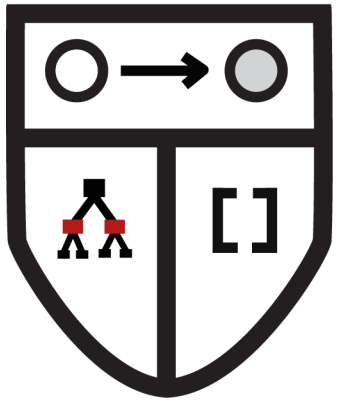


# CS 3110

## Type Systems

Prof. Clarkson

Fall 2019

Today's music: Check Yo Self by Ice Cube

# CLICKER QUESTION 1

# Review

**Previously in 3110:**

- Evaluation, i.e., formal dynamic semantics
- Small- and big-step relations
- Substitution and environment models

**Today:**

- Type systems, i.e., formal static semantics
- Type safety
- Type inference

# Evaluation errors

```
5 + false ⇏
if 5 then true else 0 ⇏
```

**Goal:** prevent evaluation errors

- analyze program before running it
- reject program (and refuse to run) if possible evaluation errors detected

Demo

# TYPE SYSTEMS

# `if` expressions [from lec 2]

Syntax:

```
if e1 then e2 else e3
```

Type checking (static semantics):

if `e1` : `bool` and `e2` : `t` and `e3` : `t`
then `(if e1 then e2 else e3)` : `t`

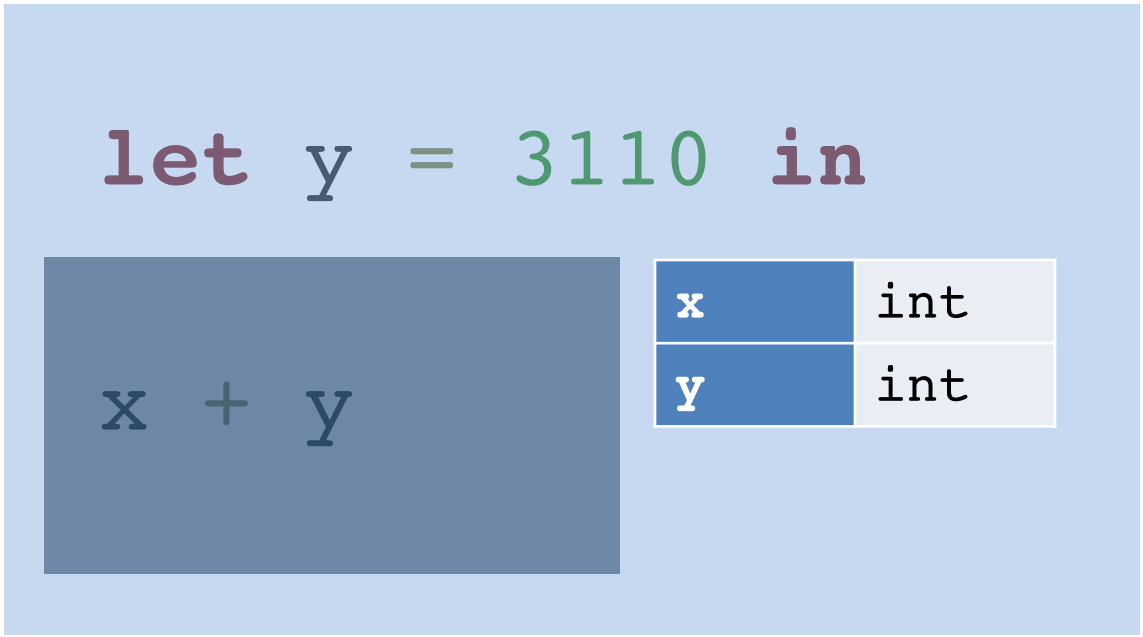
$$\mathtt{T} \vdash \mathtt{e} : \mathtt{t}$$

typing relation

# Static environment

aka typing context

```
let x = 42 in
```




| x | int |
|---|---|

# Environments

Static environment:

map from identifiers to types

| | |
|---|---|
| x | int |
| y | int |

Dynamic environment:

map from identifiers to values

| | |
|---|---|
| x | 42 |
| y | 3110 |

# Typing relation examples

$\{x{:}\texttt{int}\} \vdash x + 2 : \texttt{int}$

$\{x{:}\texttt{bool}\} \nvdash x + 2 : \texttt{int}$

$\{x{:}\texttt{int}\} \nvdash x + 2 : \texttt{bool}$

$\{\} \nvdash x : \texttt{int}$

# Typed SimPL

```
e ::= x | i | b
    | e1 bop e2
    | let x : t = e1 in e2
    | if e1 then e2 else e3

bop ::= + | * | <=

t ::= int | bool
```

