

# 6 – lines

Parametric and Implicit forms of Lines

Rasterizing Lines

*And Alb if time!*

# Parametric

$$0 \leq t \leq 1$$

$$t=0$$

$$Q(t)$$

$$t=1 \rightarrow t > 1$$
$$P_2 = (x_2, y_2)$$

$$t < 0$$
$$P_1 = (x_1, y_1)$$

$$x(t) = x_1 + t(x_2 - x_1) = x_1 + t * dx$$
$$y(t) = y_1 + t(y_2 - y_1) = y_1 + t * dy$$

$|V| = 1$

$t = \text{distance}$

$$\vdots$$
$$Q(t) = P_1 + t(P_2 - P_1) = P_1 + t(V)$$

$\rightarrow$  vector of  
the dis

# Implicit Line Equation

define function  $f(x, y)$

on line  $\rightarrow f(x, y) = 0$

off line  $\rightarrow f(x, y) \neq 0$

$$y = mx + b$$

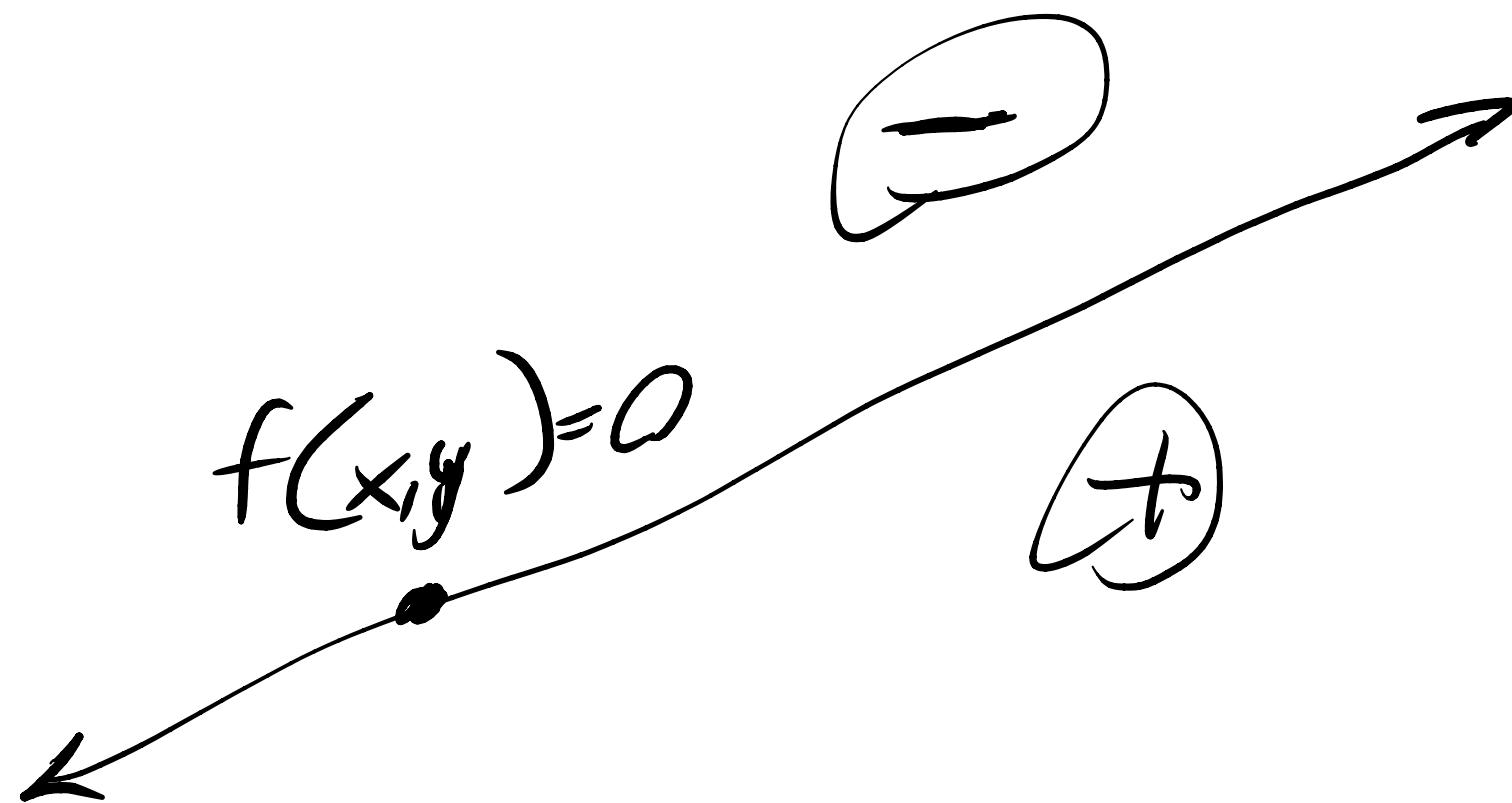
$$f(x, y) = mx + b - y$$

$$f(x, y) = ax + by + c = 0$$

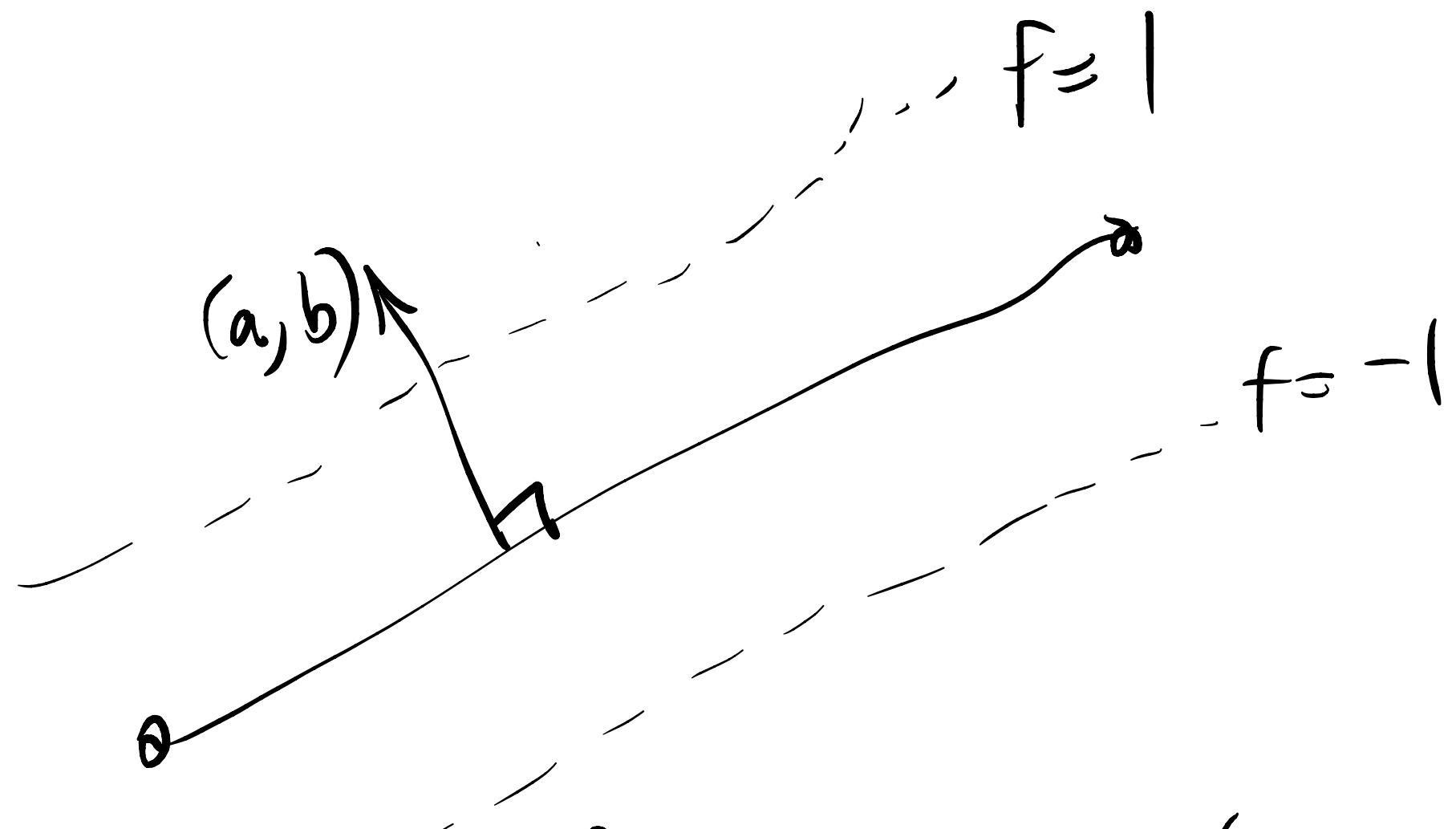
for some  $a, b, c$

$$\text{for some } \lambda \quad \lambda ax + \lambda by + \lambda c = 0$$

$a, b, c$  are coefficients of a particular line

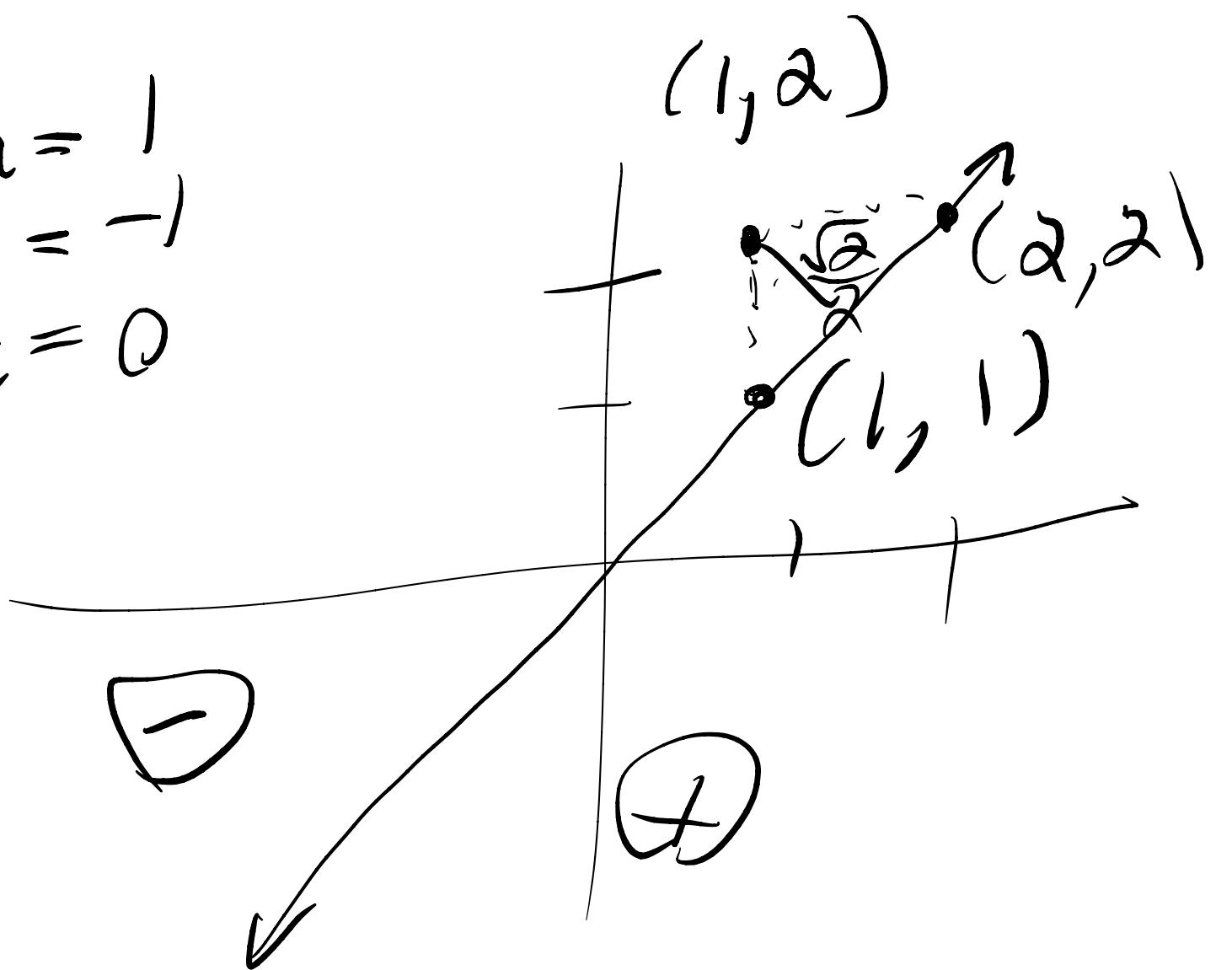


