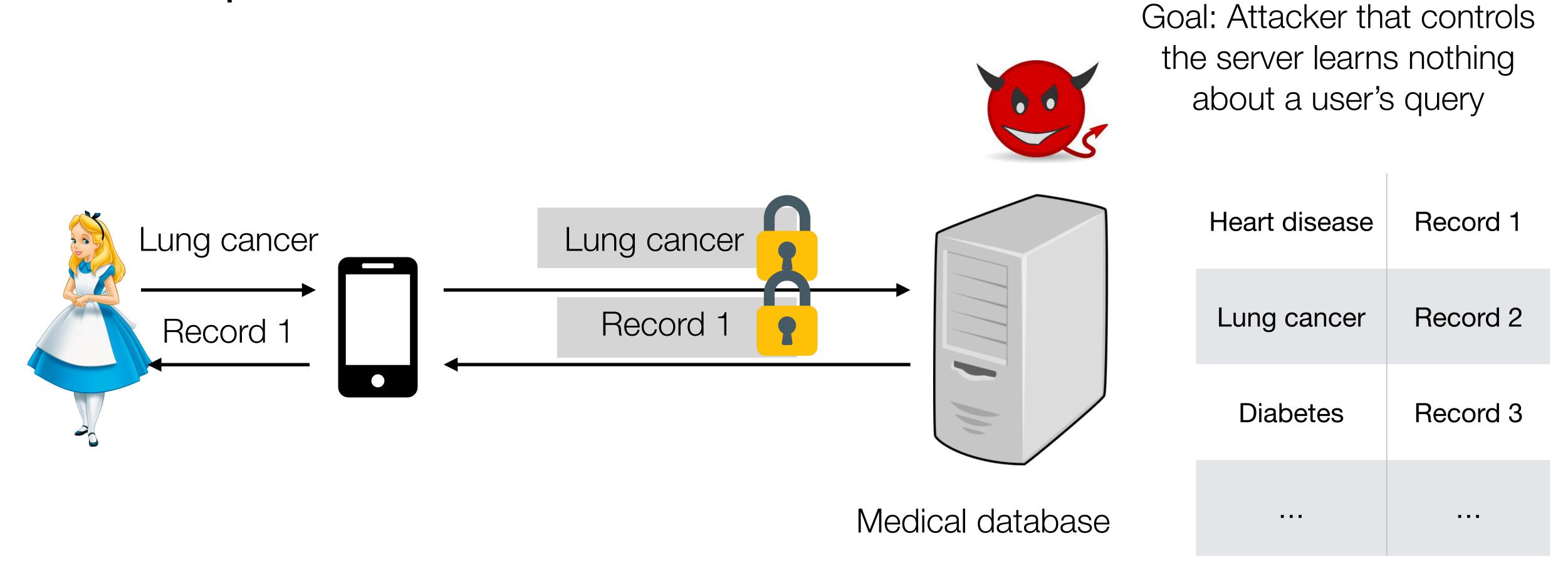
CS 350S: Privacy-Preserving Systems

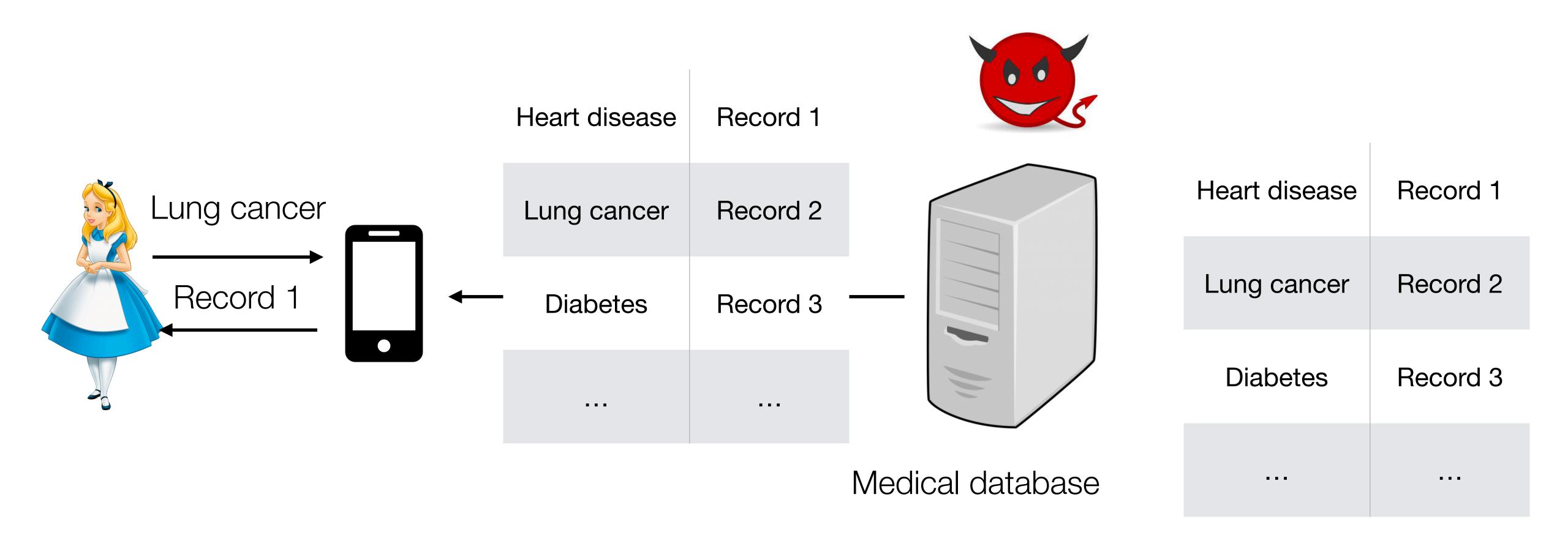
Private Information Retrieval II

Recap: Private information retrieval



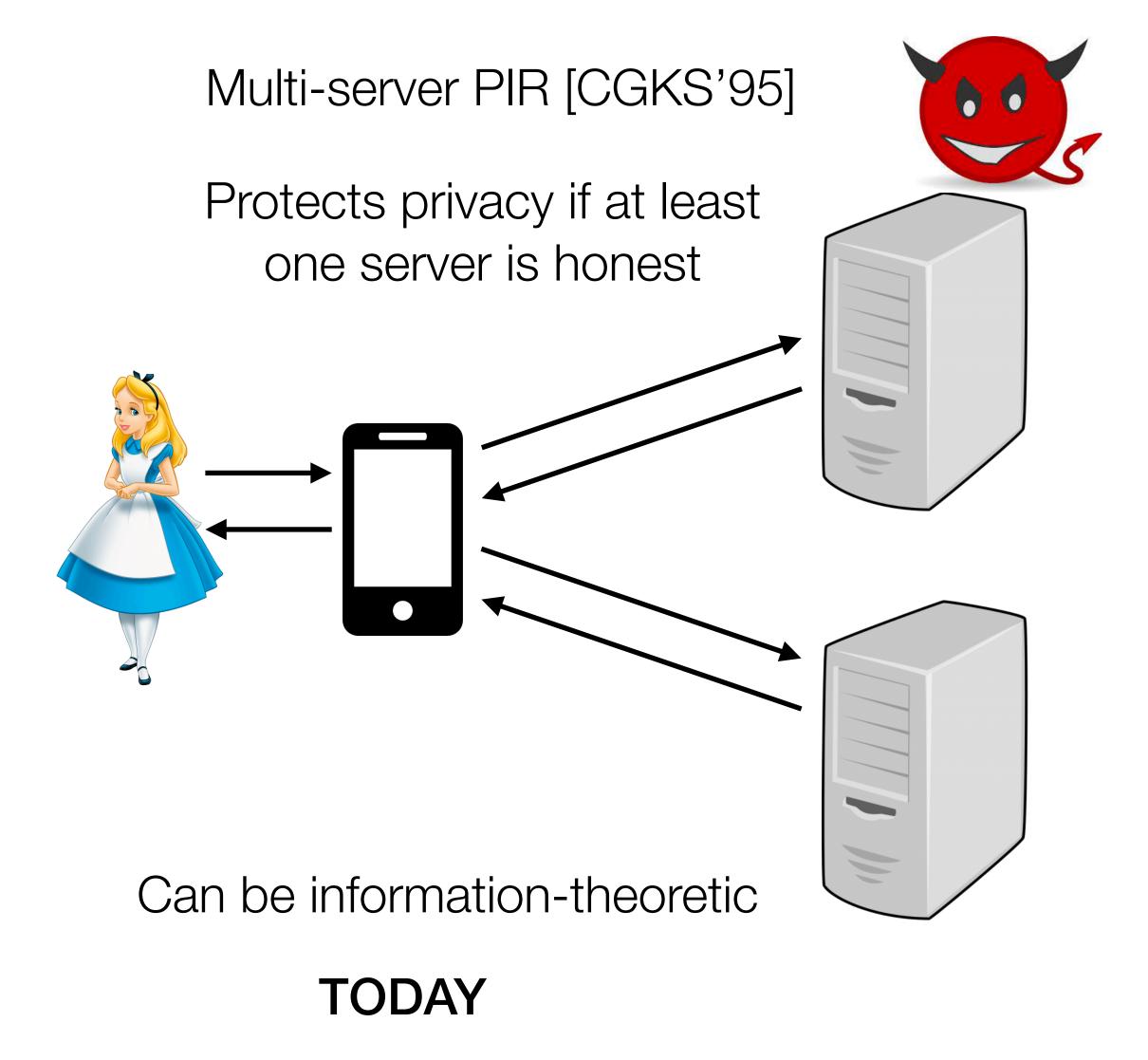
Can also use to fetch articles, images, podcasts, movies, etc. from a remote server

Trivial solution: download the whole database



Want to privately query a database with communication sublinear in database size

Two models



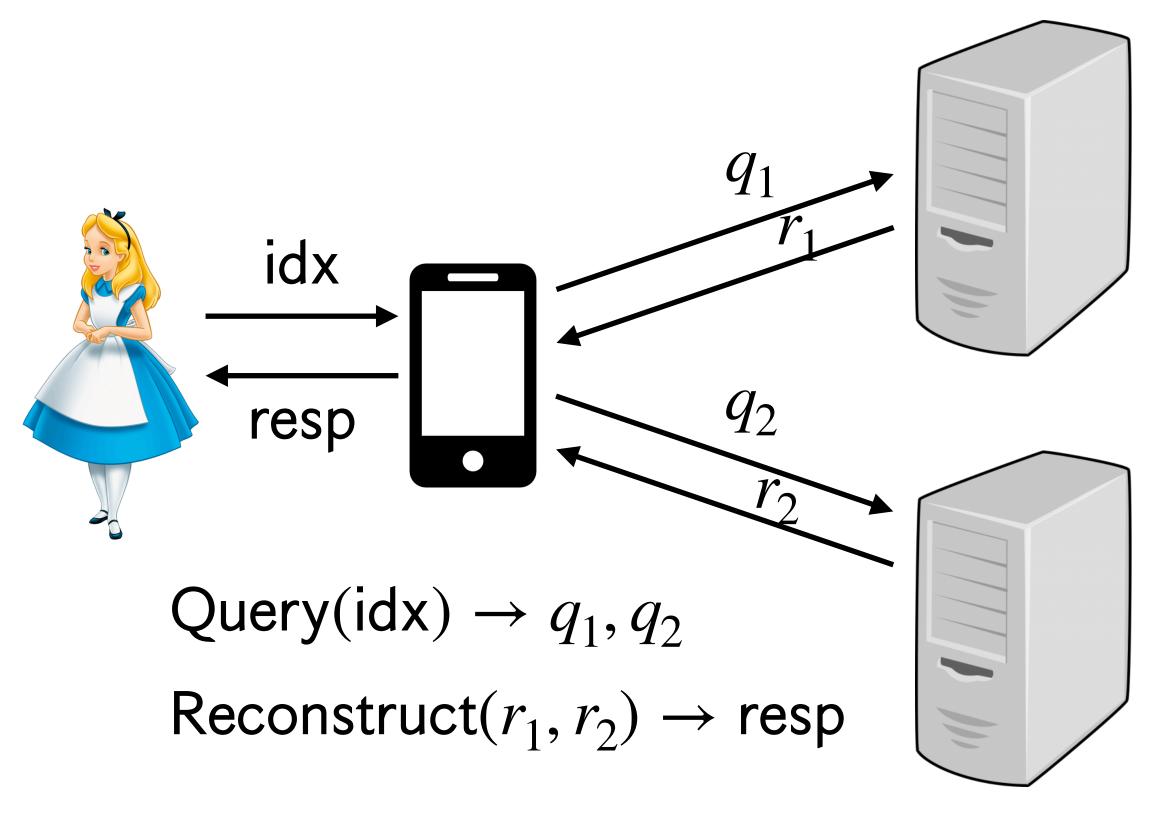
Single-server PIR [KO'97]

Protects privacy if the server is compromised

Requires a cryptographic assumption

Two-server PIR

Answer(db, q_1) $\rightarrow r_1$



For database with n elements, query space Q, values in $\{0,1\}$, and response space R

