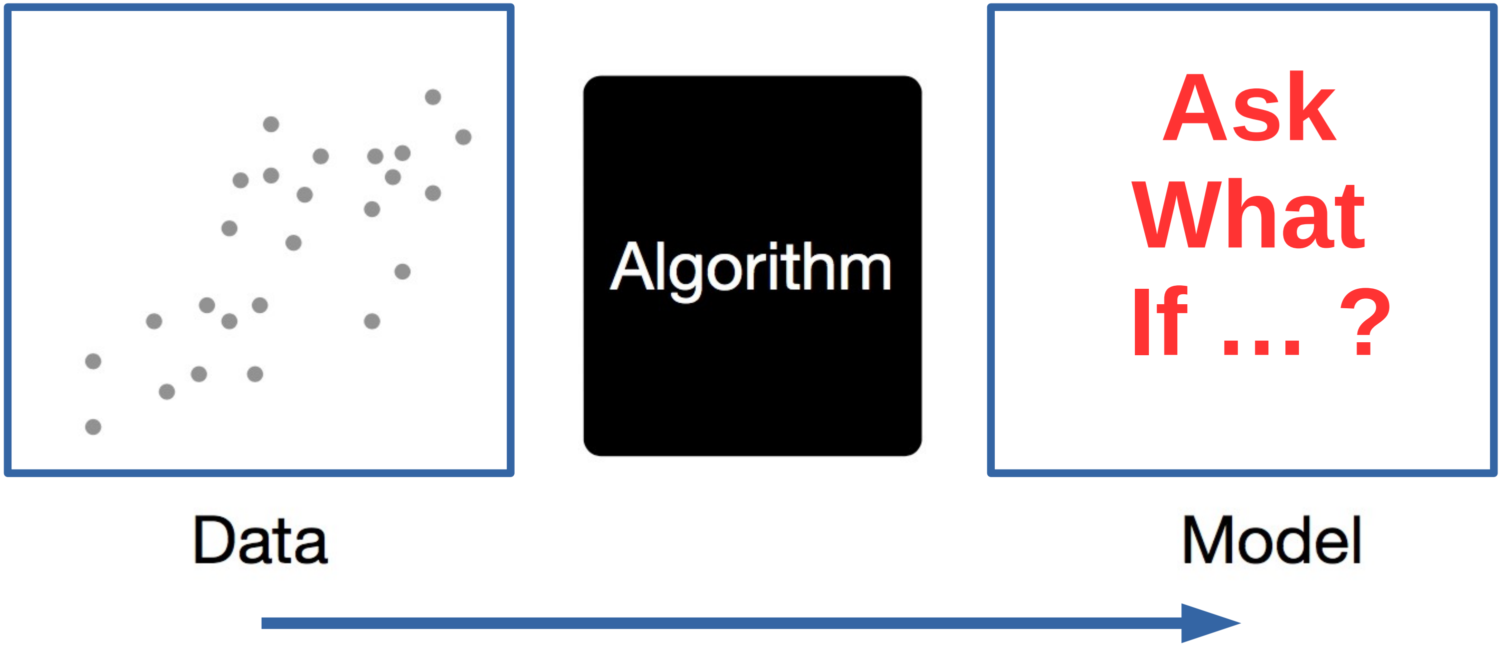
# Relationships Between Variables

- If the actual relationship is affected by other variables, data points may not fall directly on the function-line.
- **Noise**: The greater the effect of other variables, the weaker the relationship. This is normally the situation with real data.



# So, A Model, Then?

- Noise is what we get in data when not every point does *what it is supposed to do.*
- **Modeling *attempts* to *more*-correctly identify relationships in noisy data.**