If  $a^2 + b^2 = 1$  then  $f(x, y)$  gives the distance to line



$c$  is distance of the line to origin

$$\begin{aligned}a &= 1 \\b &= -1 \\c &= 0\end{aligned}$$

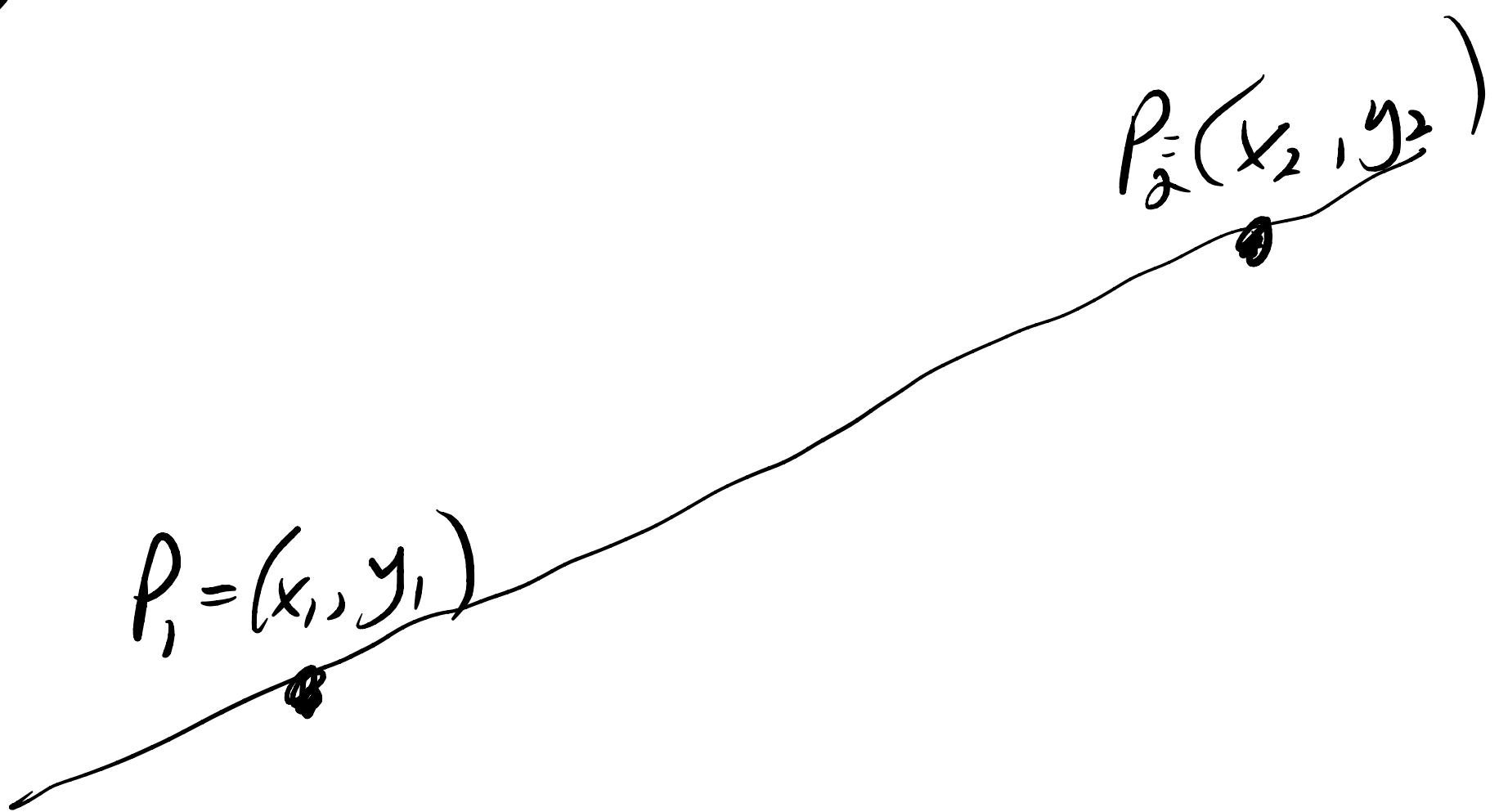


$$\begin{aligned}f(1, 2) &= 1 - 2 + 0 \\&= -1\end{aligned}$$

$$\frac{\sqrt{2}}{2} \quad 1.41 \approx .707$$

Math (2.5.)  
discusses  
implicit lines

find  $a, b, c$ ?



$$f(x, y) = ax + by + c = 0$$

$$\begin{aligned} ax_1 + by_1 + c &= 0 \\ ax_2 + by_2 + c &= 0 \end{aligned} \quad \begin{matrix} \uparrow \\ \text{subtract} \end{matrix}$$

$$a(x_1 - x_2) + b(y_1 - y_2) = 0$$

(negative  $b$ )

$$a = b \frac{(y_1 - y_2)}{(x_1 - x_2)}$$

let  $b = 1$

solve for  $a$   
solve for  $c$

## Rasterization

convert shapes to pixels

function line ( $x_0, y_0, x_1, y_1$ ) {

let  $dx = x_1 - x_0$  // assuming  $x_1 > x_0$

let  $dy = y_1 - y_0$  // ←

let length = max (  $dx, dy$  )