Query: $[n] \rightarrow Q^2$

Answer: $\{0,1\}^n \times Q \rightarrow R$

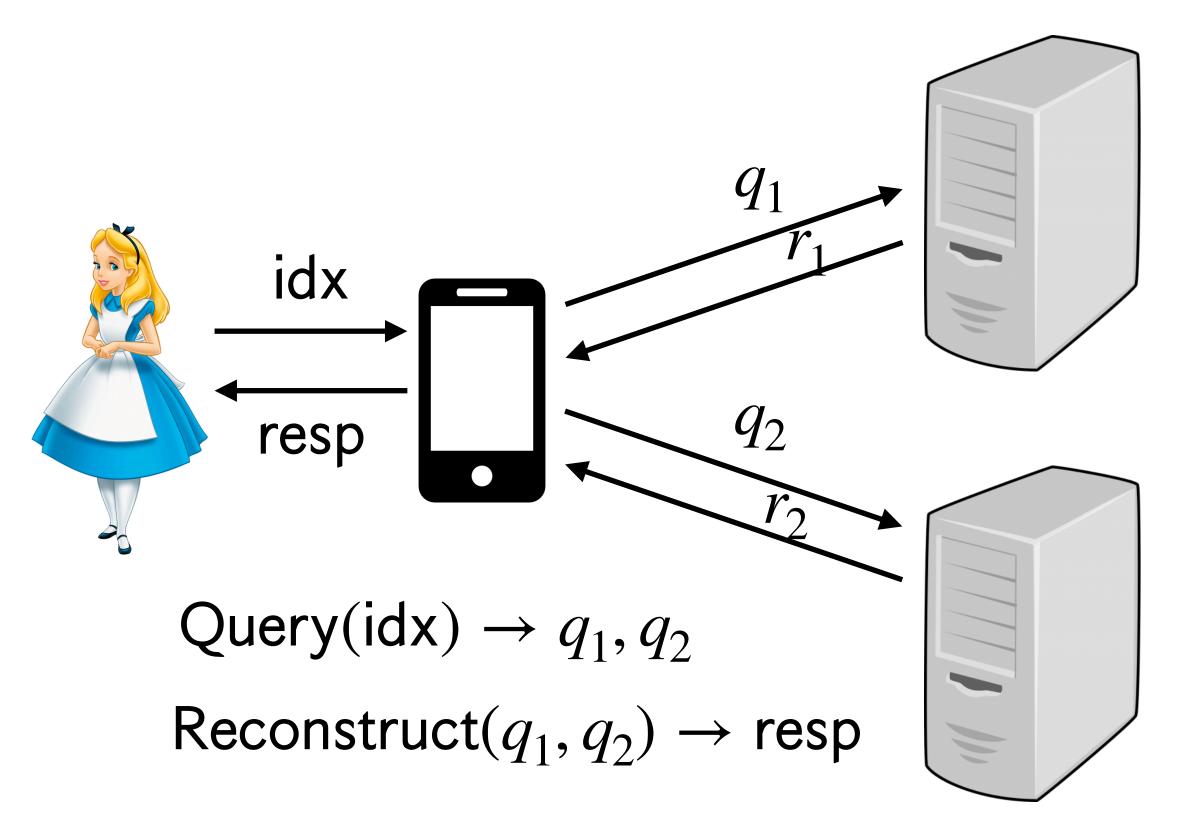
Reconstruct : $R^2 \rightarrow \{0,1\}$

Note: Query is randomized and takes security parameter as implicit input

Answer(db,
$$q_2$$
) $\rightarrow r_2$

Two-server PIR definitions

Answer(db, q_1) $\rightarrow r_1$



Correctness: For all $n \in \mathbb{N}$, $db \in \{0,1\}^n$, $idx \in [n]$, the probability that:

Query(idx) $\rightarrow q_1, q_2$ Answer(db, q_i) $\rightarrow r_i$ for $i \in \{1, 2\}$ Reconstruct(q_1, q_2) = db_{idx}

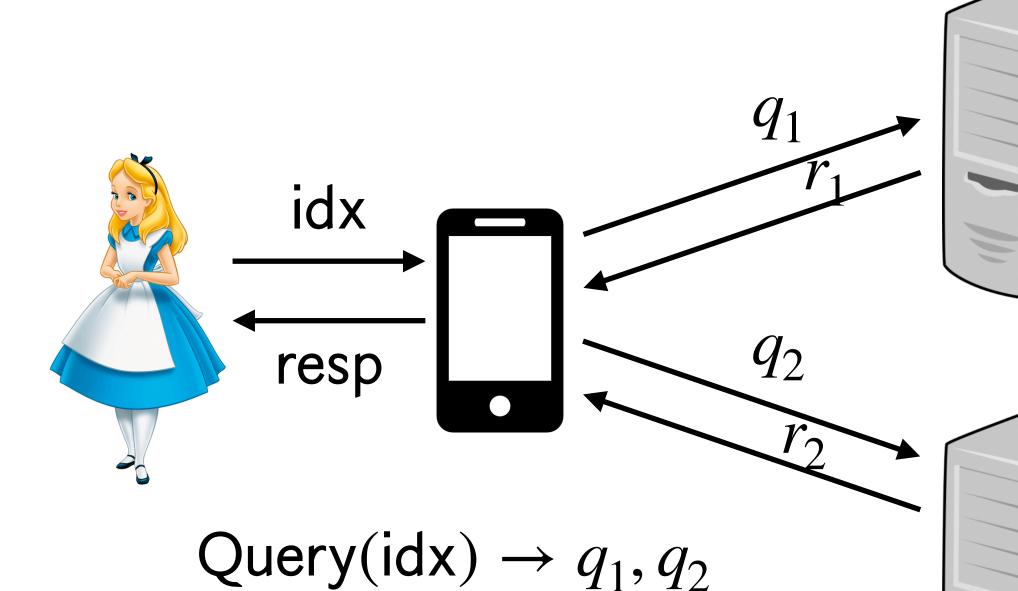
is 1.

Informally, the client gets the requested record

Answer(db, q_2) $\rightarrow r_2$

Two-server PIR definitions

Answer(db, q_1) $\rightarrow r_1$



 $Reconstruct(q_1, q_2) \rightarrow resp$

Security: For all $i, j \in [n], b \in \{1, 2\}$

 $\{q_b: \mathsf{Query}(i) \to q_1, q_2\} \approx \{q_b': \mathsf{Query}(j) \to q_1', q_2'\}$

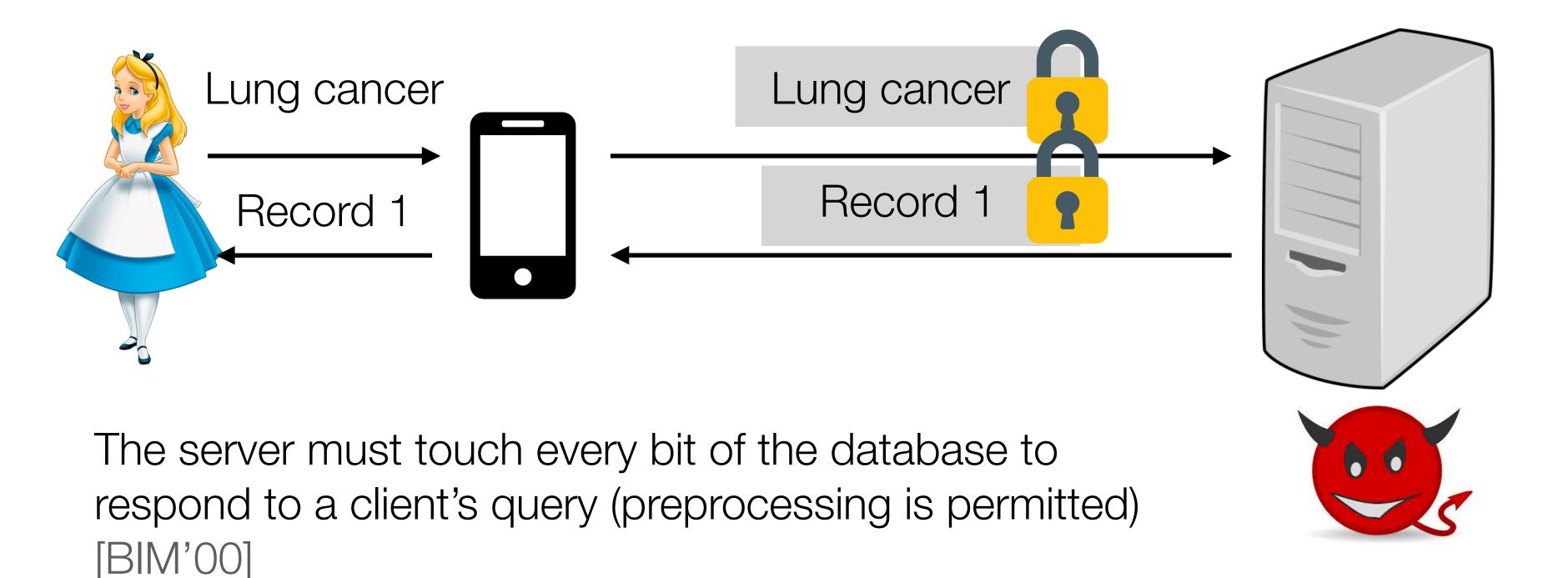
Informally, no one server can learn the client's query

Answer(db, q_2) $\rightarrow r_2$

What we can hope for with PIR

Is it possible to build a PIR scheme that only touches part of the database?

No! If query execution only touches part of the database, then an attacker can learn which part of the database is *not* accessed by the query



Heart disease	Record 1
Lung cancer	Record 2
Diabetes	Record 3

Outline

- 1. Function secret sharing
- 2. Splinter
- 3. Searching on encrypted data

Recap: secret sharing

For some group \mathbb{G} , split a value $x \in \mathbb{G}$ into secret shares $x_1, ..., x_k \in \mathbb{G}$ such that $\sum_{i=1}^{k} = x_i$

Information-theoretic privacy: Given just $[x]_b$ for $b \in [k]$, adversary learns no information about x

Operations:

- Given x, generate secret shares by randomly sampling x_1, \ldots, x_{k-1} and setting $x_k = x \sum_{k=1}^{\kappa-1} x_k$
- Given x_1, \dots, x_k , reconstruct x by computing $x = \sum_{i=1}^k x_i$

Computing on secret shares:

- Can add secret shares: [x] + [y] = [x + y]
- Can multiply by a constant: $c \cdot [x] = [c \cdot x]$ (by extension)

Function secret sharing (FSS)

[Boyle, Gilboa, Ishai]

Idea: Secret share a value rather than a function

Informal properties for n-party function secret sharing:

- Split function f into functions f_1, \ldots, f_n where $f(x) = \sum_{i=1}^n f_i(x)$
- Describe f_i using a short key K_i
- Key K_i reveals nothing about f

Function secret sharing

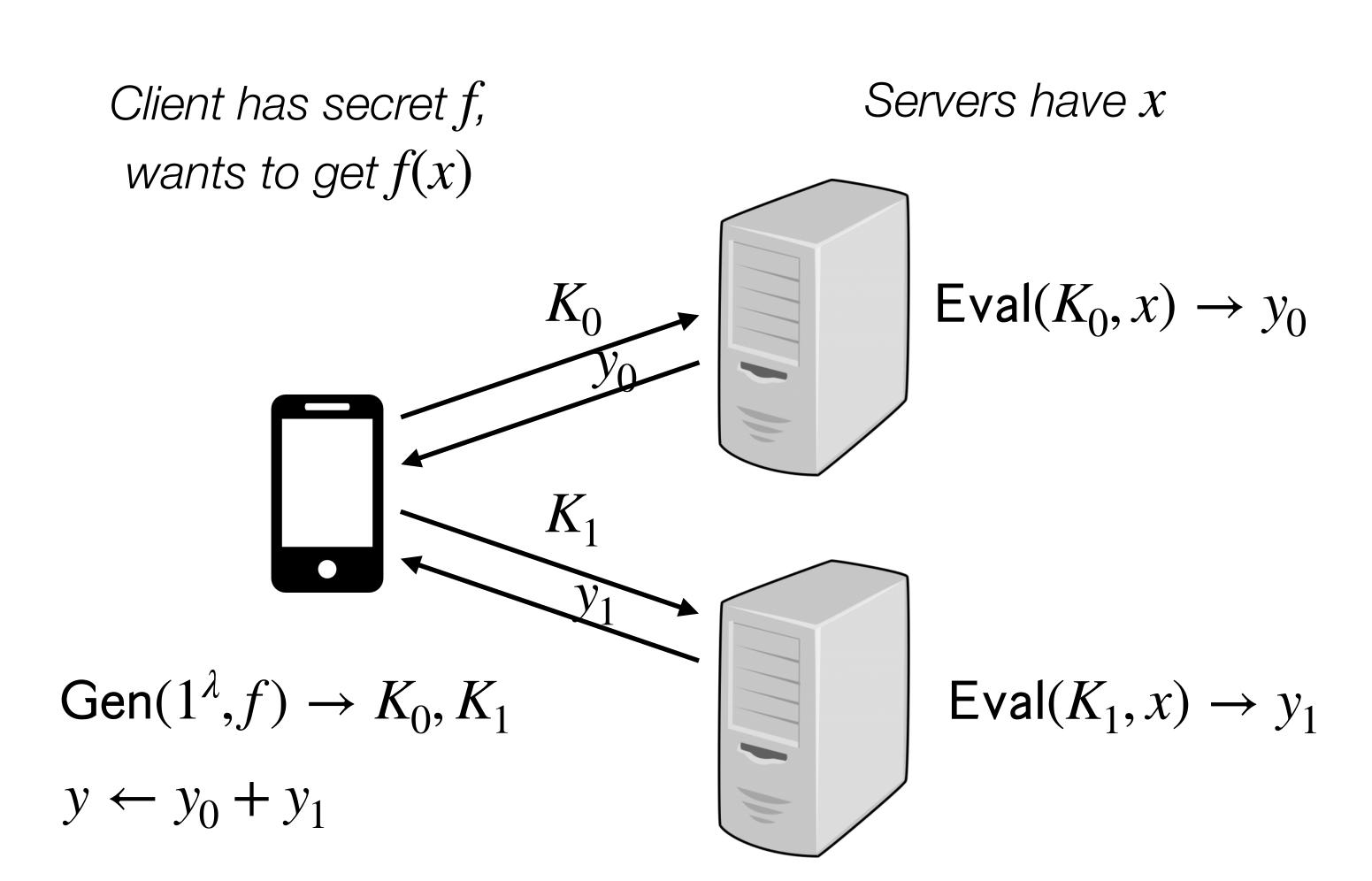
[Boyle, Gilboa, Ishai]

Security parameter λ , function f

Considering just two parties

Two algorithms:

- $\operatorname{Gen}(1^{\lambda}, f) \to K_0, K_1$
- $Eval(K_i, x) \rightarrow y_i$



FSS definitions

FSS is defined over a function class ${\mathscr F}$ with security parameter λ has the following syntax:

- $\mathrm{Gen}(1^\lambda,f)\to K_0,K_1$: On input security parameter and $f\in\mathcal{F}$, output keys K_0,K_1
- $\text{Eval}(K_i, x) \to y_i$: On input key K_i (output from Gen) and input string $x \in \mathcal{D}_f$, where \mathcal{D}_f is the input domain of function f, output a share of f(x)