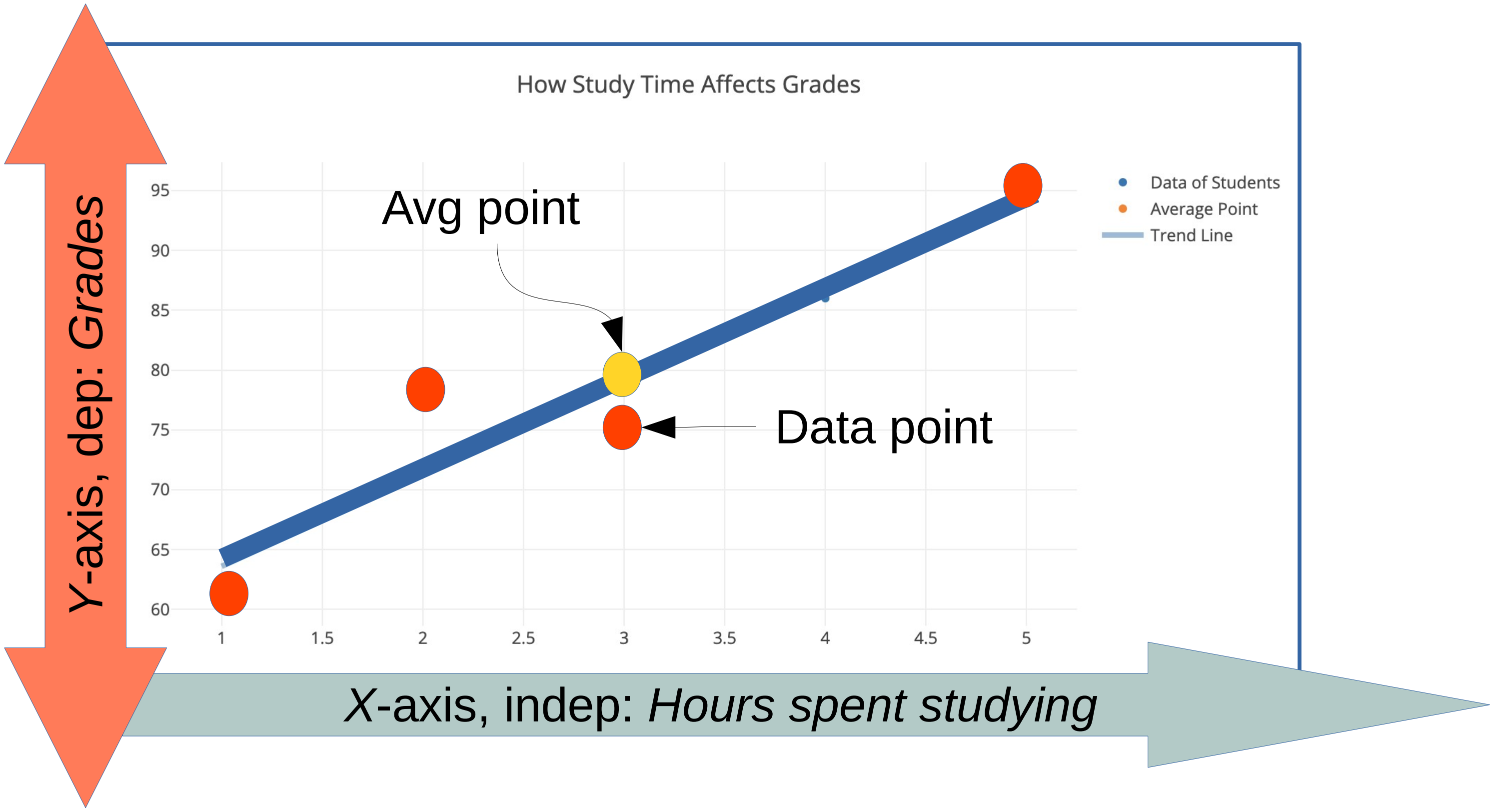


# Let’s Talk Linear Models

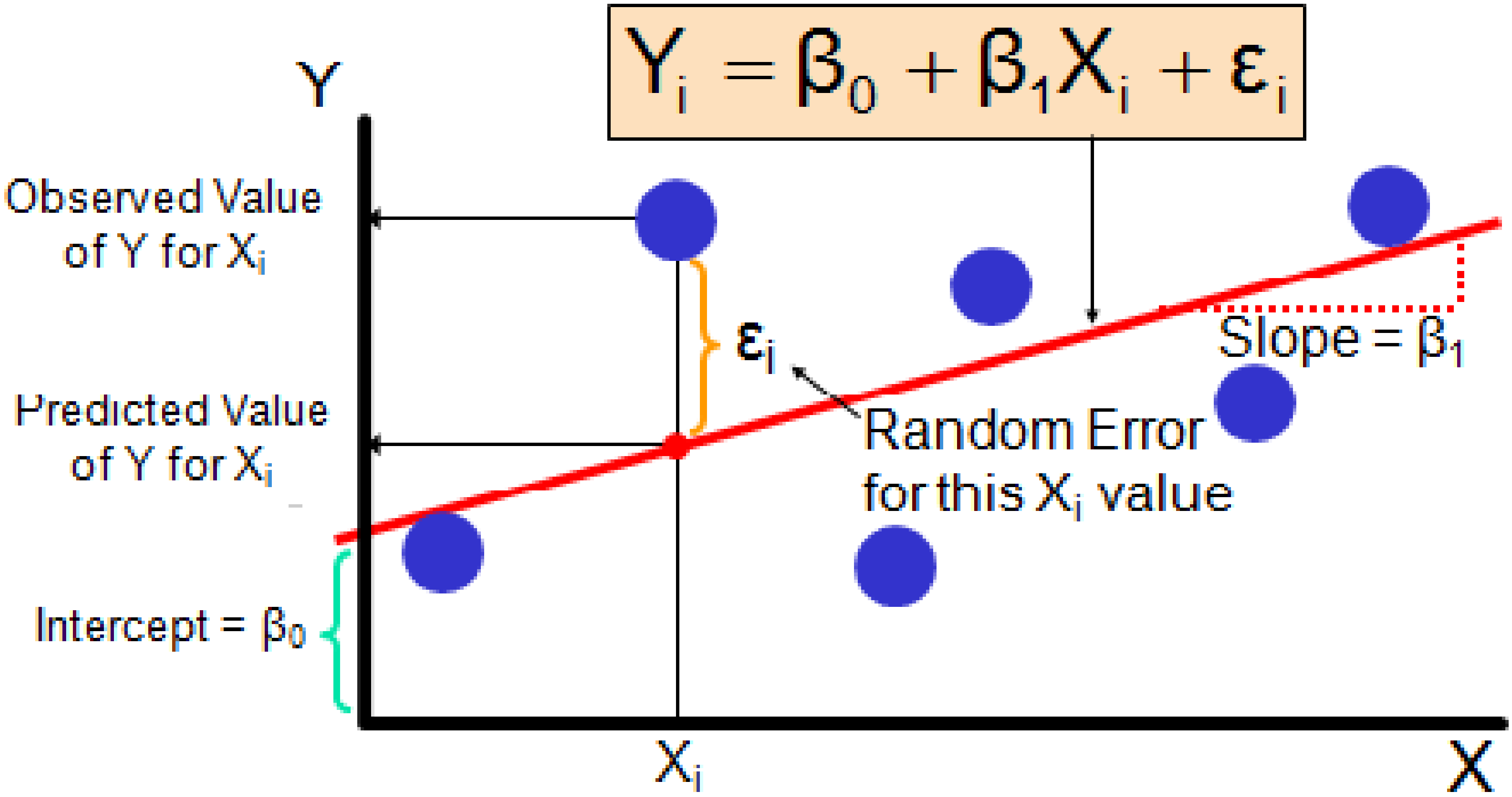
- **Linear regression**: How much do/does my **independent variable(s)** influence my **dependent variables**?
- As one variable climbs, does the other also climb (decline) at some *predicable* rate?
- Can I impose some value into my model to determine a ***what-if*** type of question which is firmly based on my data?

# Variables?

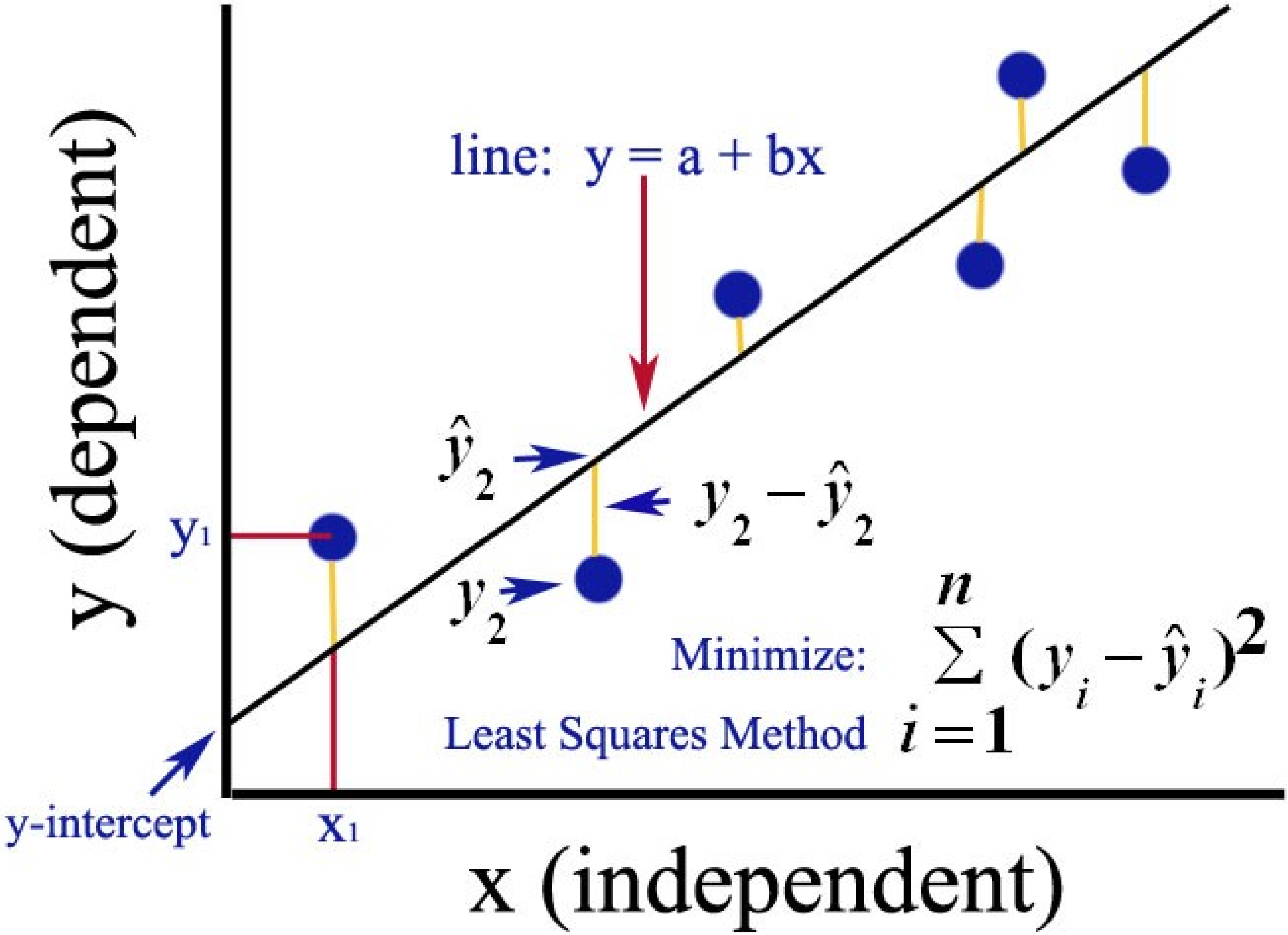
- **Independent variable:** a variable (often denoted by *x* ) whose variation does not depend on that of another (i.e., time).
- **Dependent variables:** a variables (often denoted by *y*) that depends, by some law or rule (e.g., by a mathematical function), on the values of other variables (i.e., grades).
- Example: https://chart-studio.plotly.com/~bchapman27/73.embed



