

(sorry for my writing quality)

If my laptop disconnects, DON'T leave. I will  
switch to sharing directly from the ipad.  
(should be fine, just tested for 1 hour before  
class : )

- "get big picture"



basic  $R_x, R_y, R_z$

could break down in  $R_x, R_y, R_z$   
→ "gimble lock"

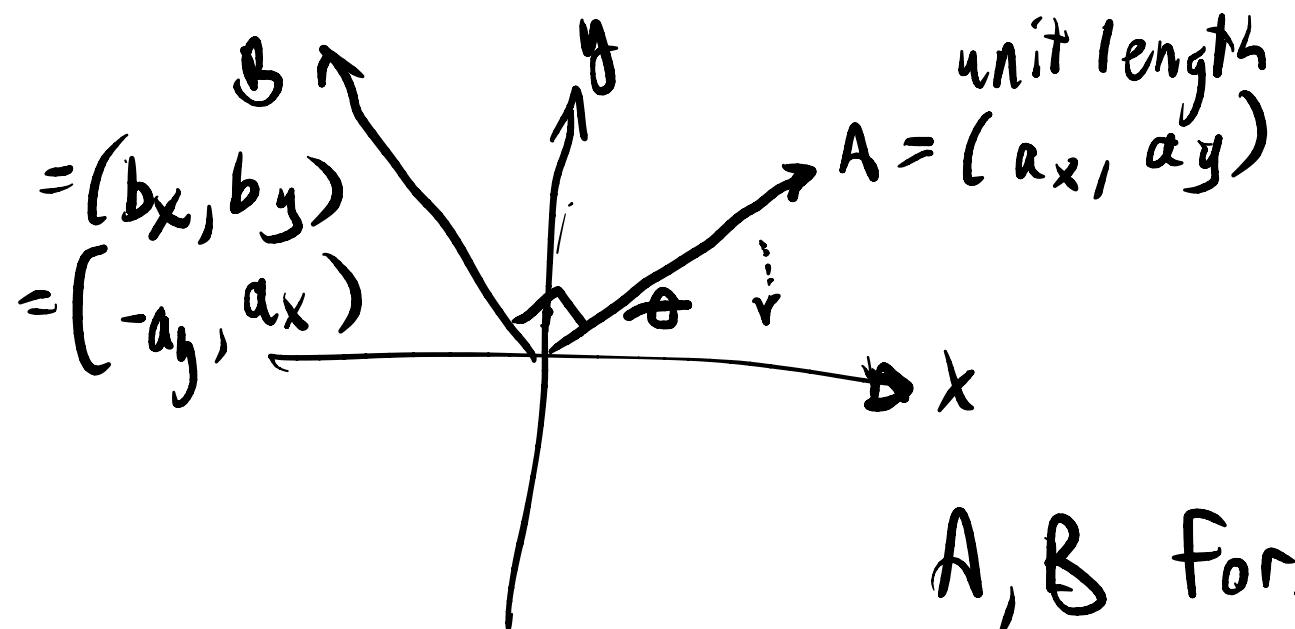
Unit Vector =  $|V|$  length 1

$$\hat{V} = \frac{V}{|V|} = \left( \frac{x}{L}, \frac{y}{L}, \frac{z}{L} \right) \quad L = \sqrt{x^2 + y^2 + z^2}$$

If  $A$  &  $B$  are orthogonal (perpendicular)