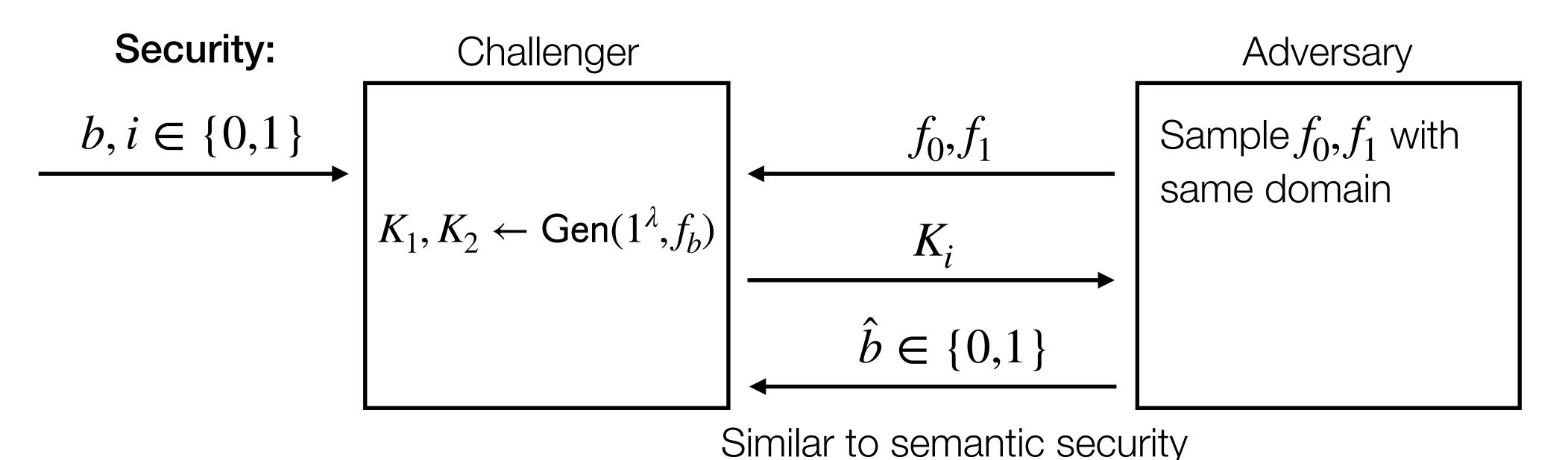
Correctness: For all $f \in \mathcal{F}, x \in \mathcal{D}_f$,

$$\Pr[K_1, K_2 \leftarrow \text{Gen}(1^{\lambda}, f) : \text{Eval}(K_0, x) + \text{Eval}(K_1, x) = f(x)] = 1$$

FSS definitions

FSS is defined over a function class ${\mathcal F}$ with security parameter λ has the following syntax:

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Functions with efficient FSS constructions

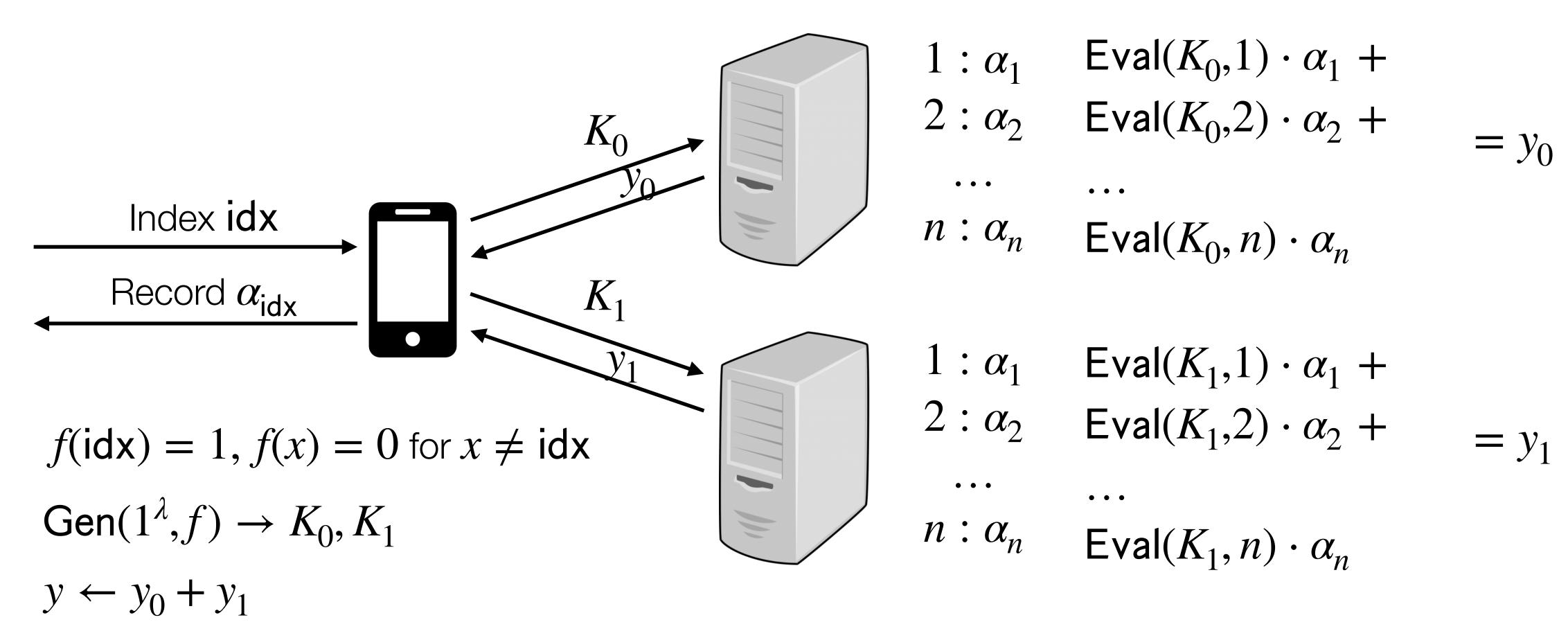
Point functions: For a, b, the corresponding point function f is defined as f(a) = b and, for all $x \neq a, f(x) = 0$

- Corresponding FSS construction: distributed point function (DPF)

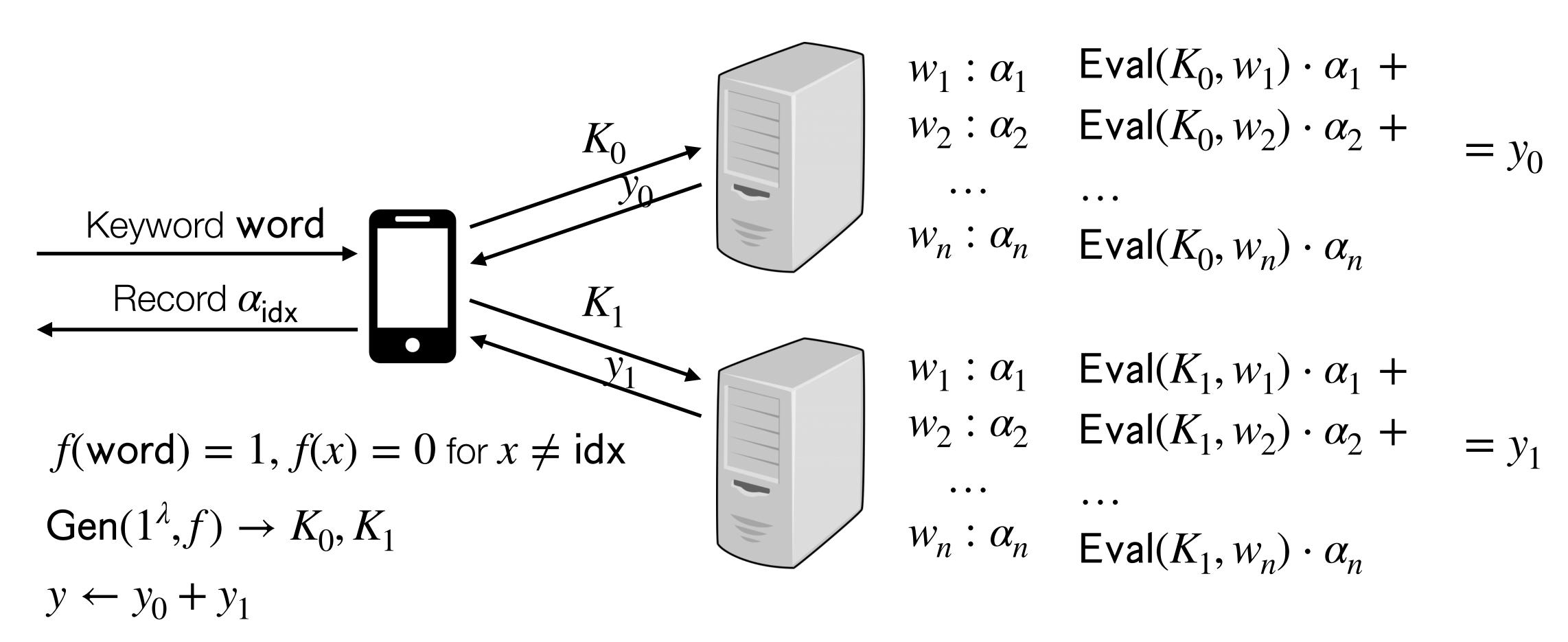
Comparison functions: For a, b, the corresponding comparison function f is defined as f(x) = b for x < a and f(x) = 0 for $x \ge a$

- Corresponding FSS construction: distributed comparison function (DCF)
- Can generalize to > and intervals

PIR from distributed point functions



PIR by keywords from DPFs



Costs for DPFs and DCFs

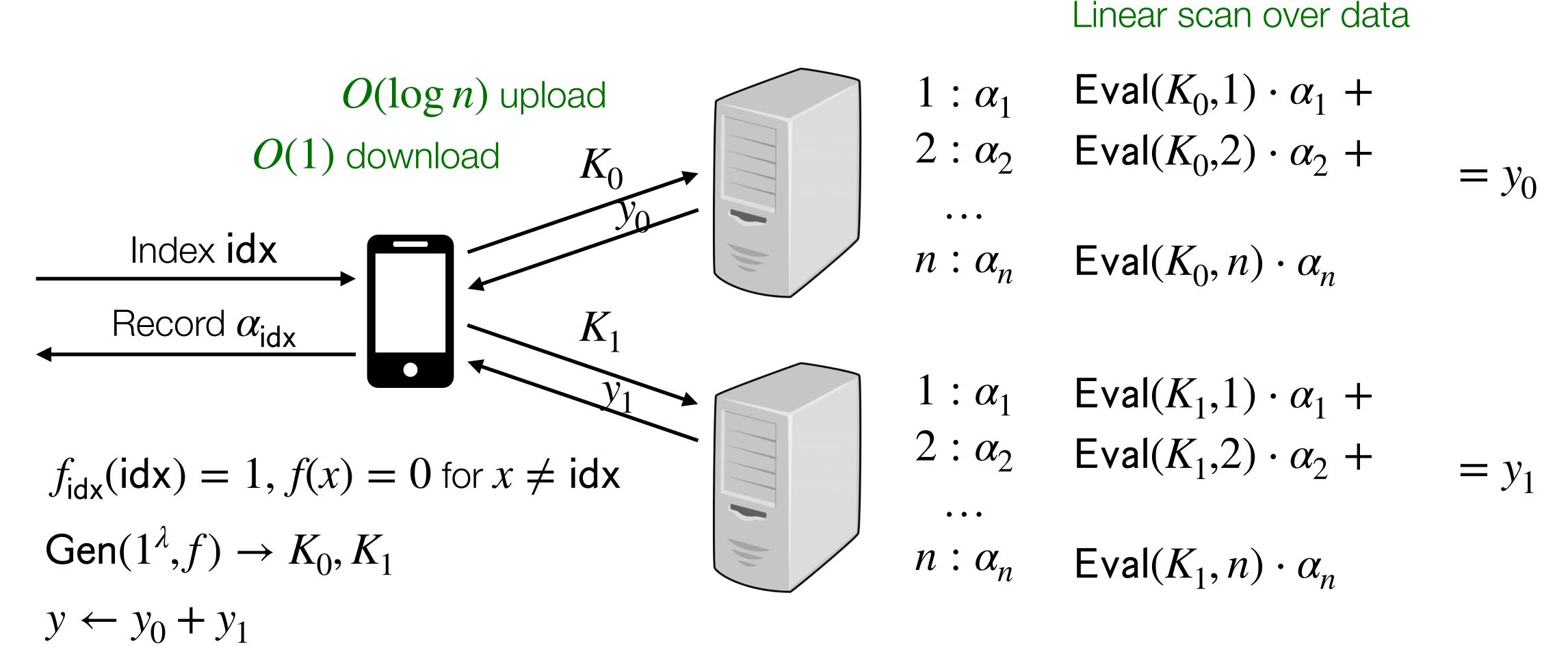
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For $\mathcal{D} = \{0,1\}^m$, key size is O(m)

Only AES operations

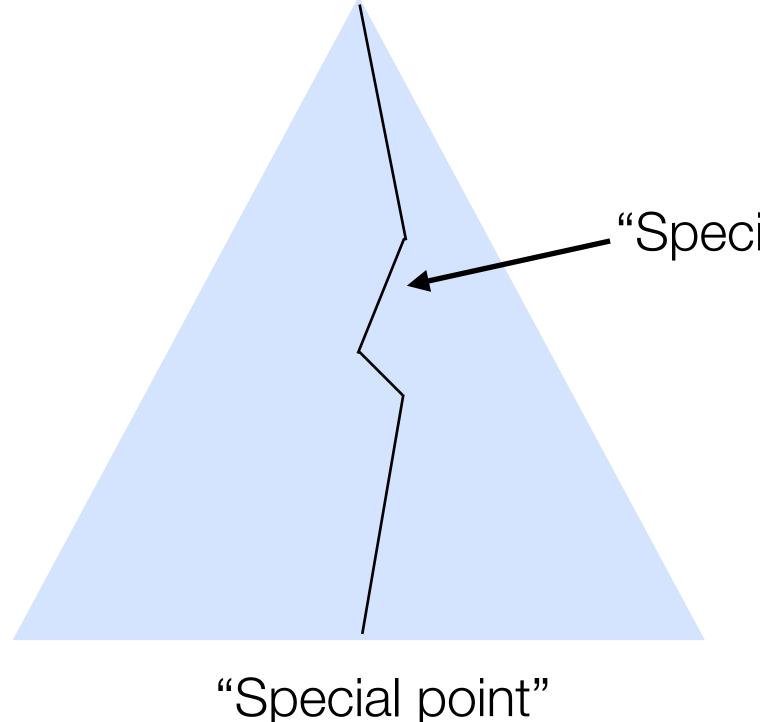
Costs for PIR from DPFs



DPF construction intuition

Observation: If K_0 outputs y_0 , K_1 outputs y_1 , and $y_0 = y_1$, then if we assemble shares as $y_0 - y_1$, (y_0, y_1) are shares of 0.

Construction idea: Each server can use its key to construct a tree that is identical to the other server's tree (secret shares of 0) *except* along a path from the root to the non-zero ("special") point.