# Let's Talk Linear Models

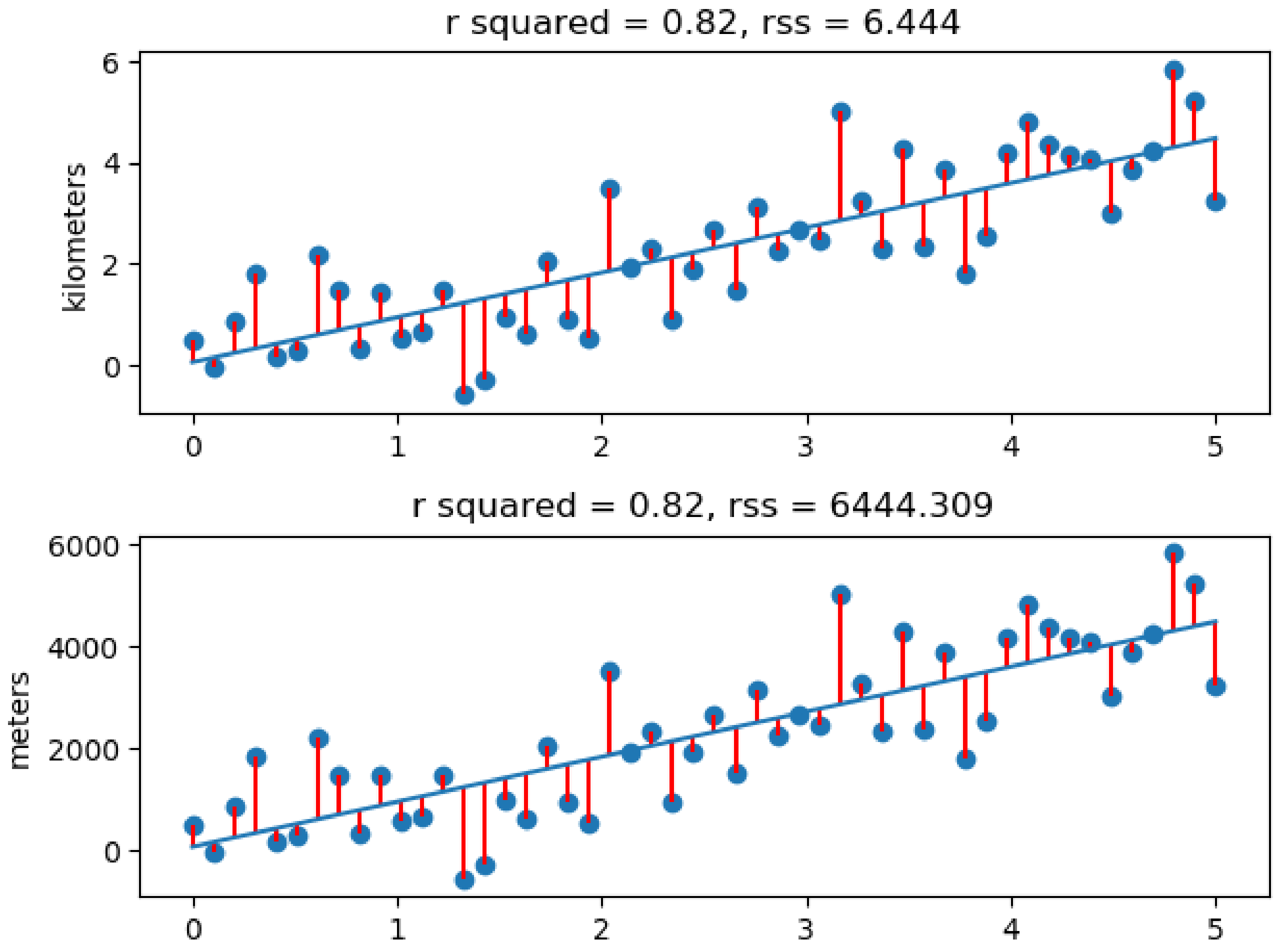


# Another Linear Model



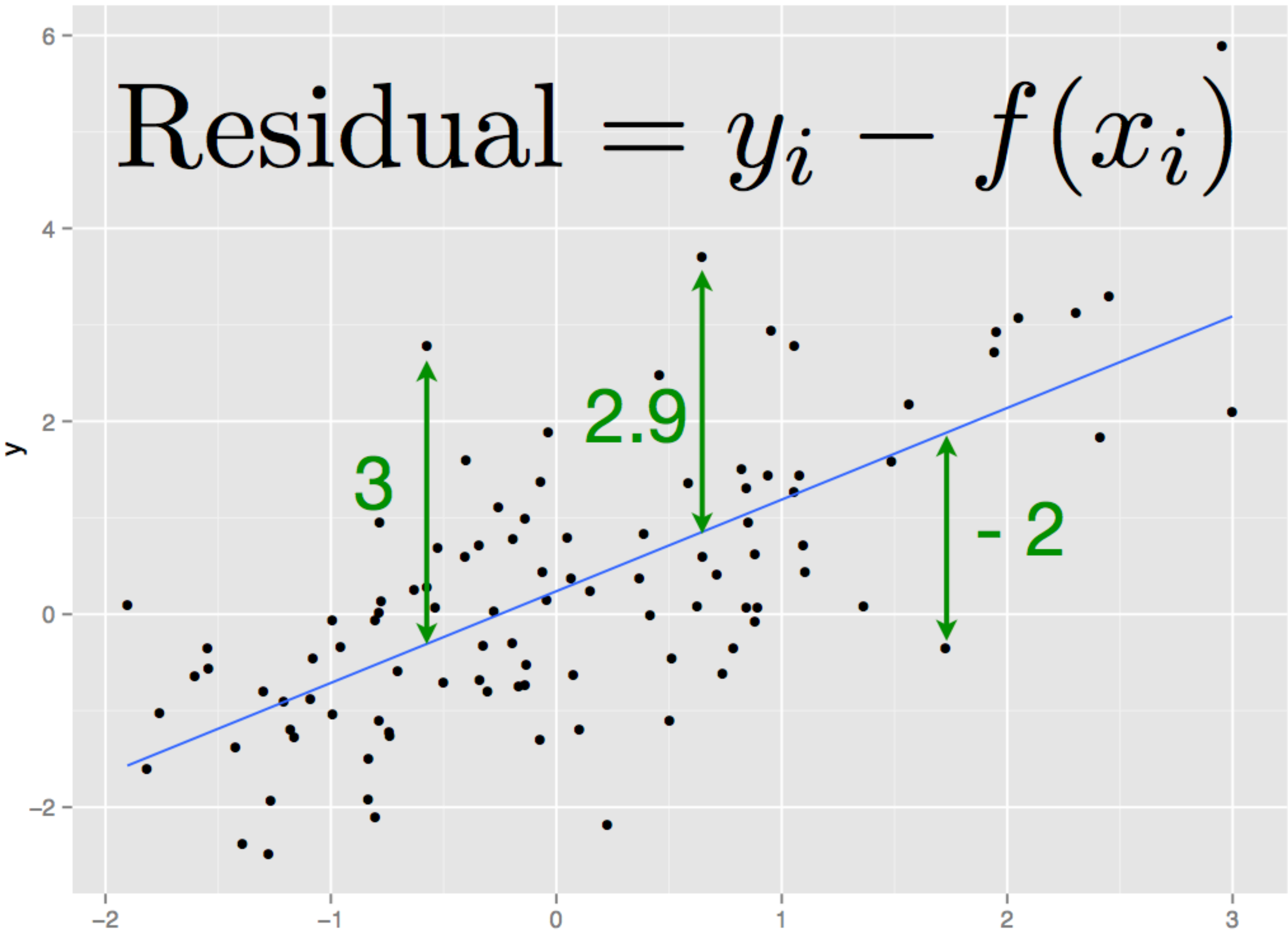
# A Difference Between the Expected and the Actual Data

- The residual sum of squares (**RSS**) is a measure of the discrepancy between the data and an estimation model.
- A small RSS value indicates a tight fit of the model to the data.



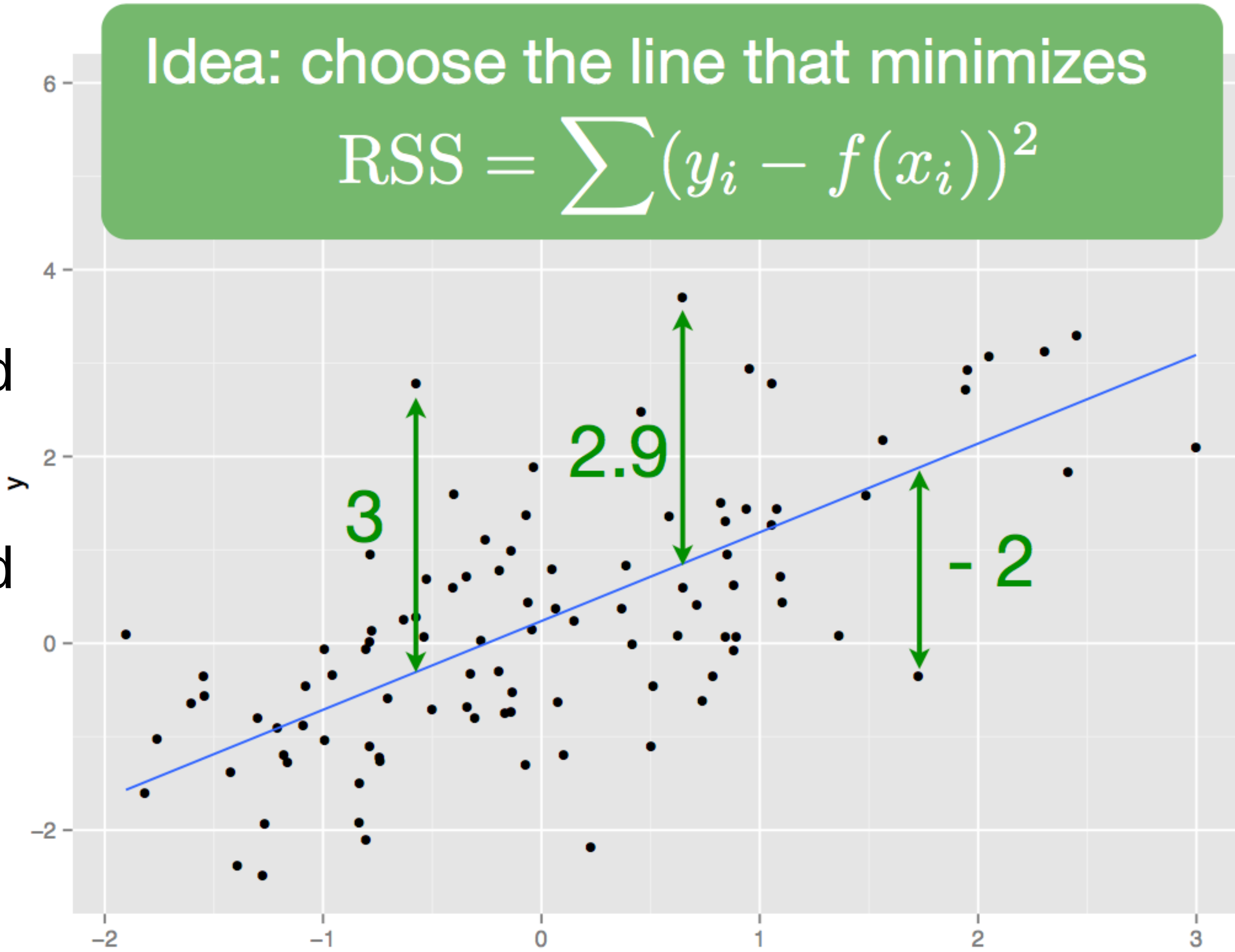
# Draw a Line Through Data

- A *residual* of an observed value is the difference between the observed value and the estimated value of the quantity of interest



# Draw a Line Through Data

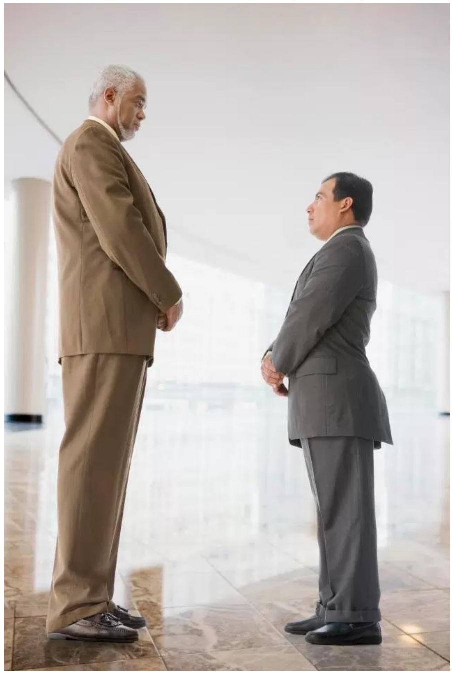
- Residual sum of squares (RSS), also known as the sum of squared residuals (SSR) or the sum of squared errors of prediction (SSE)



# Types of Questions to Address With Data

**Q1: Is crime influenced by yearly temperature?**

File: crime.csv



Q2: What influence is there on earning potential and personal height?