# Values and variables

$$\mathtt{env} \vdash \mathtt{i} : \mathtt{int}$$

$$\mathtt{env} \vdash \mathtt{b} : \mathtt{bool}$$

$$\mathtt{env} \vdash \mathtt{x} : \mathtt{env(x)}$$

# Binary operators

**env ⊢ e1 + e2 : int**

if

**env ⊢ e1 : int**

**env ⊢ e2 : int**

**env ⊢ e1 * e2 : int**

if

**env ⊢ e1 : int**

**env ⊢ e2 : int**

**env ⊢ e1 <= e2 : bool**

if

**env ⊢ e1 : int**

**env ⊢ e2 : int**

# If expressions

**`env ⊢ if e1 then e2 else e3 : t`**

if

**`env ⊢ e1 : bool`**

**`env ⊢ e2 : t`**

**`env ⊢ e3 : t`**

# Let expressions

$$\texttt{env} \vdash \texttt{let x : t1 = e1 in e2 : t2}$$

if

$$\texttt{env} \vdash \texttt{e1 : t1}$$

$$\texttt{env[x} \mapsto \texttt{t1]} \vdash \texttt{e2 : t2}$$

Demo

# CLICKER QUESTION 2

# TYPE SAFETY

# Preventing evaluation errors

Evaluation of an expression **e** is stuck if:

- **e** is not a value, and
- **e** $\not\rightarrow$

Purpose of type system:
guarantee no expression ever gets stuck

# Type safety

Type safety **means never getting stuck**

Type safety = progress + preservation

- **Progress:** can always step (unless already value)
- **Preservation:** stepping never changes type

# Preservation

if **env** $\vdash$ **e : t**

and **e** $\longrightarrow$ **e′**

then **env** $\vdash$ **e′ : t**

$$\{\} \vdash (10 + 1) + (5 + 6) : \text{int}$$
$$(10 + 1) + (5 + 6) \longrightarrow 11 + (5 + 6)$$
$$\{\} \vdash 11 + (5 + 6) : \text{int}$$

# Progress

if `{ }` $\vdash$ `e : t`

then `e` is a value or

there exists an `e'` such that `e` $\longrightarrow$ `e'`

$$\{\} \vdash (10 + 1) + (5 + 6) : \text{int}$$

$$\{\} \nvdash x : \text{int}$$

# Type safety proof sketch

**Claim:** Well-typed programs don't get stuck.

**Proof:** by induction on number of steps to reach a value.

**Base case:** value. Zero steps.

Already done, hence not stuck.

**Inductive case:** not a value.

- By progress: can take one step.
- By preservation: still well-typed.
- IH applies: one step taken.

QED.

# TYPE INFERENCE

# Typed SimPL, without annotations

```
e ::= x | i | b
    | e1 bop e2
    | let x = e1 in e2
    | if e1 then e2 else e3

bop ::= + | * | <=

t ::= int | bool
```

# Guess and check

```
let x = e1 in e2
```

- Infer type **`t1`** of **`e1`**
- Put **`{x:t1}`** in static environment
- Use that to type check **`e2`**

Demo

# OCaml type inference

Based on Hindley-Milner algorithm

- Never infers the wrong types
- Never fails to infer types
- Usually runs in linear time

*(for the curious: see textbook 10.5, but we aren't covering it this semester unless I bump a lecture at the end of the semester)*

# Robin Milner

1934-2010

Awarded 1991 Turing Award for

*"…ML, the first language to include polymorphic type inference and a type-safe exception handling mechanism…"*

# Upcoming events

- [today] A6 released
- [tonight] MS1 due: no late submissions

*This is true to type.*

**THIS IS 3110**