"Special path"

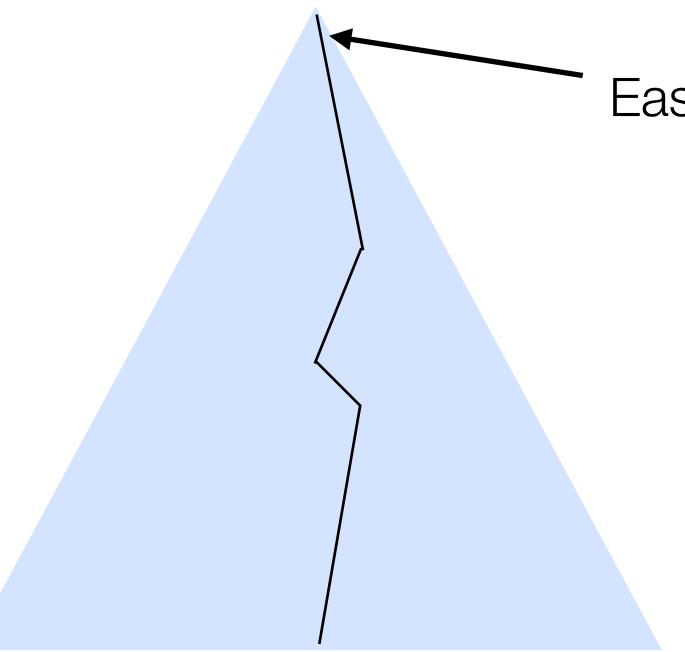
Each node is associated with a seed and a control bit

Invariant: Seed and control bit are identical across the two evaluations, except along the special path

DPF construction intuition

Construction idea: Each server can use its key to construct a tree that is identical to the other server's tree (secret shares of 0) *except* along a path from the root to the non-zero ("special") point.

Invariant: Seed and control bit are identical across the two evaluations, except along the special path



Easy to maintain invariant at root (always on special path)

What about when we leave the special path?

Idea: use a "correction word" to set the node values to the same value when leaving the special path

- $[s] \oplus [t] \cdot CW$ where s is the seed, t is the control bit, and CW is the correction word — correction conditioned on t

Applications of FSS beyond PIR

Privately writing to a shared database

- Want to hide which element the client is writing to
- Applications to metadata-hiding messaging (see later in class) [Riposte]

Private aggregate statistics

- Privately collect histograms
- Compute most popular set of strings without revealing all strings [Poplar]

Multiparty computation

- Execute some computation across parties, but without each party revealing its inputs to the other parties [BGI19]

Outline

- 1. Function secret sharing
- 2. Splinter
- 3. Searching on encrypted data

Splinter

[Wang, Yun, Goldwasser, Vaikuntanathan, Zaharia]

Gap between PIR and what applications need:

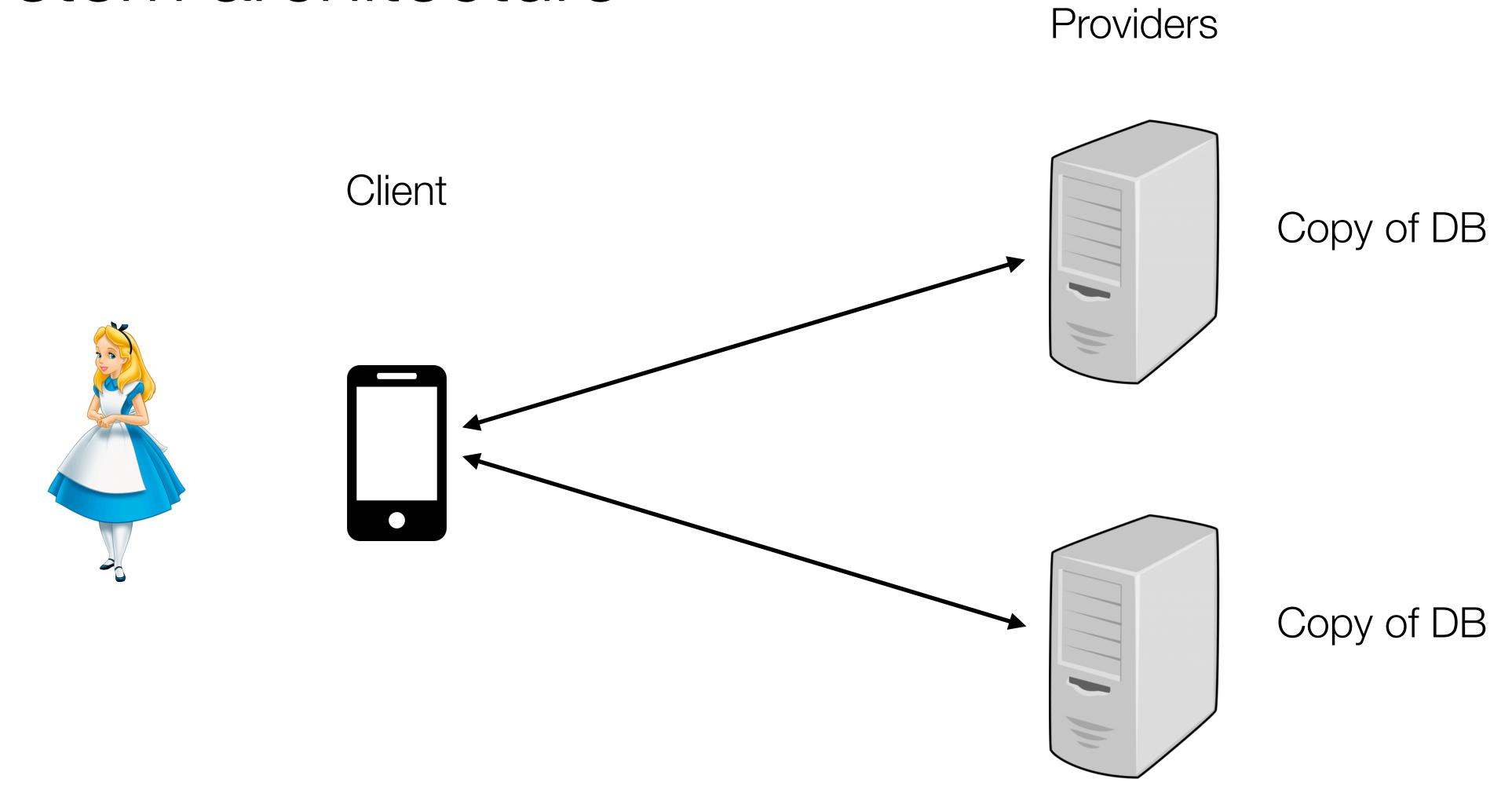
- Private information retrieval: Privately look up a record in a database by some index
- Applications need: Private SQL-like queries based on filters

Splinter helps to bridge this gap

SELECT TOP 10 flights from flights WHERE source = SFO AND dest = EWR ORDER BY price

SELECT AVG(price) WHERE month=3
AND origin=SFO AND dest=EWR

System architecture



Security properties

Providers Client Copy of DB Copy of DB Goal: protect user's query as long as one server is honest

Semihonest adversary

Security properties

Splinter hides some query parameters, but not the query structure

Column names, aggregation function, and other features of query structure are not hidden from the server (only the "?" are hidden)

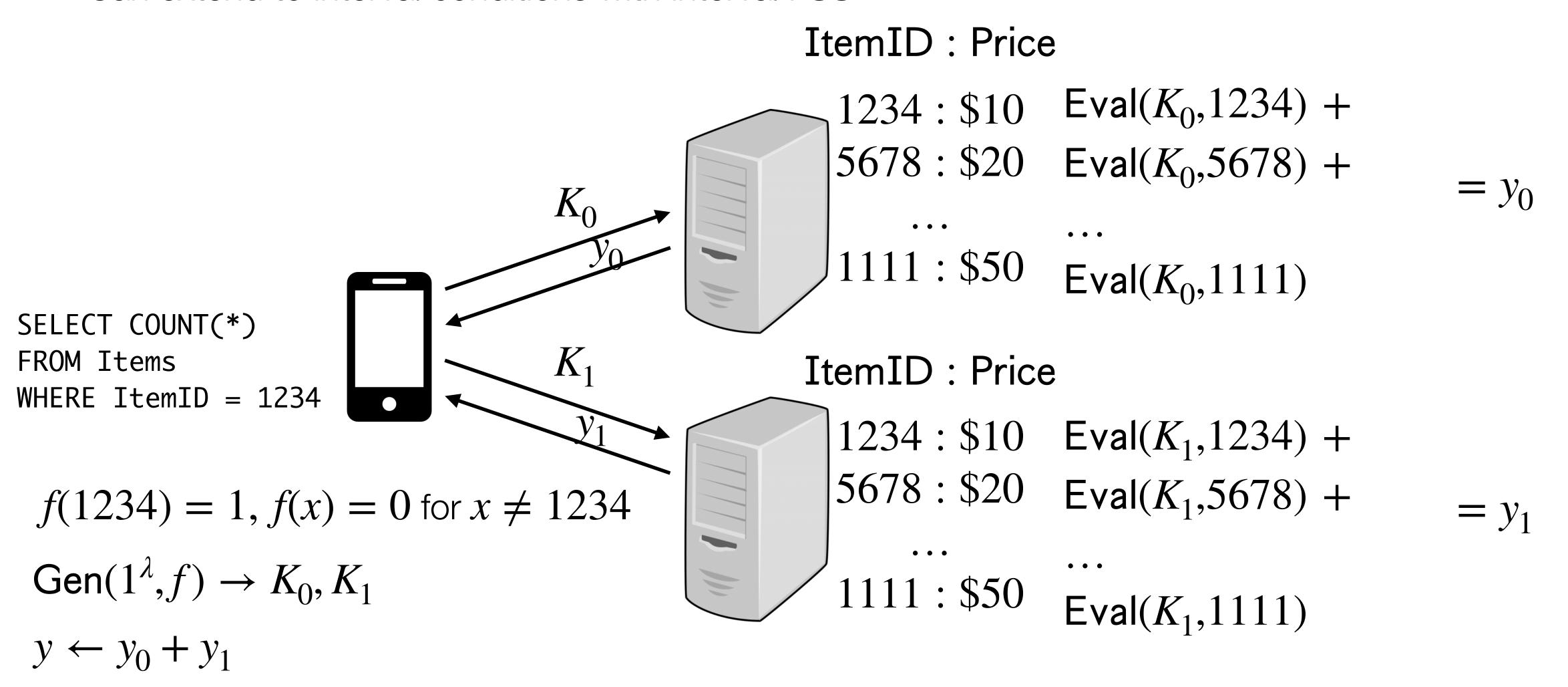
```
SELECT TOP 10 flights from flights
WHERE source = ? AND dest = ?
ORDER BY price
```

Queries supported

```
Query format:
     SELECT aggregate<sub>1</sub>, aggregate<sub>2</sub>, ...
     FROM table
     WHERE condition
     [GROUP BY expr_1, expr_2, ...]
aggregate:
      • COUNT | SUM | AVG | STDEV (expr)
      • MAX | MIN (expr)
      • TOPK (expr, k, sort_expr)
      • HISTOGRAM (expr, bins)
condition:
      \bullet expr = secret
      • secret_1 \leq expr \leq secret_2
      • AND of '=' conditions and up to one interval
      • OR of multiple disjoint conditions
        (e.g., country="UK" OR country="USA")
expr: any public function of the fields in a table row
     (e.g., ItemId + 1 or Price * Tax)
```

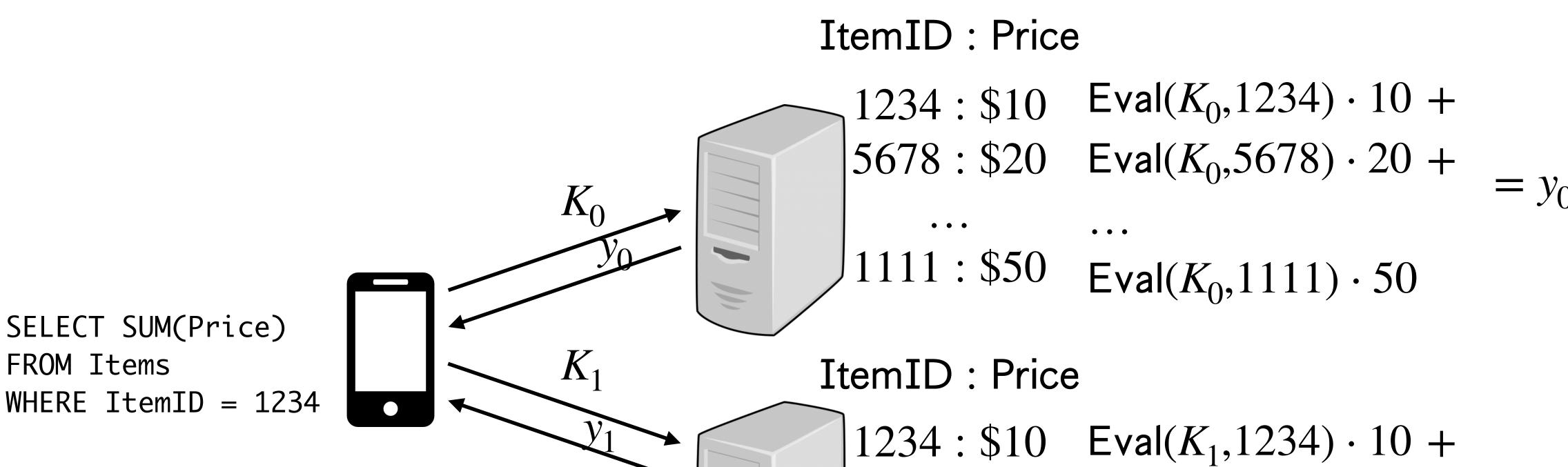
Count queries

Can extend to interval conditions with interval FSS



Sum queries

Can extend to interval conditions with interval FSS



$$f(1234) = 1, f(x) = 0 \text{ for } x \neq 1234$$

$$\mathsf{Gen}(1^{\lambda},f) \to K_0,K_1$$

$$y \leftarrow y_0 + y_1$$

1234: \$10 Eval
$$(K_1, 1234) \cdot 10 + 5678$$
: \$20 Eval $(K_1, 5678) \cdot 20 + y$