$$A \cdot B = 0$$

$$V_1 \times V_2 \Rightarrow V_3$$



want to rotate  $\theta$  to have A lie on x-axis

A, B form orthonormal basis

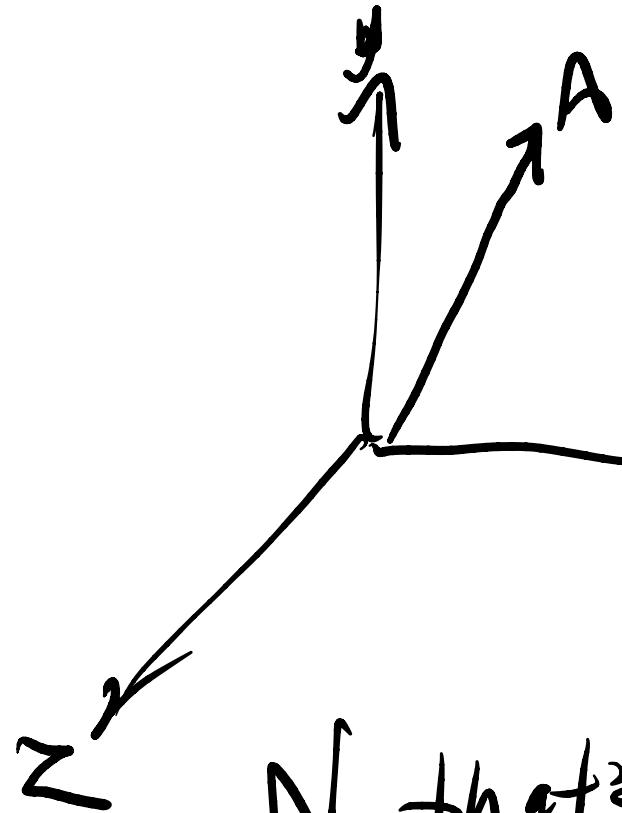
$$R = \begin{bmatrix} a_x & a_y & 0 \\ b_x & b_y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_x & a_y & 0 \\ -a_y & a_x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R \cdot A = R \begin{bmatrix} ax & ay & 0 \\ -ay & ax & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} dx \\ dy \\ 1 \end{bmatrix} \\ \begin{bmatrix} dx^2 + ay^2 + P \\ ax \cdot dy + Q \\ ax \cdot ay + Q \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = R_T \cdot [1, 0, 1]$$

$$R \cdot A \rightarrow [1, 0, 1]$$

$$R \cdot B \rightarrow [0, 1, 1]$$

$$R^{-1} = R^T = \begin{bmatrix} ax & -ay & 0 \\ ay & ax & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$A = (a_x, a_y, a_z)$$

Goal: rotate (theta,  $a_x, a_y, a_z$ )

$N$  that's not parallel to  $A$

$$B = \frac{A \times N}{|A \times N|}$$

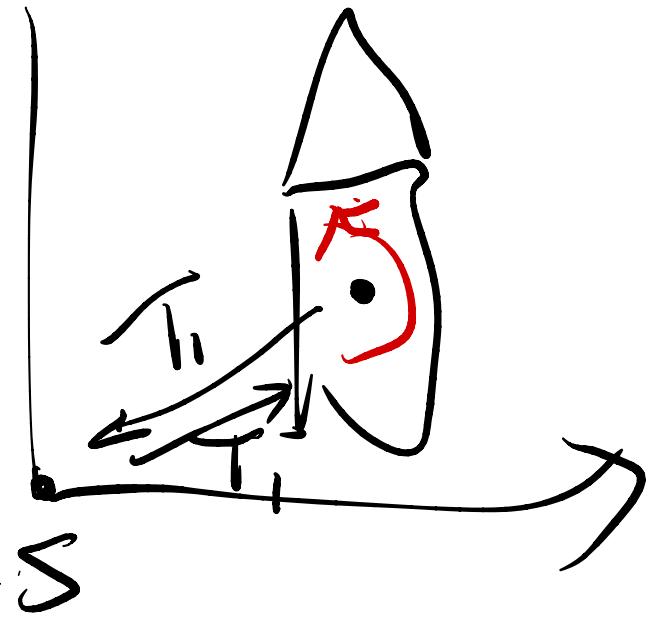
$$C = A \times B$$

$B \perp A$ , unit length

$A, B, C$  form an orthonormal basis in 3D

$$\begin{aligned} R &= R_3 R_2 R_1 \\ &= R_1^{-1} R_2 R_1 \end{aligned}$$

$$R_1 = \begin{bmatrix} a_x & a_y & a_z & 0 \\ b_x & b_y & b_z & 0 \\ c_x & c_y & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$S_{\text{orig}} = T_1^{-1} S T_1 \leftarrow$$