let  $x_{inc} = dx / length$

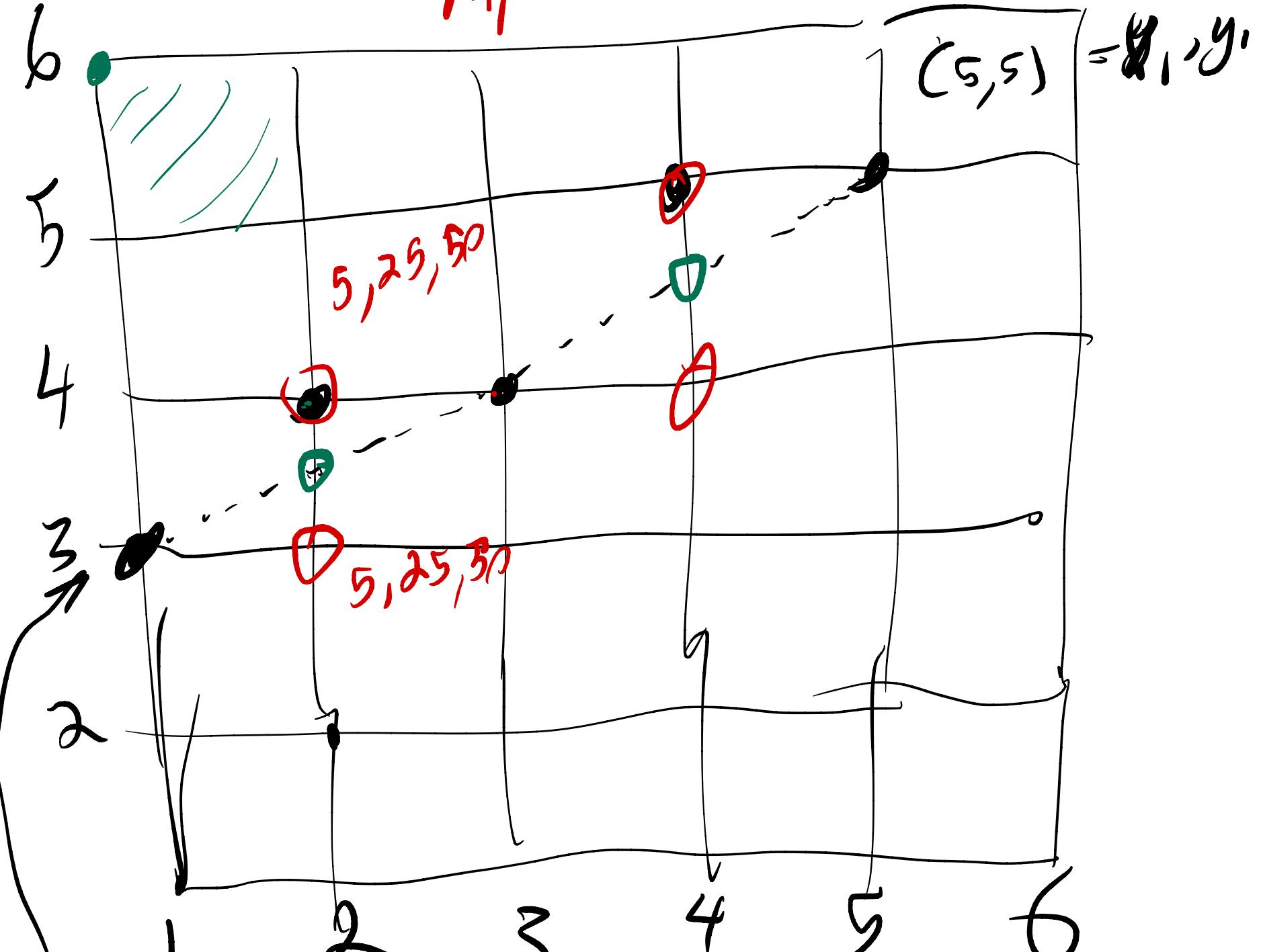
let  $y_{inc} = dy / length$

let  $x = x_0$

let  $y = y_0$

```
for (let i = 0; i < length; i++) {  
    drawPixel (round(x), round(y), 10, 50, 100)  
    x += xinc  
    y += yinc  
}
```

assumption: black background



$$(1,3) = (x_0, y_0)$$

"aliasing"

$$dx = 5 - 1 = 4$$

$$dy = 5 - 3 = 2$$

length = 4

$$x_{inc} = 1$$

$$y_{inc} = 0.5$$

$$x = 1, \quad y = 3$$