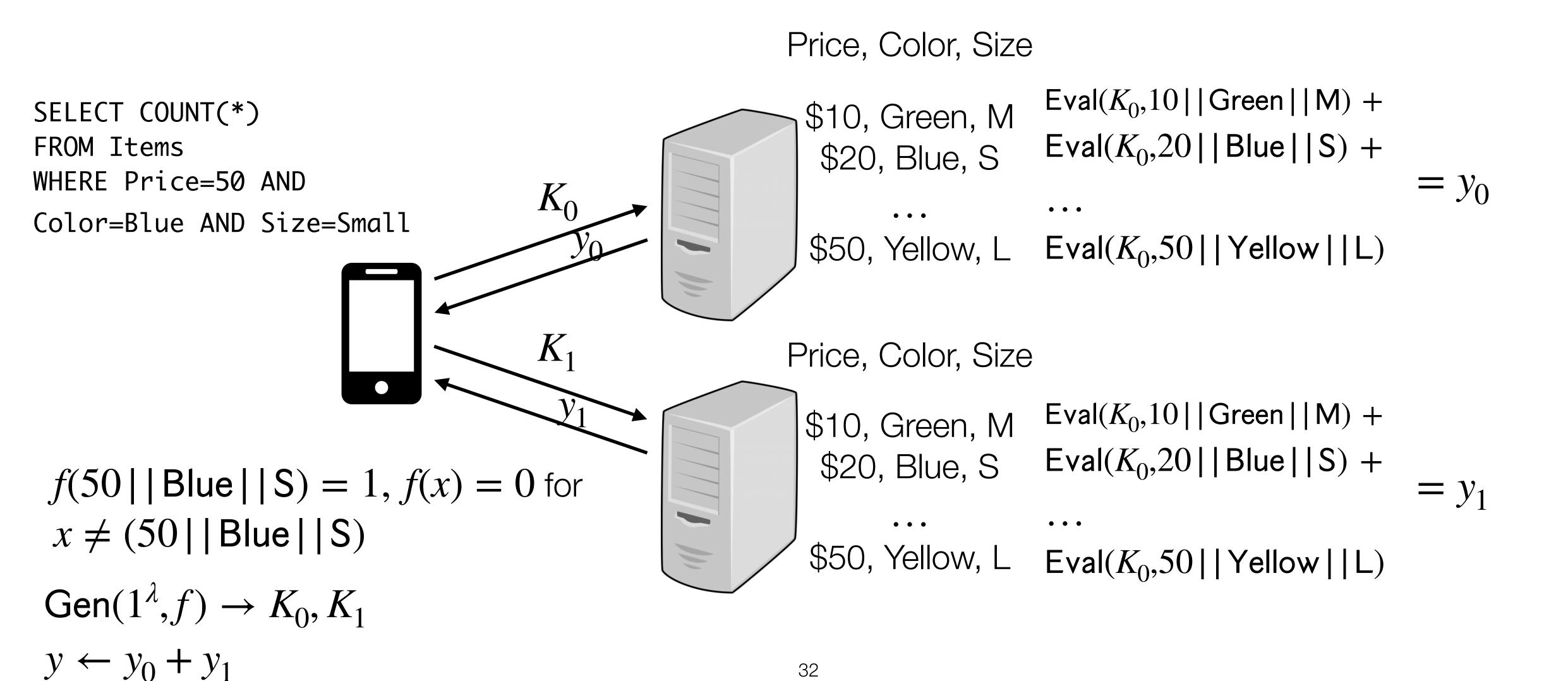
• • •

1111: \$50 $Eval(K_1, 1111) \cdot 50$

Extending to other linear aggregation functions

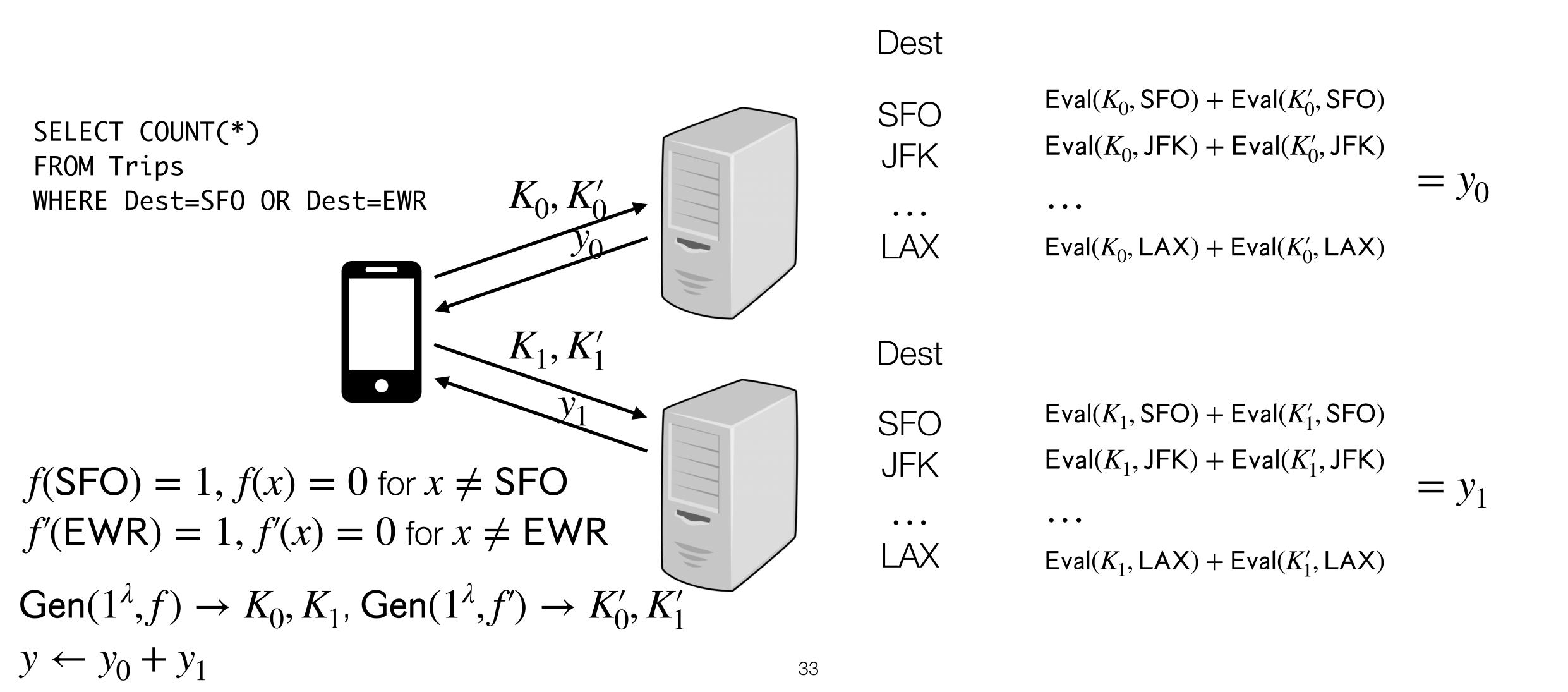
- AVG(x): Fetch SUM(x) and COUNT(x) and use to compute the mean
- STDEV(x): Fetch SUM(x) and COUNT(x), along with SUM(x^2) and COUNT(x^2)

More complex conditions: AND



32

More complex conditions: Disjoint OR



Max/min for interval conditions

SELECT MAX(Price) FROM Items WHERE 9 <= Rating <= 10

Setup: build an array A ordered by ratings

Round 1: Find indices i, j in A where all $rating \in [9,10]$ using keys for interval function for $rating \in [0,8]$, $rating \in [0,9]$

Round 2: Select maxes from power-of-2 sized intervals that cover exactly records i through j in A

	A[36]							
A	3	5	1	2	4	1	0 1	
Size-2 intervals	5 2			1	1			
Size-4 intervals	5			4				
Size-8 intervals	5							

See paper for max for single conditions

Max/min for disjoint OR

SELECT MIN(Price) FROM Flights WHERE Dest=SFO OR Dest=EWR

Setup: build an array A ordered by ratings

Binary search over A in decreasing power-of-2 sizes to find the smallest index where Dest=SFO or Dest=EWR

- SELECT COUNT(*) FROM A WHERE (Dest=SFO OR Dest=EWR) AND index ∈ [start, end]
- To search over interval, take advantage of fact that intervals are power-of-2 aligned (match on first x bits)

Use a DPF to retrieve the price for the entry with the smallest index satisfying Dest=SFO OR Dest=EWR

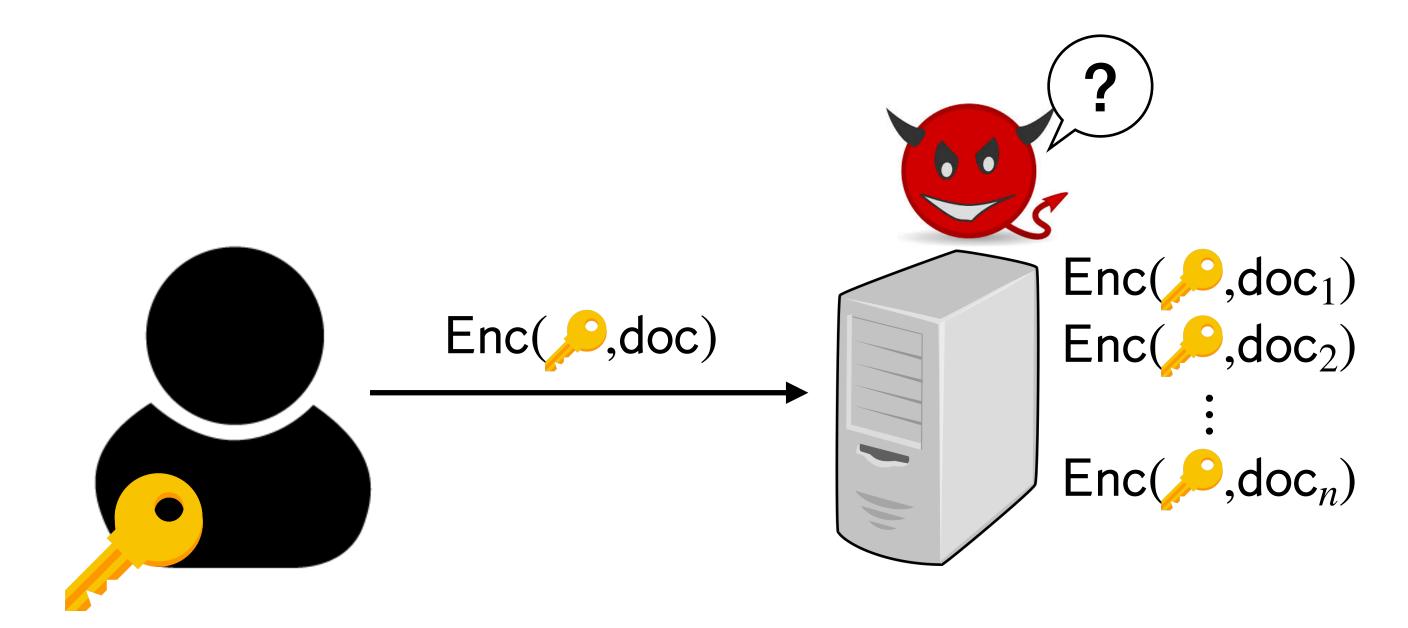
[For small number of conditions, can also run one query for each clause and combine at the client]

Outline

- 1. Function secret sharing
- 2. Splinter
- 3. Searching on encrypted data

End-to-end encrypted filesystems

Provide strong security guarantees if attacker compromises server.