form  $N$  if ( $a_x$  is very small)  
 $N = (1, 0, 0)$

$R_2 = \text{rotateX}(\theta)$

else  
 $N = (0, 1, 0)$

$R_3 = R_i^{-1}$  ← matrix inversion

For rotation Matrices

$R_i^{-1} = R_i^T$

$R = R_3 R_2 R_1$

$\text{rotate}(\theta, t)$

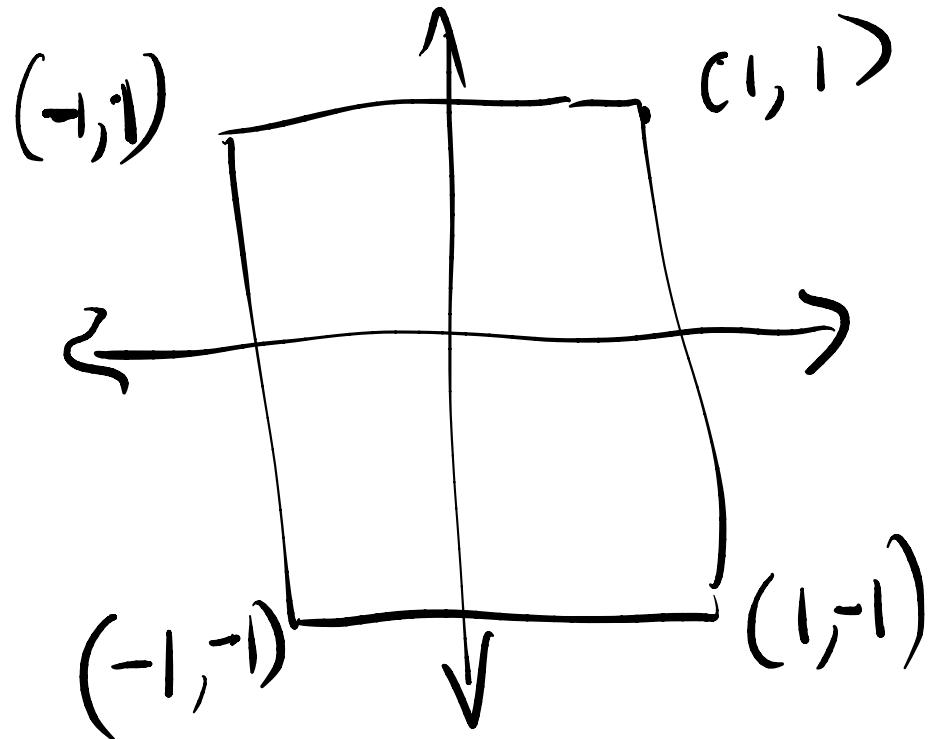
$$Q = (s, x, y, z)$$

interpolation problem as well  
matrices  $\Rightarrow$  difficult because we can't  
interpolate

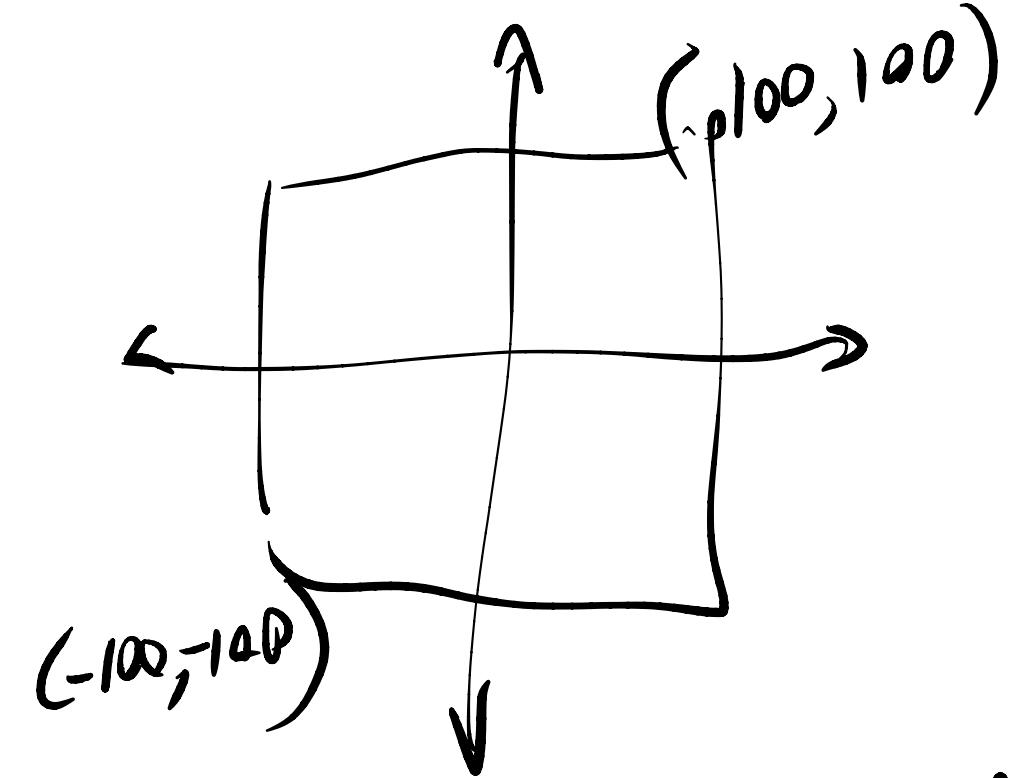
$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix} \dots \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