File: wages.csv

# Crime Data Set


- Is there a relationship between crime and temperature? State statistics from 2009.

```
rm(list = ls()) # remove old vars in memory
library(tidyverse)
# open the crime dataset from the data.
c <- file.choose() # set the filename
crime <- read.csv(c) # load and read the data.
```

# Crime Data Set

```
View(crime) #or

tibble::as_tibble(crime) # dataframe
```

```
         state   abbr   low   murder   tc2009
         <chr>  <chr> <int>    <dbl>    <dbl>
1      Alabama     AL   -27      7.1   4337.5
2       Alaska     AK   -80      3.2   3567.1
3      Arizona     AZ   -40      5.5   3725.2
4     Arkansas     AR   -29      6.3   4415.4
5   California     CA   -45      5.4   3201.6
6     Colorado     CO   -61      3.2   3024.5
7  Connecticut     CT   -32      3.0   2646.3
8     Delaware     DE   -17      4.6   3996.8
9      Florida     FL    -2      5.5   4453.7
10     Georgia     GA   -17      6.0   4180.6
...
```

Yearly low temp

Murder rate

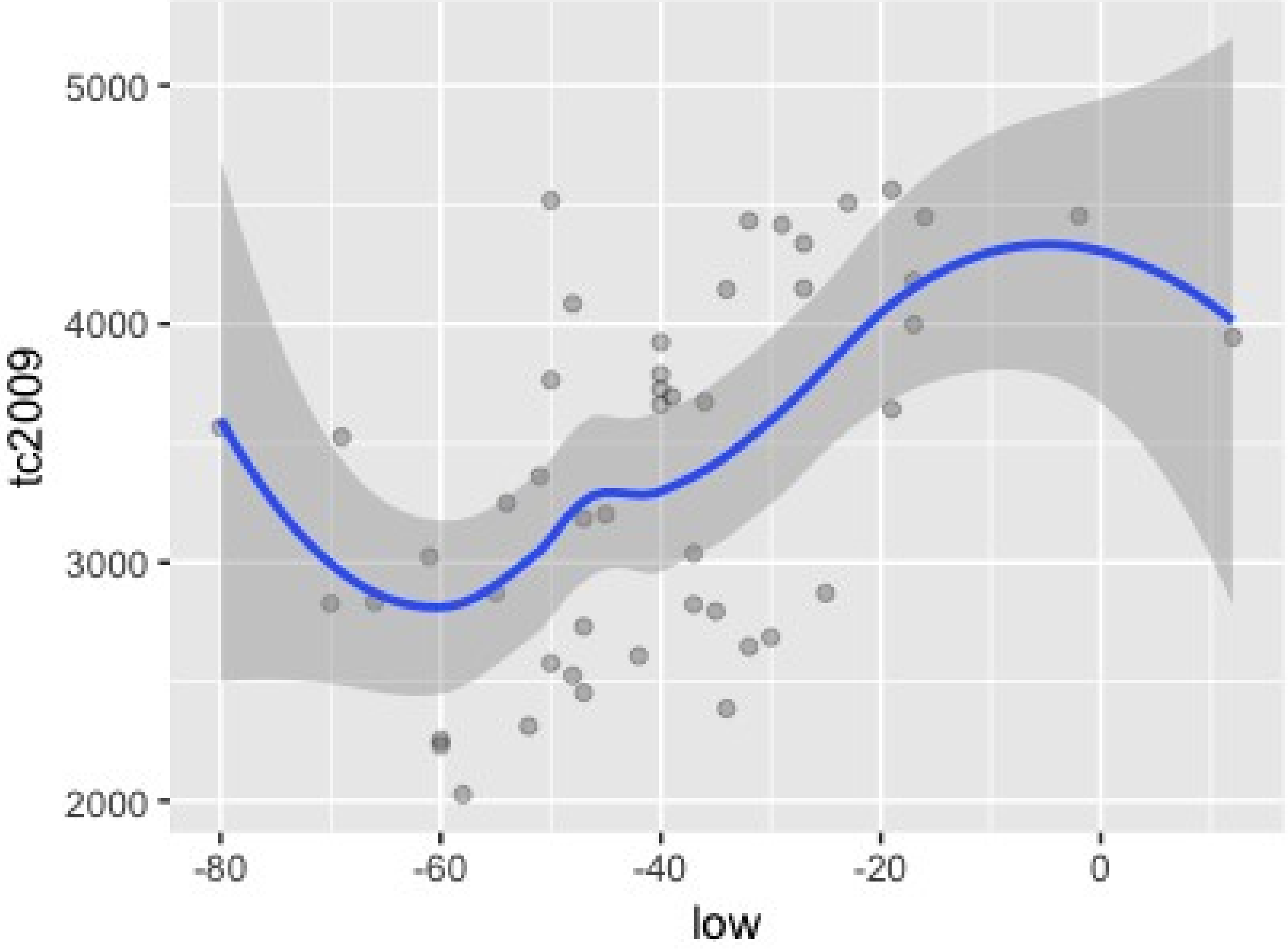
Training data

# Exploratory Plots

```
#plot with general trend line

crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth()


#plot with linear model line

crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) +
geom_smooth(method = lm)
```

# No Model: Just General Trends

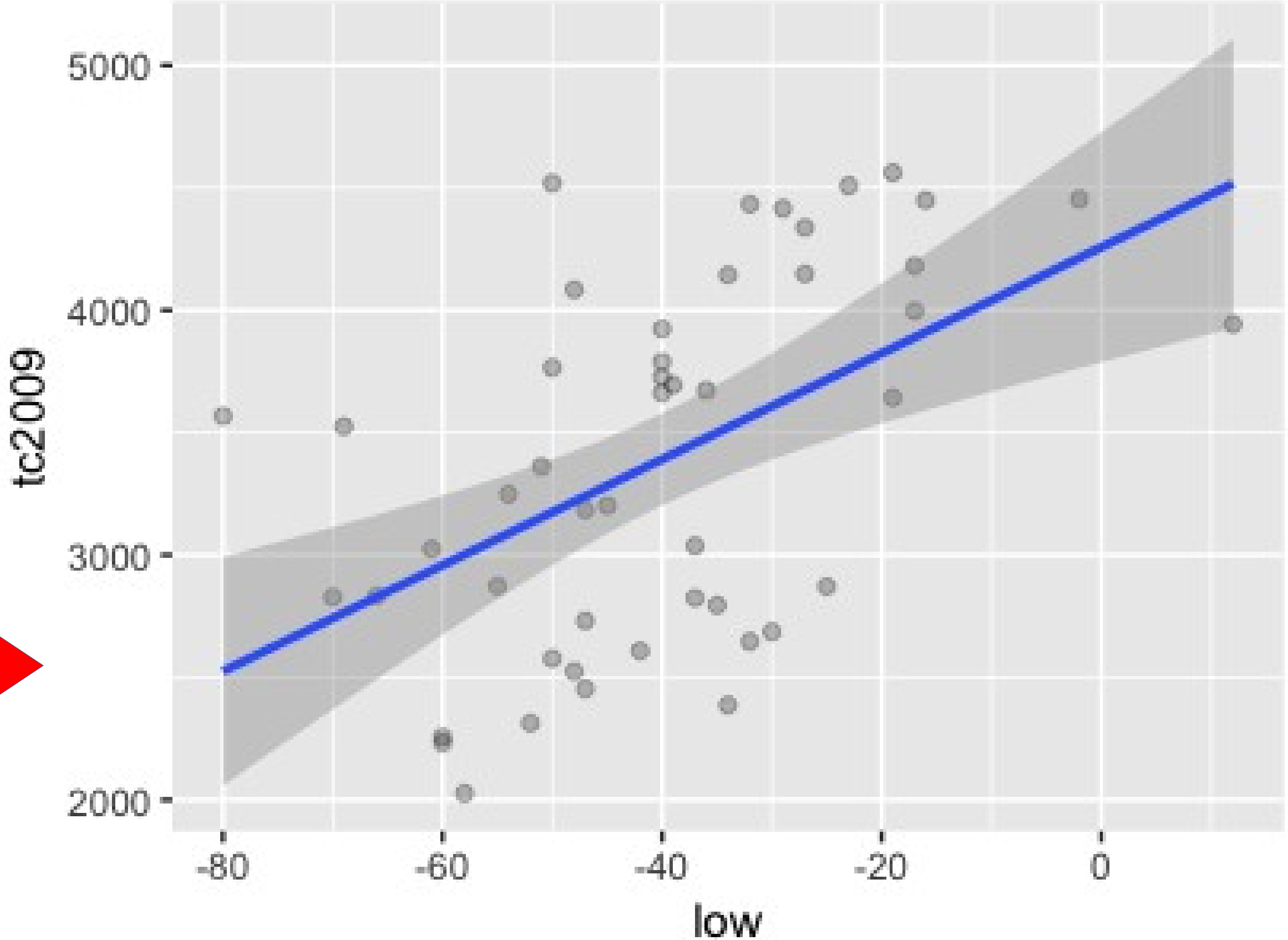
```
crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth()
```