Users expect the ability to search

Apple

Doc 1

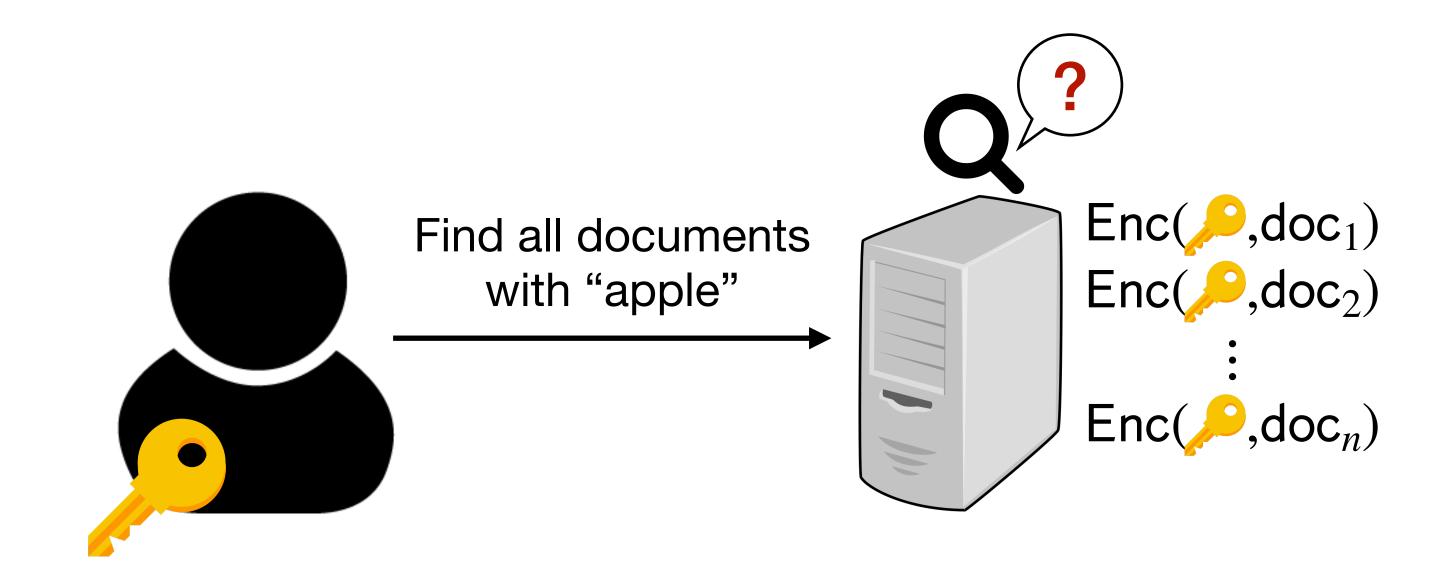
Doc 7

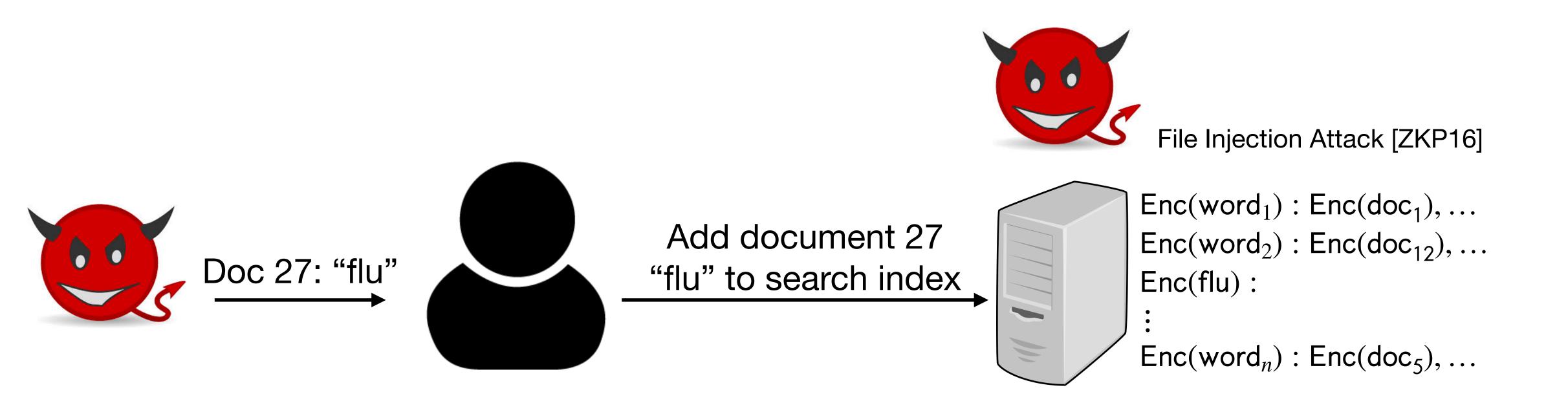
Doc 21

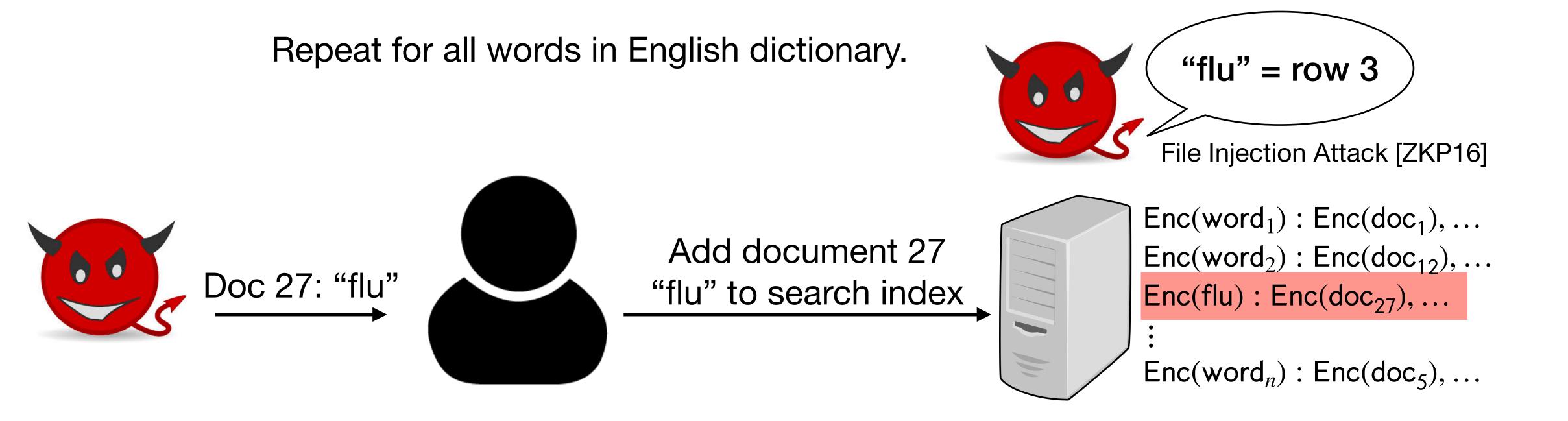
Doc 53

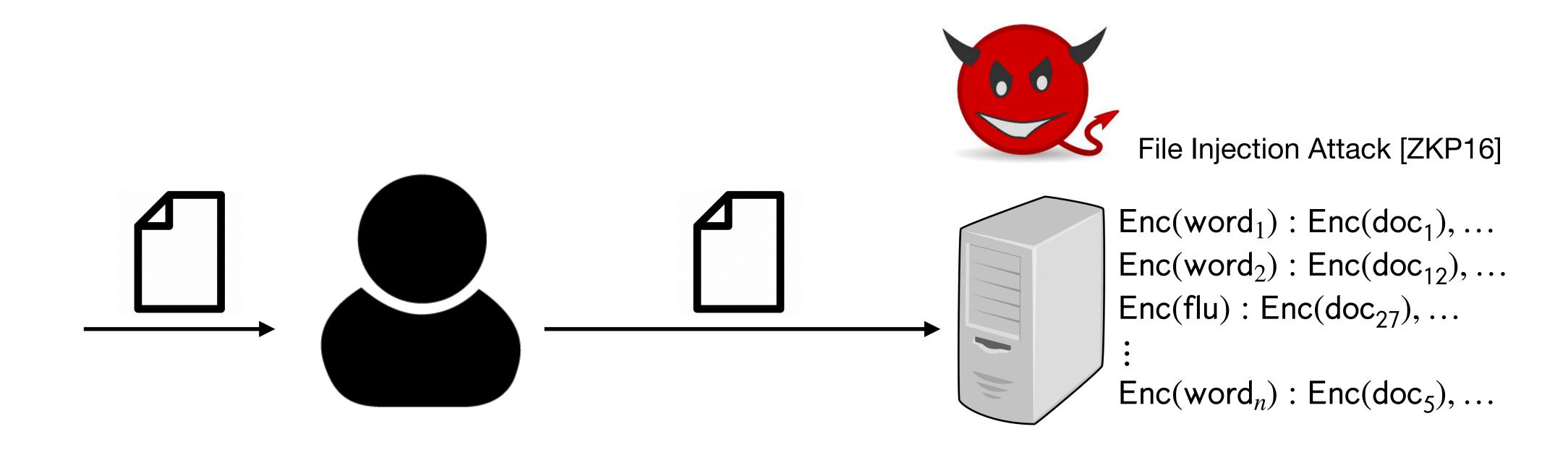
Search for end-to-end encrypted filesystems

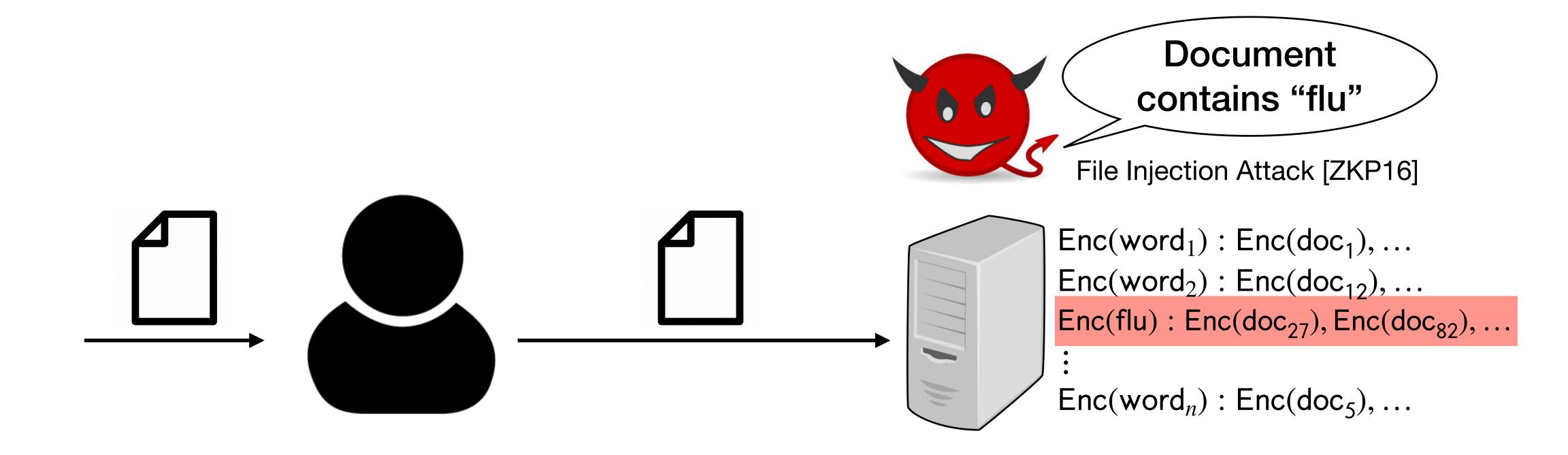
Challenge: server cannot decrypt data to search.







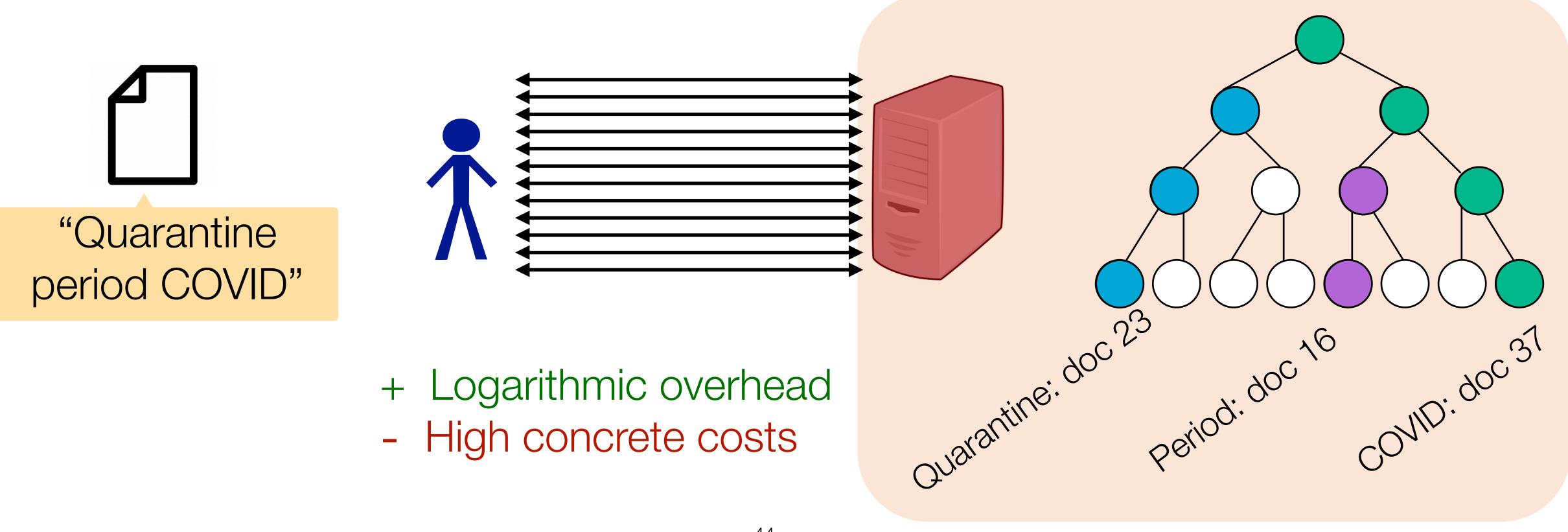




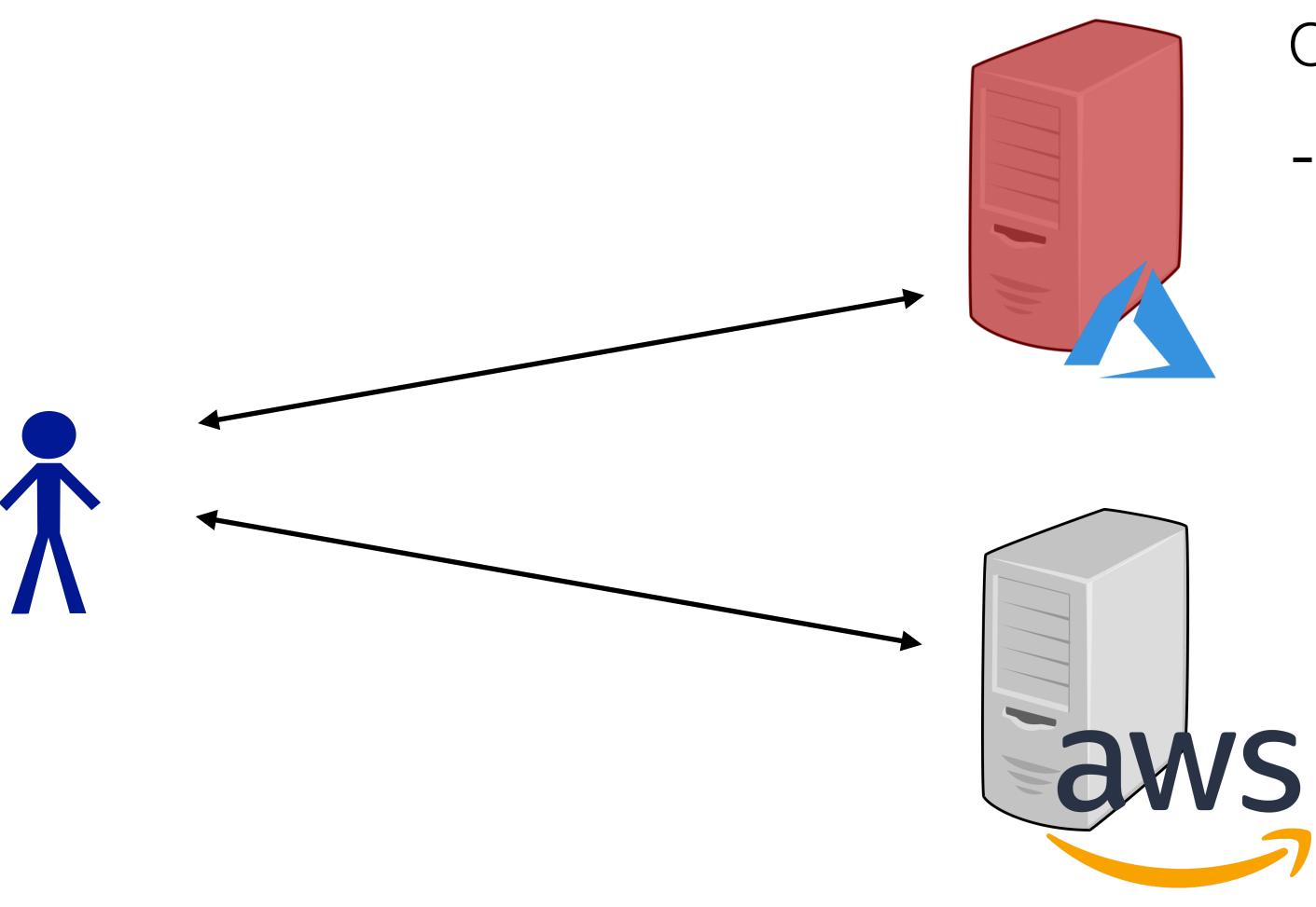
... and many more attacks

Oblivious RAM solutions are slow for encrypted search

Client can read/write server data without revealing data location [GO96, PathORAM]