at 0.5      if I do  
pointwise  
interp.





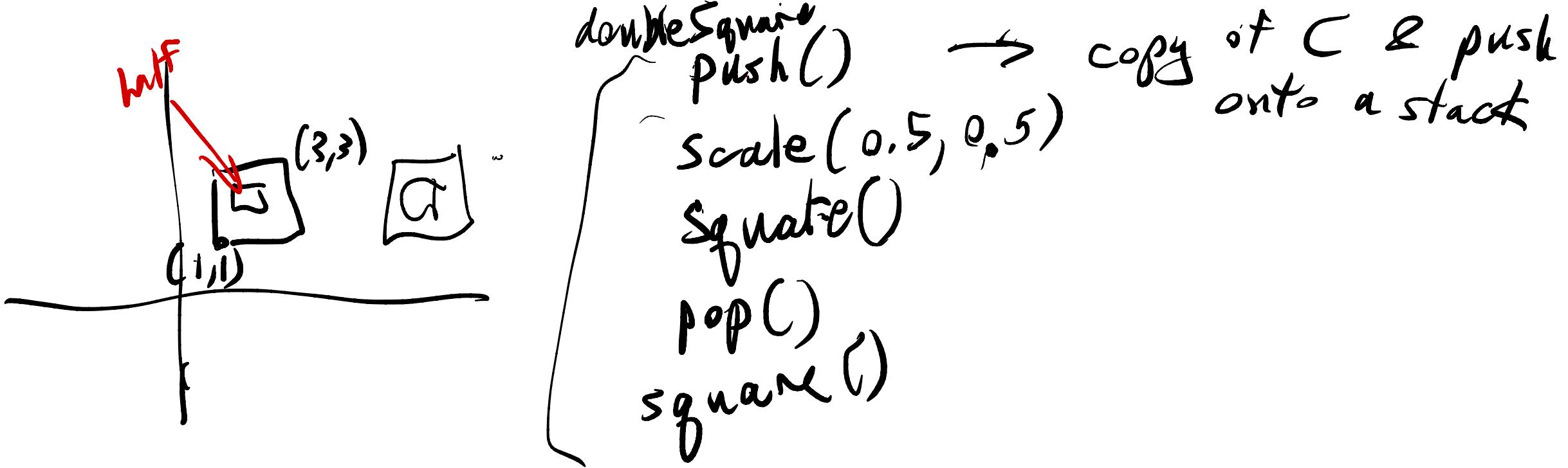
function Square () {  
begin Shape();  
startPoint( 1, 1 )  
lineTo( -1, 1 )  
lineTo( -1, -1 )  
lineTo( 1, -1 )  
end Shape()  
}