**This is the data's general pattern**

# Linear Model: Predictions

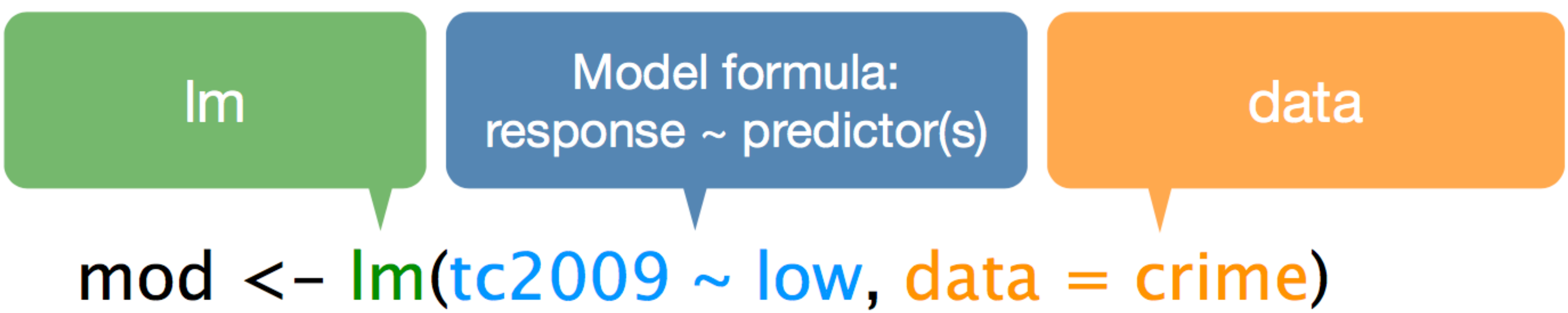
```
crime %>% ggplot(aes(x = low, y = tc2009)) +
geom_point(alpha = I(1/4)) + geom_smooth(method = lm)
```




**The linear model and is used for predictions**

# The Linear Model

- How much does ***low*** *(indep Var)* influence ***tc2009*** *(dep Var)*
- Linear model syntax



# Models Use Formulas

- Formulas only need to include the response and predictor variables

$$y = f(x) = \alpha + \beta x + \epsilon$$

#Syntax to Build the linear model:

$$y \sim x$$

# Models Use Formulas

- R formulas are expressions built with ~ (tilda)

tc2009 ~ low

# Note: ***tc2009*** is **independent** variable and ***low*** is **dependent** variable

class(tc2009 ~ low)

# gives: [1] "formula",

meaning, $tc2009 = f(x) = low*x + b$

# Types of Formulas

```
response ~ explanatory

dependent ~ independent

outcome ~ predictors
```

# Build Your Model!

```
mod <- lm(tc2009 ~ low, data = crime)
```

Dependent ~ independent

```
Call:
lm(formula = tc2009 ~ low, data = crime)

Coefficients:
(Intercept)          low
    4256.86        21.65
```

# Intercept and Coefficient

```
mod
```

```
> mod

Call:
lm(formula = tc2009 ~ low, data = crime)

Coefficients:
(Intercept)          low
    4256.86        21.65
```

# Coef

- Shows the model's coefficients (I.e., intercept, slopes)

```
coef(mod)
coefficients(mod)
# (Intercept)           low
#  4256.86158      21.64725
```

$\alpha$ $\beta$

# Interpreting Models

Linear models are very easy to interpret

$$y = \alpha + \beta x + \epsilon$$

$\alpha$ is the expected value of y when x is 0.

$\beta$ is the expected increase in y associated with a one unit increase in x

# Coefficients: For Prediction

```
coef(mod)
coefficients(mod)
# (Intercept)          low
#  4256.86158     21.64725
```

The best estimate of
tc2009 for a state with low = -10 is
4256.86 + 21.6 * (-10) = 4040.86

**(x,y) ← (-10, 4040.86)**

# Coefficient Calculator

**This function is now my data!!**

Based on our training using data,
if $x$ = -10, then $y$ = 4040.86

The best estimate of
tc2009 for a state with low = -10 is
4256.86 + 21.6 * (-10) = 4040.86

I can predict *y*,
based on values of x!

**Due to error, there is a slight difference between this value and our own value.**

# Coefficient Calculator Function

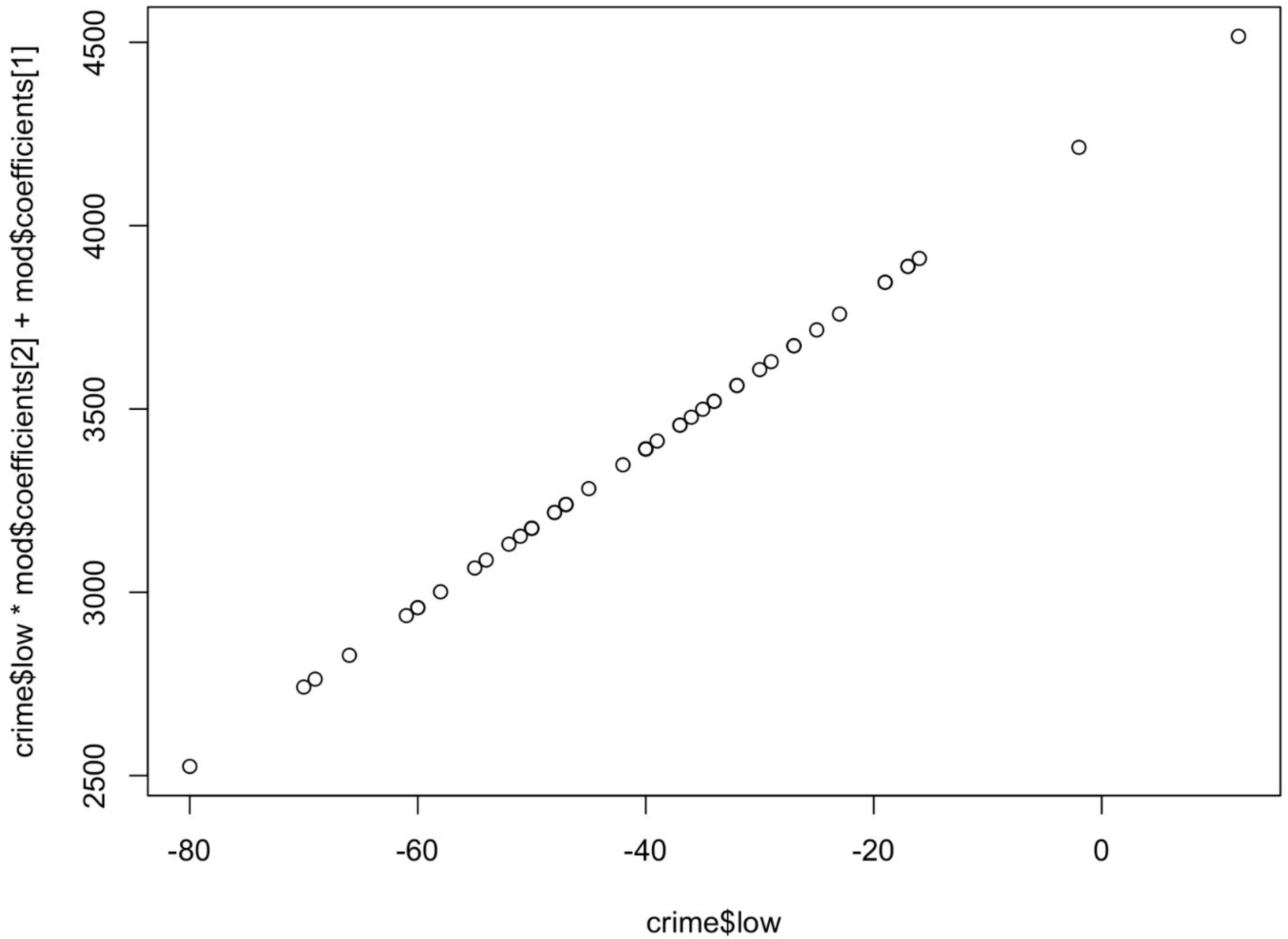
```
# Function to compute estimated y value for an entered x
value

tellMeY <- function(x_int){

  cat("  intercept :",mod$coefficients[1] )
  cat("\n  slope     :",mod$coefficients[2] )
  y = mod$coefficients[1] + x_int * mod$coefficients[2]
  cat("\n Model predicts y = ",y, "from x = ",x_int)
}

# what if x = -10?
tellMeY(-10) # note: x = -10
```

```
> # what if x = -10?
> tellMeY(-10) # note: x = -10
  intercept : 4256.862
  slope     : 21.64725
 Model predicts y =  4040.389 from x =  -10
>
```