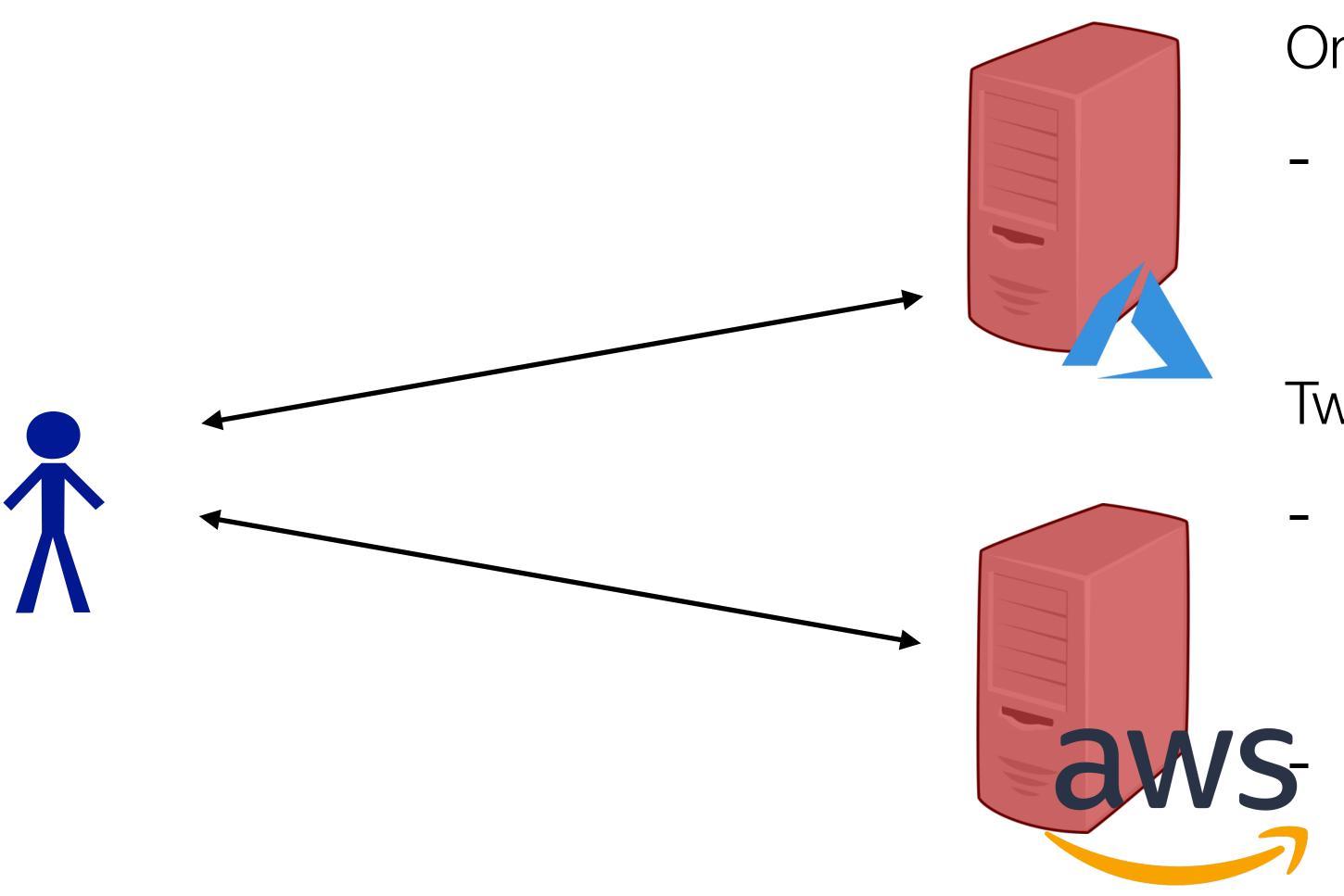
Security properties



One malicious server:

- Attacker can't learn anything about queries or document contents from memory accesses

Security properties



One malicious server:

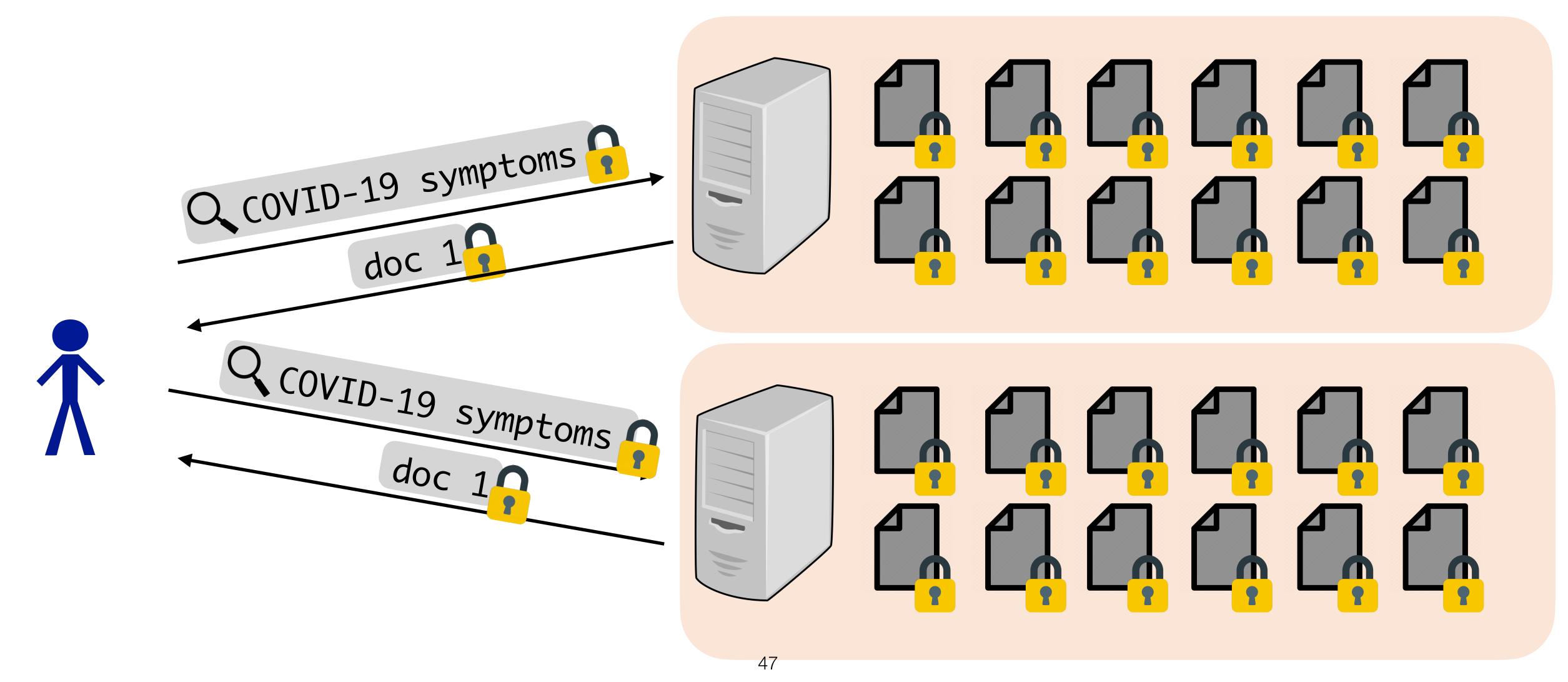
- Attacker can't learn anything about queries or document contents from memory accesses

Two malicious servers:

 Attacker can't immediately learn document contents (but can infer info from memory accesses)

DORY: Efficient search without search-access-pattern leakage

[Dauterman, Feng, Luo, Popa, Stoica]



Searchable encryption schemes

[Song, Wagner, Perrig], ...

Queries require accessing part of search index

Leak info about query and data

DORY

Queries require accessing entire search index

→ ok if concretely fast

Leak nothing about query or data

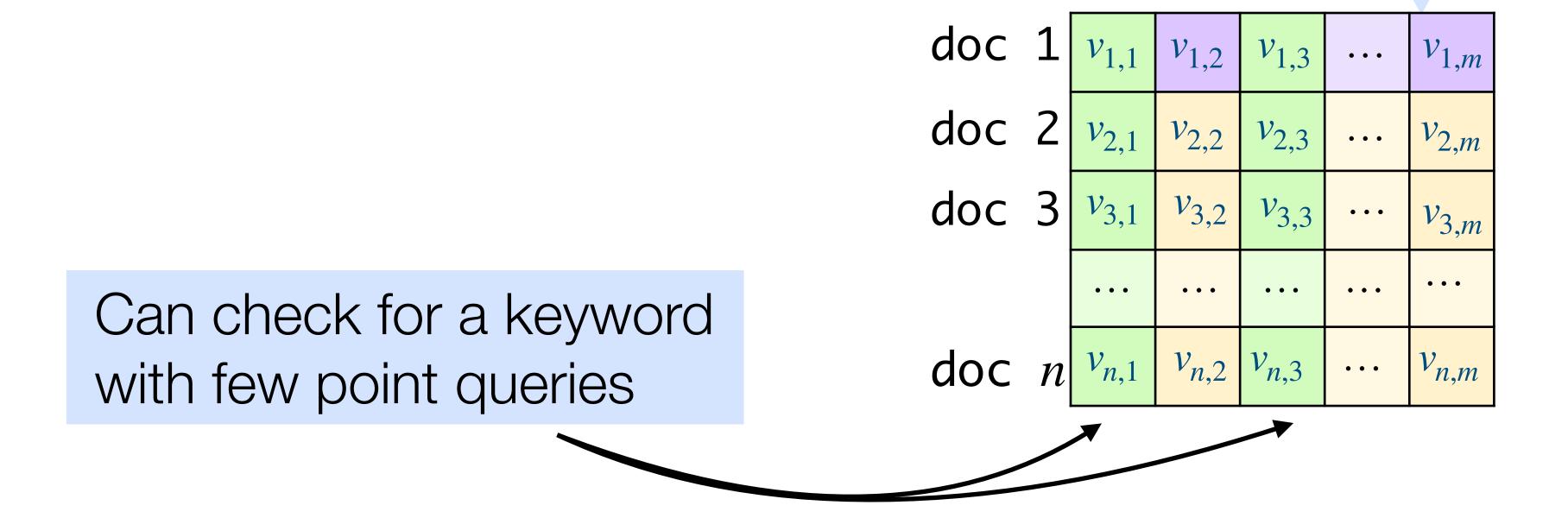
Building the search index

Bloom filter containing keywords in doc 1

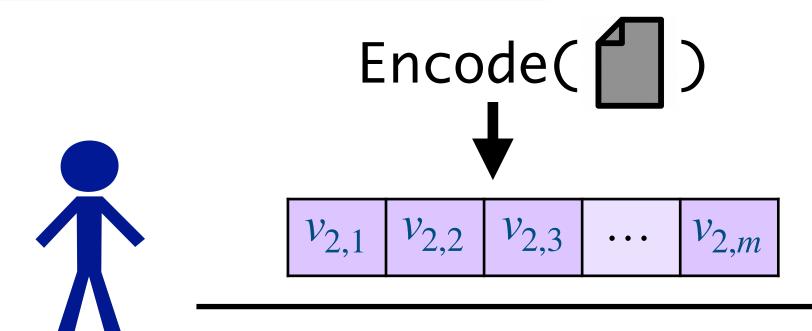
doc	1	$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	• • •	$v_{1,m}$
doc	2	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	• • •	$v_{2,m}$
doc	3	$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	• • •	$v_{3,m}$
		• • •	• • •	• • •	• • •	• • •
doc	n	$v_{n,1}$	$v_{n,2}$	$v_{n,3}$	• • •	$v_{n,m}$