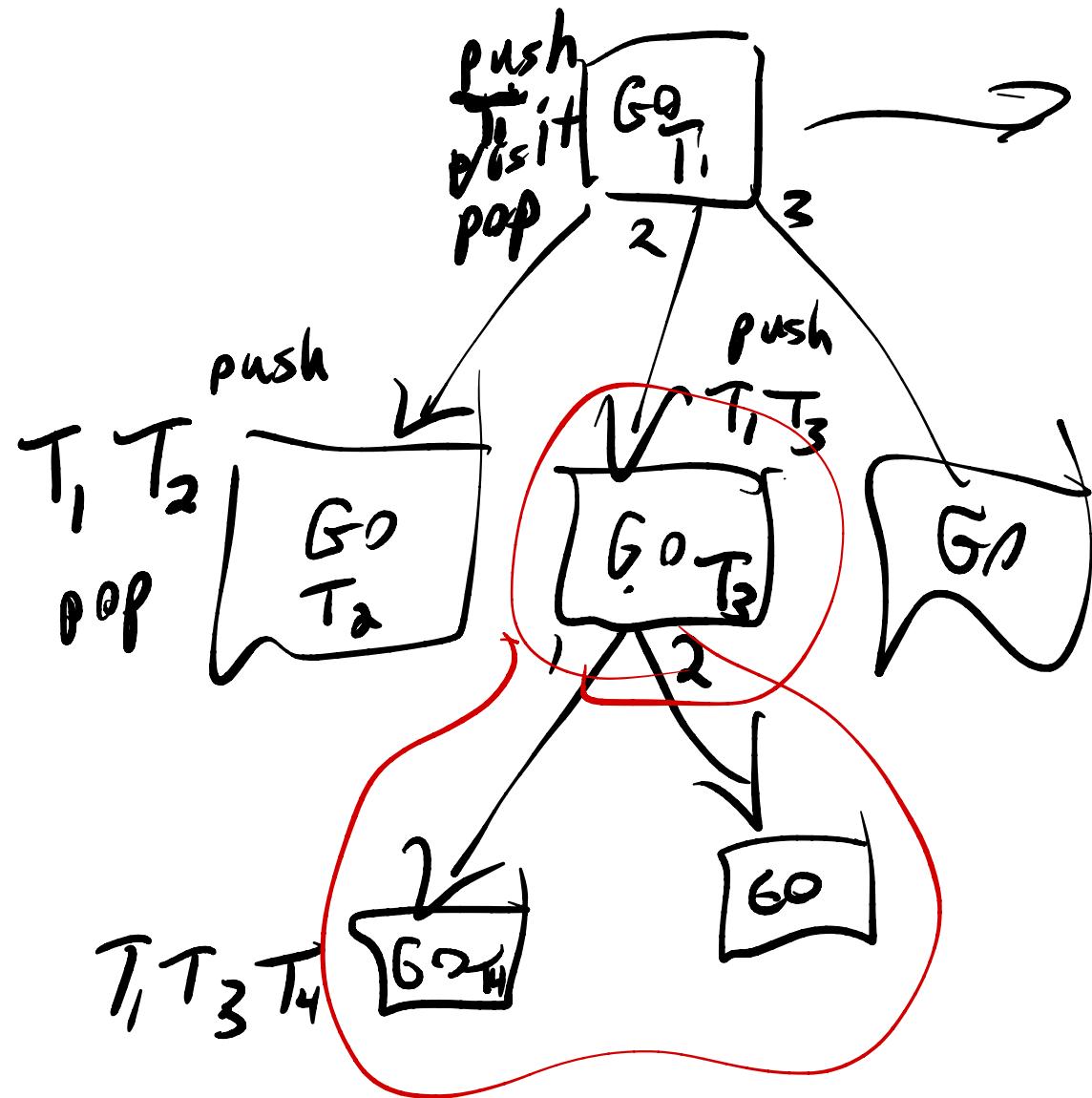
```
init()    // C=I  
scale(100,100) // C=I · S  
Square()
```

$C$  = current transform

All vertices  $(x, y)$  are transformed by  $C$   $P' = C \cdot P$



push  
translate(2, 2)  
double square  
pop



$GO$ 's,  $Object3D$ , ...  
 base object  
 transformation  
 name, other properties, ...  
 children

# Two kinds of Graphics Libraries

→ Immediate Mode

→ Canvas, WebGL, OpenGL

"the commands I issue are executed immediately"

→ Retained Mode

→ Three.js, Unity, ...

Build a graph

The system renders

With a SG.

Limited to a "graph"