*y* = *intercept* + *slope* * *x*
= alpha + beta**x*
= *b* + *mx*

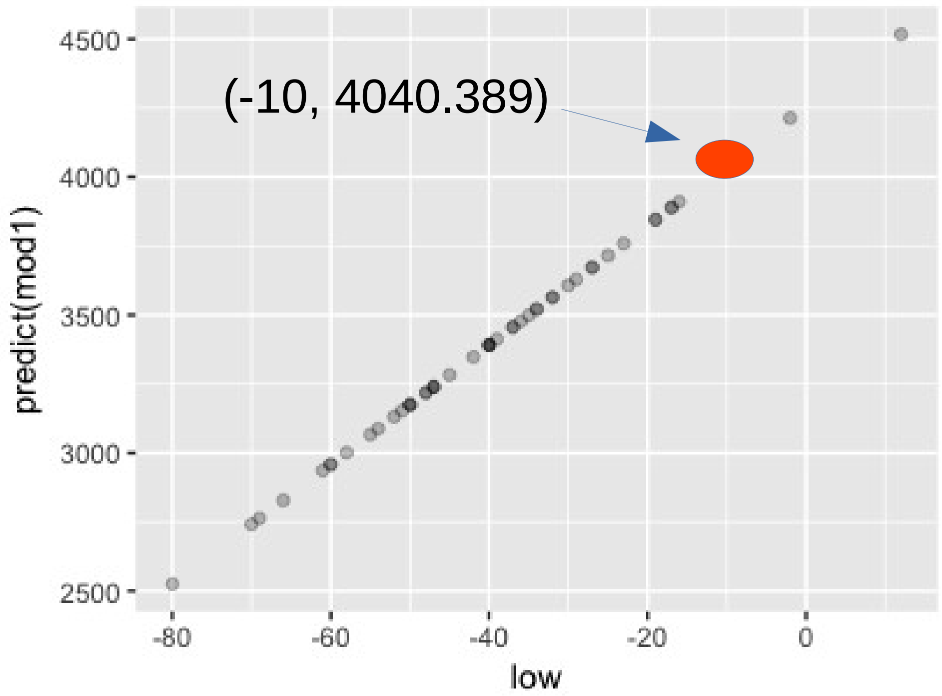
# Forecasting **Trends** With a Simple Line

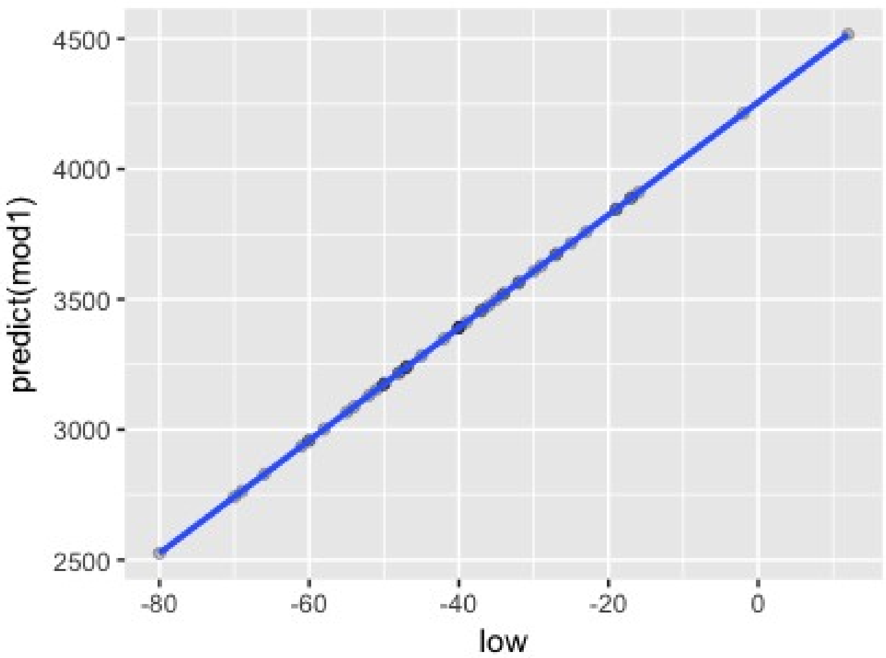
```
plot(crime$low, crime$low*mod$coefficients[2] + mod$coefficients[1] )
```



# Forecasting with `predict()`

```
?predict

crime %>% ggplot(aes(x = low, y = predict(mod))) +
geom_point(alpha = I(1/4))

crime %>% ggplot(aes(x = low, y = predict(mod))) +
geom_point(alpha = I(1/4)) + geom_smooth()
```



# Aside: intercept terms

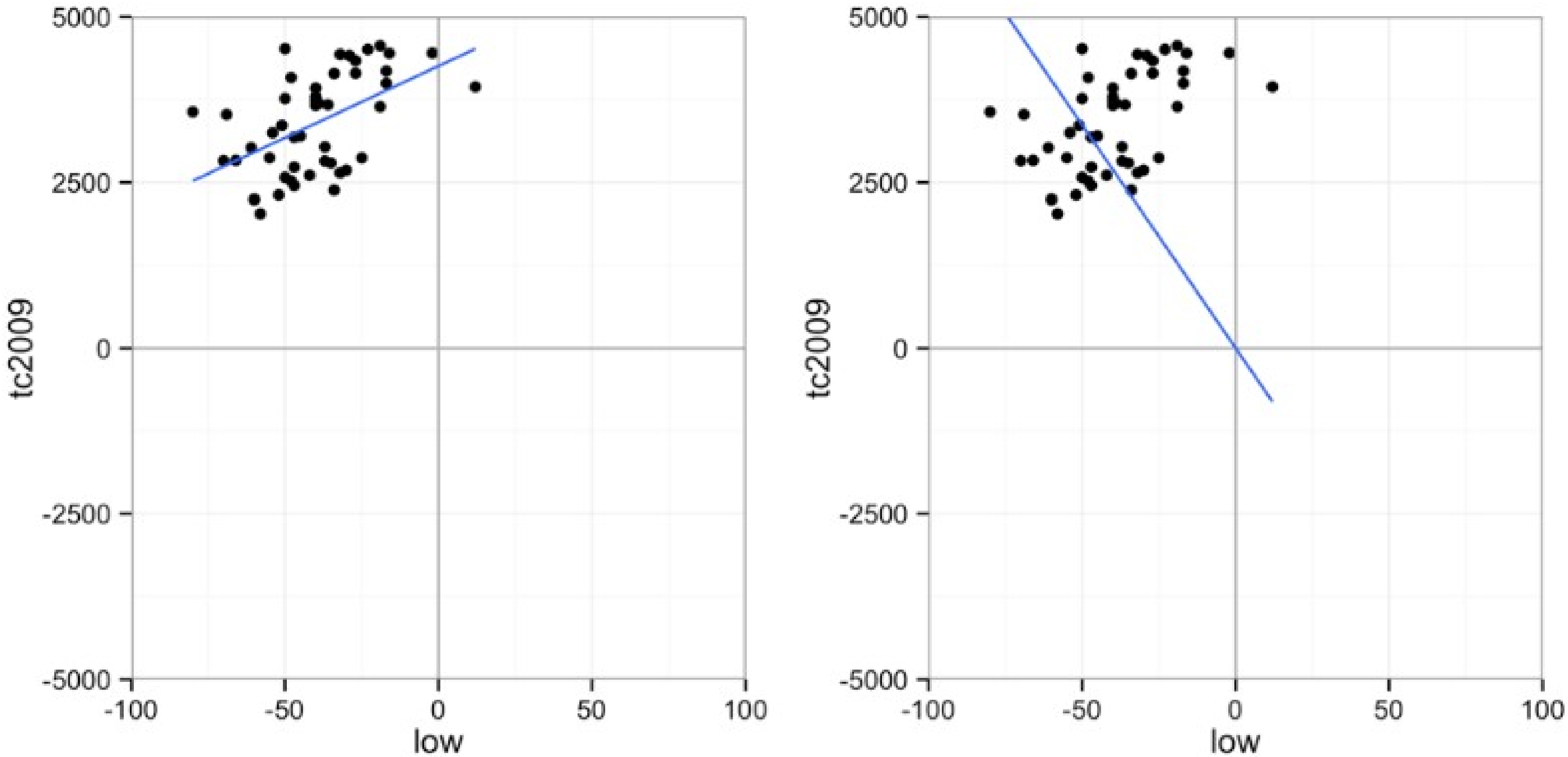
R includes an intercept term in each model by default

$$y = \alpha + \beta x + \epsilon$$

```
y ~ x
```

# Study at $x = 0$?

## (Does $x = 0$ make sense here?)




**With a** **Without a**

Every linear model has a y intercept. Including a lets this term vary. Not including a forces the intercept to (0, 0).

# Study at $x = 0$?

# (Does $x = 0$ make sense here?)

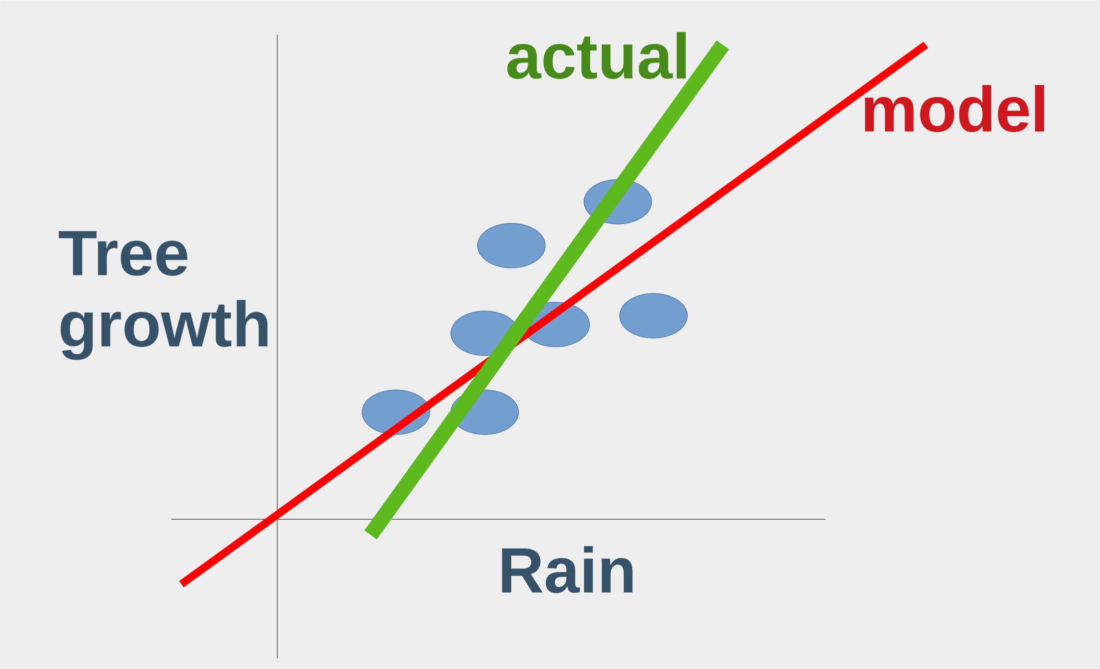
- The *y*-intercept is the place where the regression line crosses the y-axis (where $x = 0$), and is denoted by $b$ from $y = mx + b$
- **Meaningful interpretation**: Sometimes the *y*-intercept is relevant (and sometimes it is not)
- No meaning for the *y*-intercept when data is not present near the point where $x = 0$ (and the model suggests that data is present at this point)

# Study at $x = 0$?
# (Does $x = 0$ make sense here?)

**Ex: A model where rain (*x*) is used to predict tree growth (*y*)**

**If *rain* = 0, then *tree_growth* = 0**

**Intercept not relevant: The regression line may cross *y*-axis at some other point (other than zero)**



# An Intercept Term: To Use or Not?

FYI: You can explicitly ask for an intercept by including the number one, 1, as a formula term. You can remove the intercept by including a zero or negative 1.

```
# equivalent - includes intercept
lm(tc2009 ~ 1 + low, data = crime)
lm(tc2009 ~ low, data = crime)

# equivalent - removes intercept
lm(tc2009 ~ low - 1, data = crime)
lm(tc2009 ~ 0 + low, data = crime)
```

# Let’s Test An Intercept

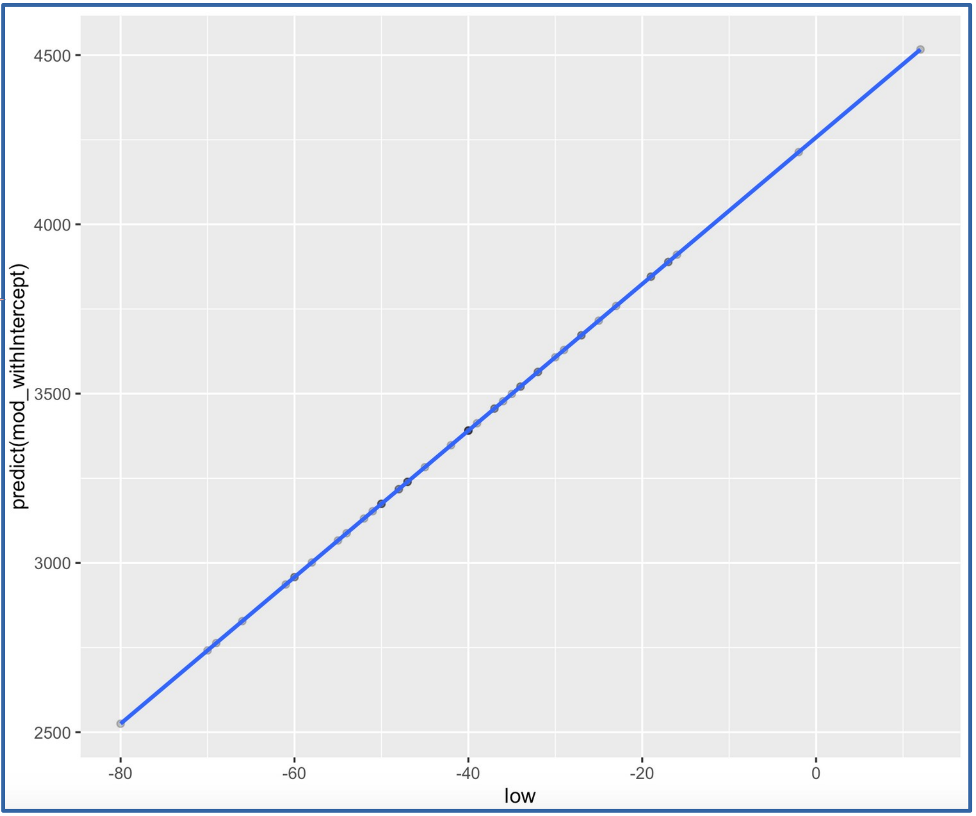
## Add the intercept

# equivalent - includes intercept

mod_withIntercept <- lm(tc2009 ~ 1 + low, data = crime)

crime %>% ggplot(aes(x = low, y = predict(mod_withIntercept))) + geom_point(alpha = I(1/4)) + geom_smooth()

Does this represent your data?

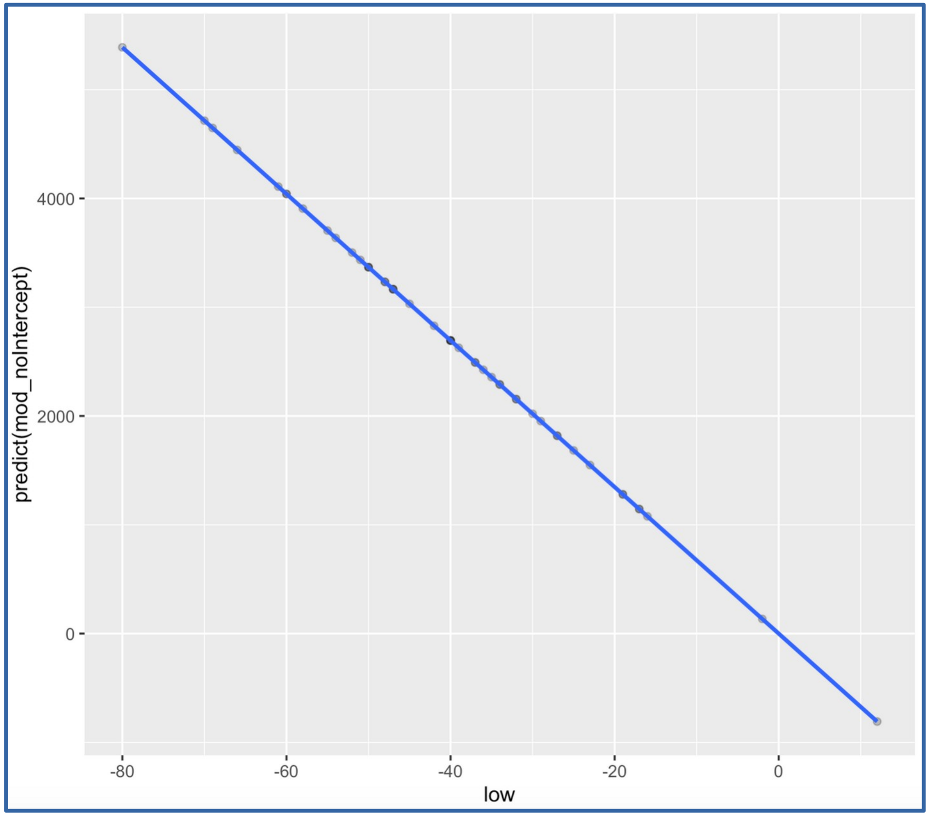


# Let’s Test An Intercept

## Remove the intercept

```
# equivalent - removes intercept

mod_noIntercept <- lm(tc2009 ~ low - 1, data = crime)

crime %>% ggplot(aes(x = low, y =
predict(mod_noIntercept))) + geom_point(alpha = I(1/4)) +
geom_smooth()
```

Does this represent your data?



# Results: summary(mod)

```
> summary(mod)

Call:
lm(formula = tc2009 ~ low, data = crime)

Residuals:
     Min       1Q   Median       3Q      Max 
-1134.36  -647.13    98.03   533.62  1344.30 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4256.86     233.44  18.236  < 2e-16 ***
low            21.65       5.33   4.061 0.000188 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 649.9 on 46 degrees of freedom
Multiple R-squared:  0.2639,	Adjusted R-squared:  0.2479 
F-statistic: 16.49 on 1 and 46 DF,  p-value: 0.000188
```

# R-squared Value

- Goodness of fit of a model: The R2 coefficient of determination describes how well the regression predictions approximate the real data points.

R2 = 1, indep variable(s) predict dep variables

R2 = 0, no prediction

```
Residual standard error: 649.9 on 46 degrees of freedom
Multiple R-squared:  0.2639,    Adjusted R-squared:  0.2479
F-statistic: 16.49 on 1 and 46 DF,  p-value: 0.000188
```

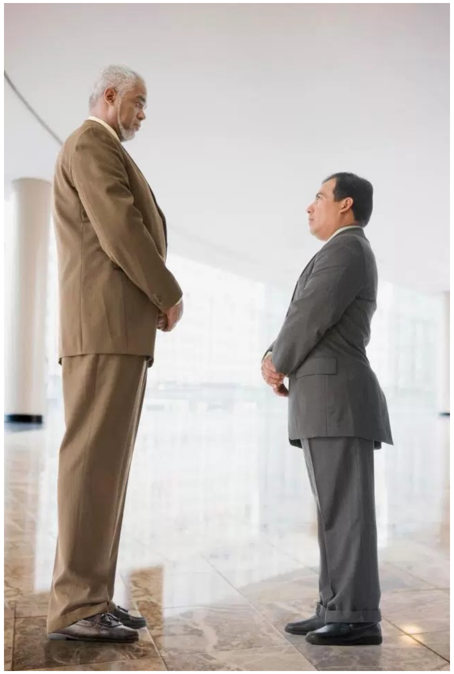
# Extracting Info

```
#Create a simple model
myX <- 0:100
myY <- myX + 1
mod <- lm(myY ~ myX)
summary(mod)
```

# Types of Questions to Address With Data

Q1: Is crime influenced by yearly temperature?

File: crime.csv



Q2: What influence is there on earning potential and personal height?

File: wages.csv

# Consider This!

- Try making a model with the other data set to determine what influence height has on earning potential.

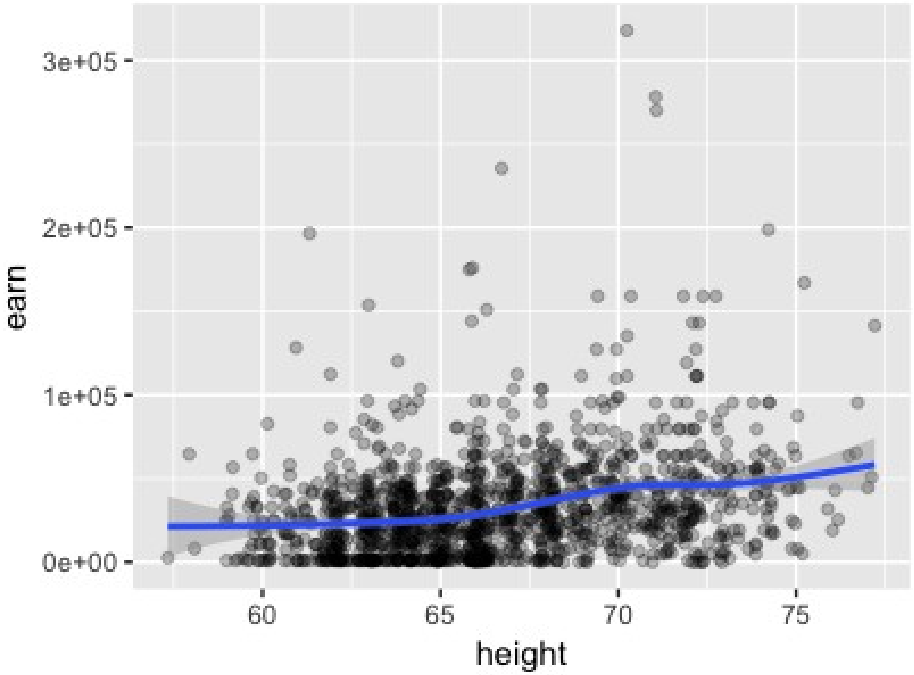
# Load the Wages Data

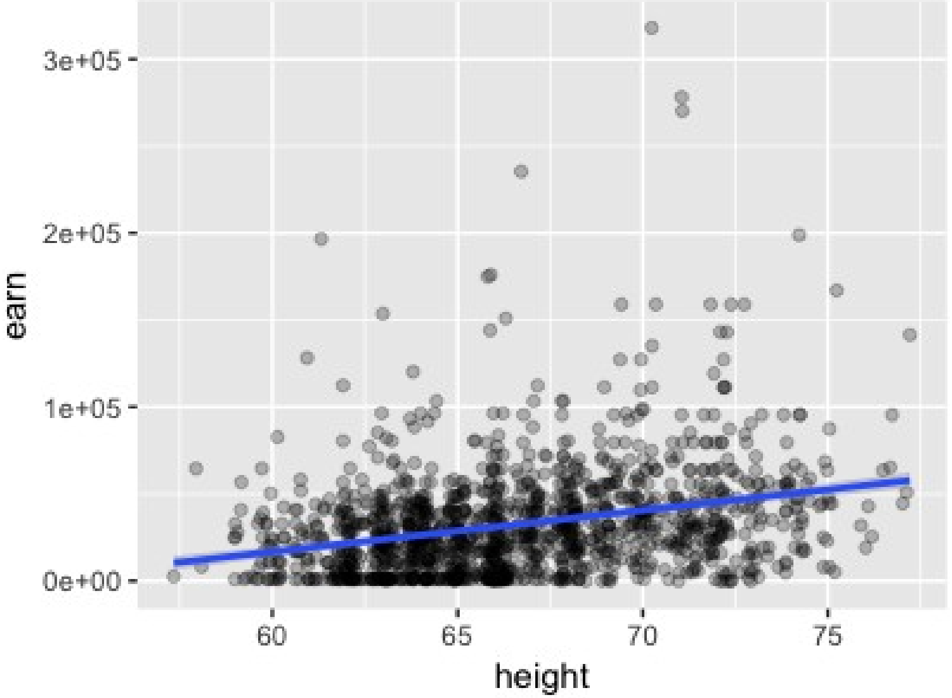
Fit a linear model to the wages data set that predicts ***earn*** with ***height***.

```
rm(list = ls()) # remove old vars

# open the wages.csv dataset from the data.

w <- file.choose() # set the filename

wages <- read.csv(w) # load and read the data.
```

# Do *Tall* People *Earn* More?

```
wages %>% ggplot(aes(x = height, y = earn)) + geom_point(alpha = I(1/4)) + geom_smooth() # add a line

wages %>% ggplot(aes(x = height, y = earn)) + geom_point(alpha = I(1/4)) + geom_smooth(method = lm) # linear model line

# Try switching the x's and y's for another view.
```



# Correlations:
## Earn and Height

```
# Find correlations using the "pearson" method
cor(wages$earn, wages$height, method = "pearson")
```

```
> # Find correlations using the "pearson" method
> cor(wages$earn, wages$height, method = "pearson")
[1] 0.2916002
```

# Make a Model

```
hmod <- lm(earn ~ height, data = wages)
summary(hmod)
```

Where **dependent** var is ***earn***

And **independent** var is ***height***

$$y = \alpha + \beta x + \epsilon$$

# Summary of Model

- summary(hmod)

```
> summary(hmod)

Call:
lm(formula = wages$earn ~ wages$height)

Residuals:
   Min     1Q Median     3Q    Max
-47903 -19744  -5184  11642 276796

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)   -126523      14076  -8.989   <2e-16 ***
wages$height     2387        211  11.312   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 29910 on 1377 degrees of freedom
Multiple R-squared:  0.08503,   Adjusted R-squared:  0.08437
F-statistic:   128 on 1 and 1377 DF,  p-value: < 2.2e-16
```

# *Earn* Regressed Over *height*

```
hmod <- lm(earn ~ height, data = wages)
coef(hmod)
## (Intercept)      height
## -126523.359    2387.196
```

$$earn = \alpha + \beta \times height + \epsilon$$

$$\boxed{earn = -126523.36 + 2387.20 \times height + \epsilon}$$

# An Estimation

The best estimate of earn for someone 68 inches tall is

$$earn = -126523.36 + 2387.20 \times \quad 68 \quad + \epsilon$$

$$earn = 35806.24$$

# Build a Model

- Fit a linear model to the wages data set
- How do we interpret the results?

Q: What happens when we regress *earn* over *race*?

# Summary

```
rmod <- lm(earn ~ race, data = wages)
coef(rmod) # get the model's y-intercepts and slopes
```

```
coef(rmod)
# (Intercept) racehispanic    raceother    racewhite
#    28372.09     -2886.79      3905.32      4993.33
```

**summary(rmod)**

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)     28372       2781  10.204   <2e-16 ***
racehispanic    -2887       4515  -0.639   0.5227    
raceother        3905       6428   0.608   0.5436    
racewhite        4993       2929   1.705   0.0885 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

# Estimates From Coefficients

```
coef(rmod)
# (Intercept) racehispanic   raceother   racewhite
#    28372.09     -2886.79     3905.32     4993.33
```

The estimate for a white person is
28372.09 + 4993.33 = 33365.42

The estimate for a other person is
28372.09 + 3905.32 = 32277.41

The estimate for a hispanic person is
28372.09 + -2886.79 = 25485.30

The estimate for a black person is
28372.09 = 28372.09

# What Do Plots Also Indicate??


```
ggplot(data = wages) +
  geom_point(mapping = aes(y = earn, x = height, color = race )) +
  geom_smooth(mapping = aes(y = earn, x = height )) + facet_wrap(~race)
```