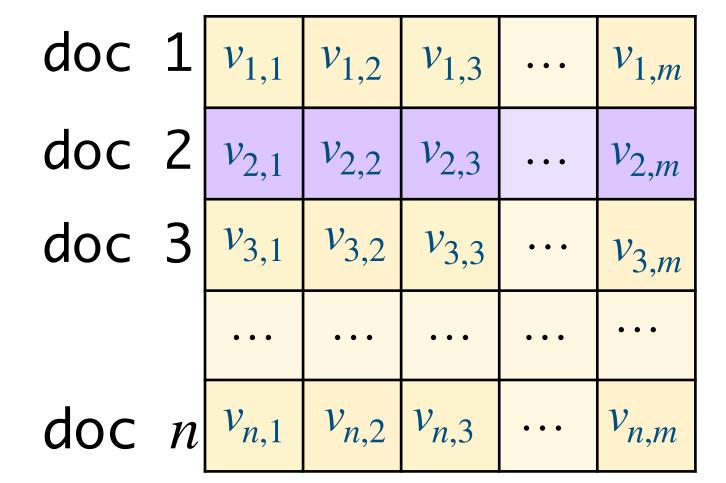
Building the search index

Bloom filter containing keywords in doc 1

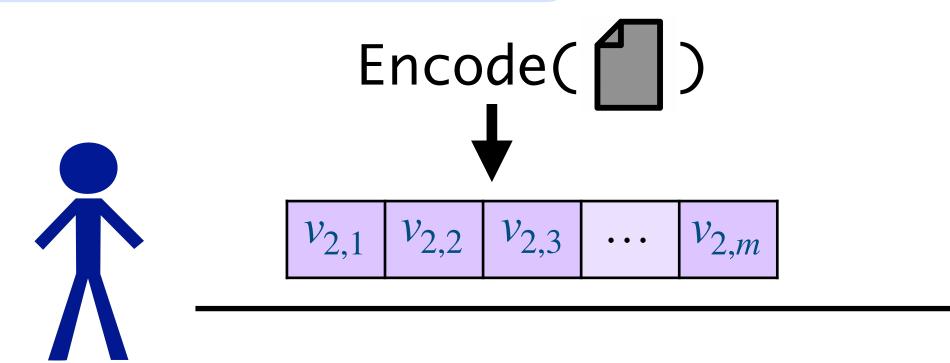


Update: write a row

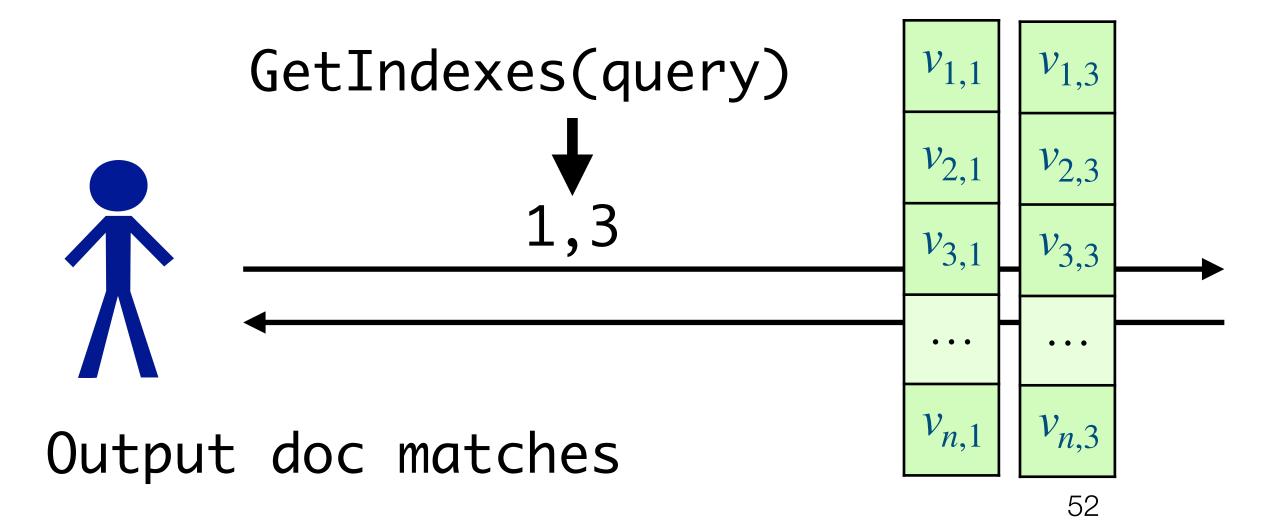




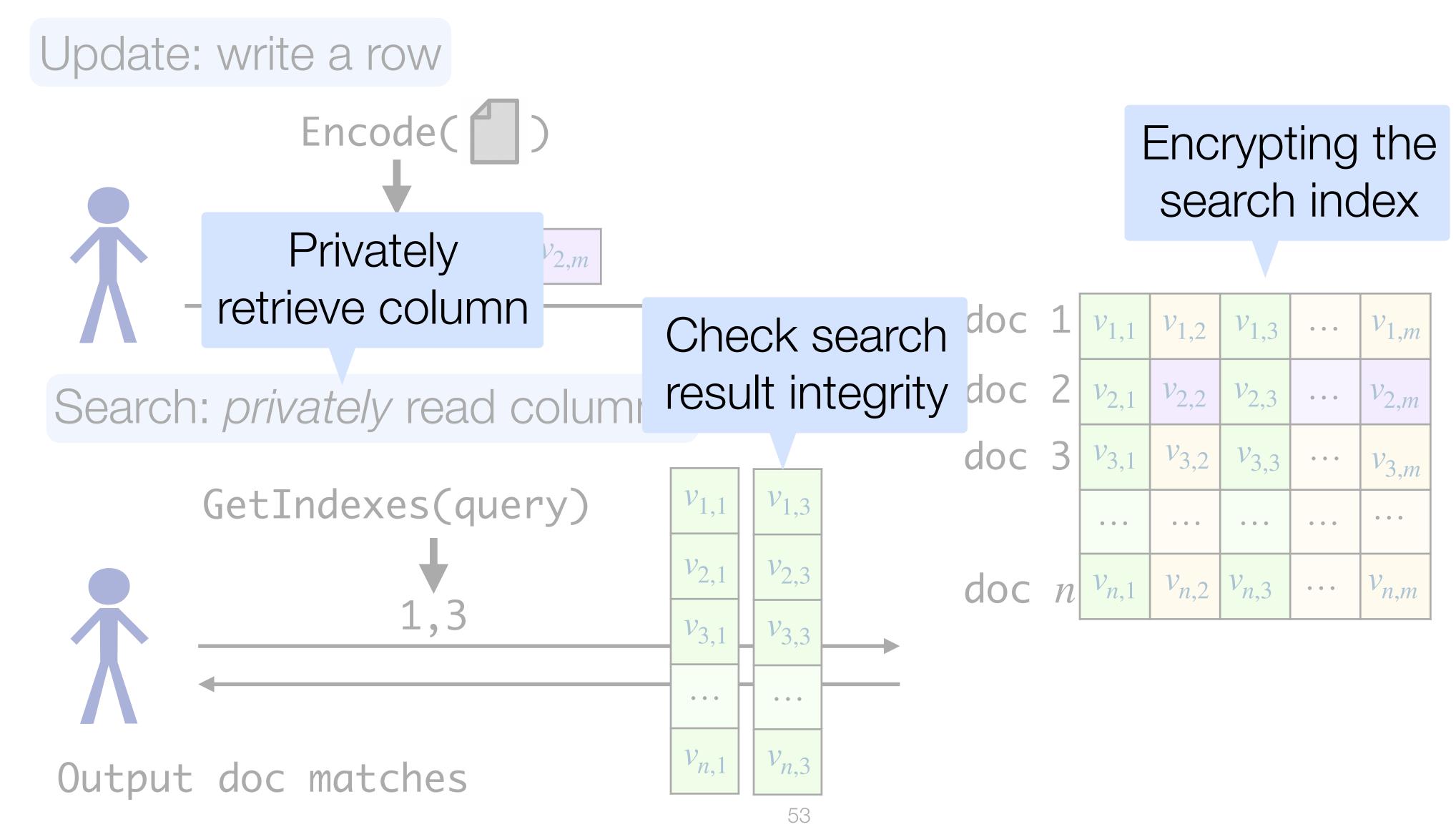
Update: write a row

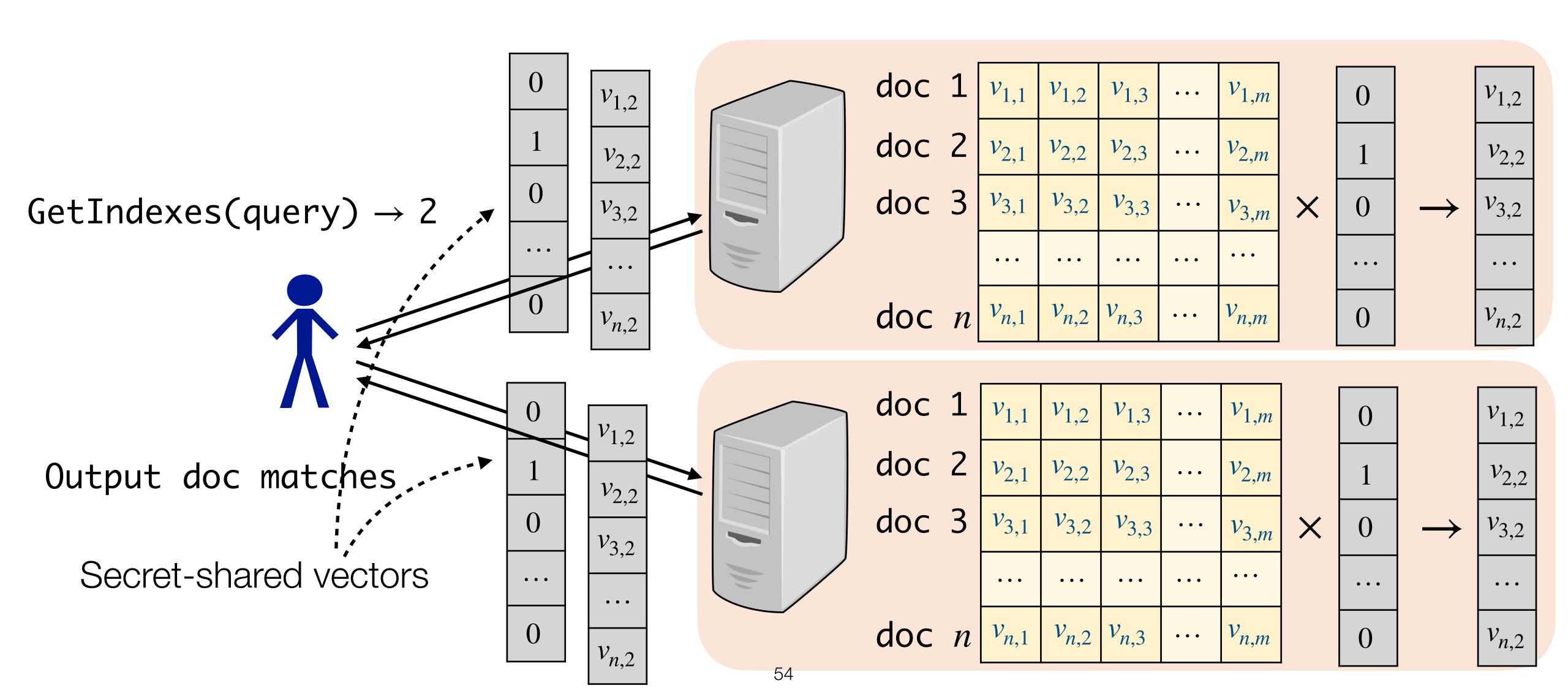


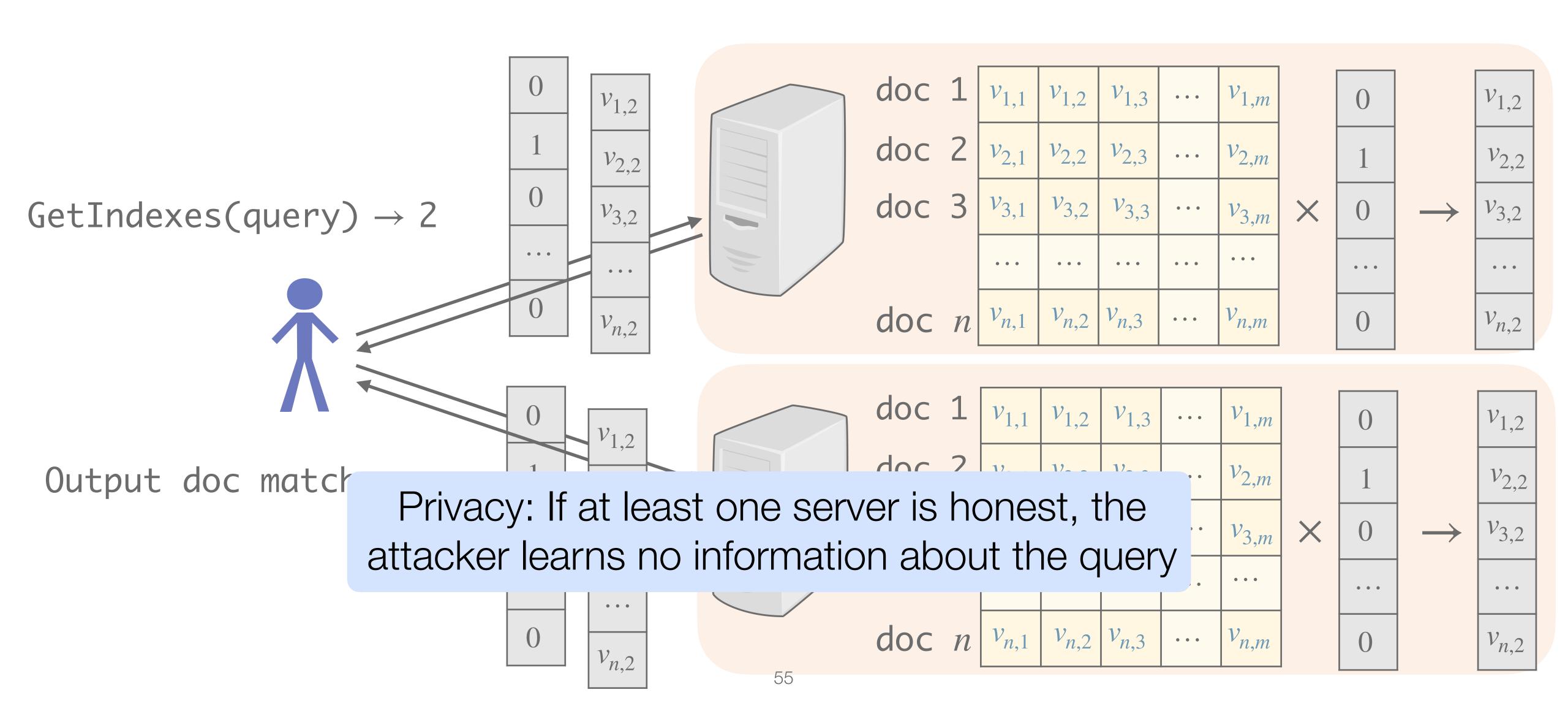
Search: privately read columns

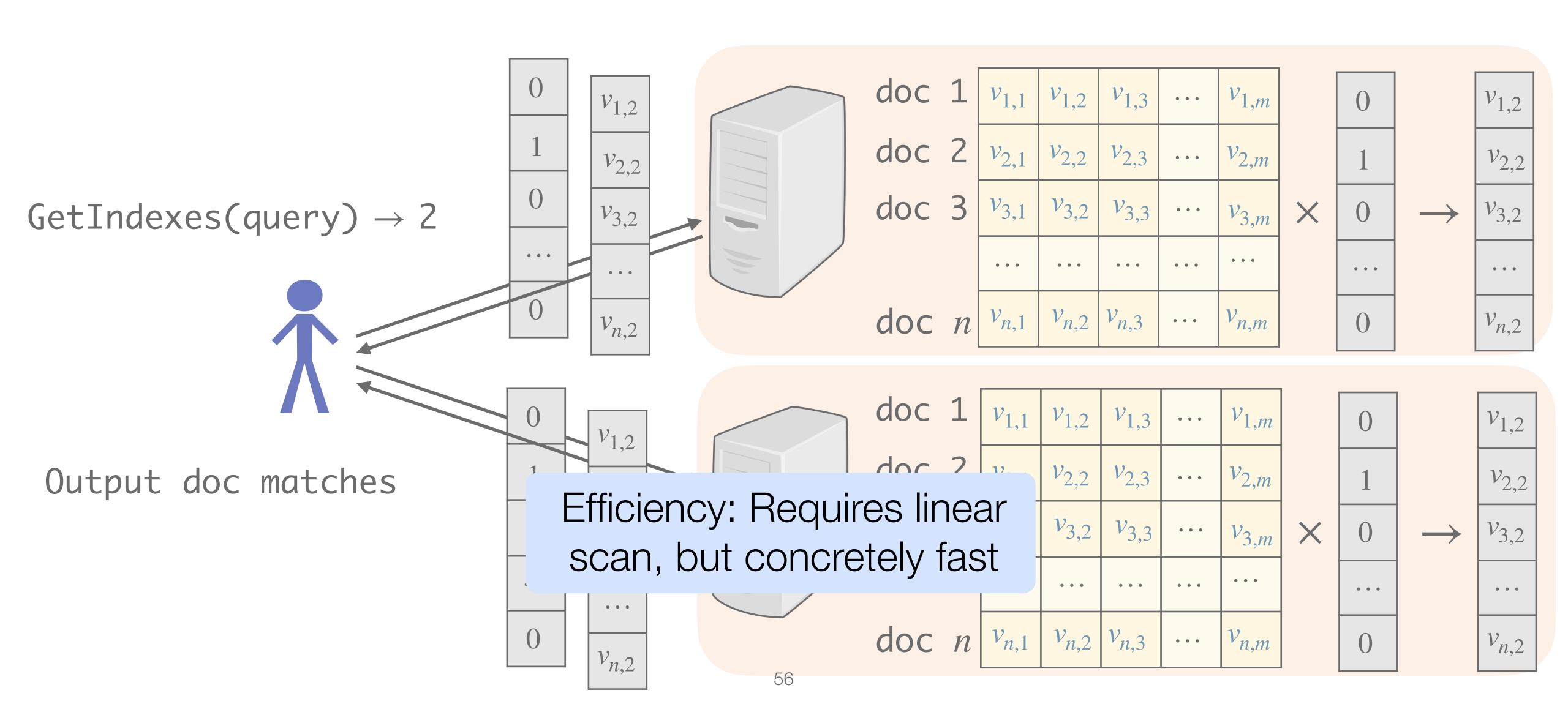


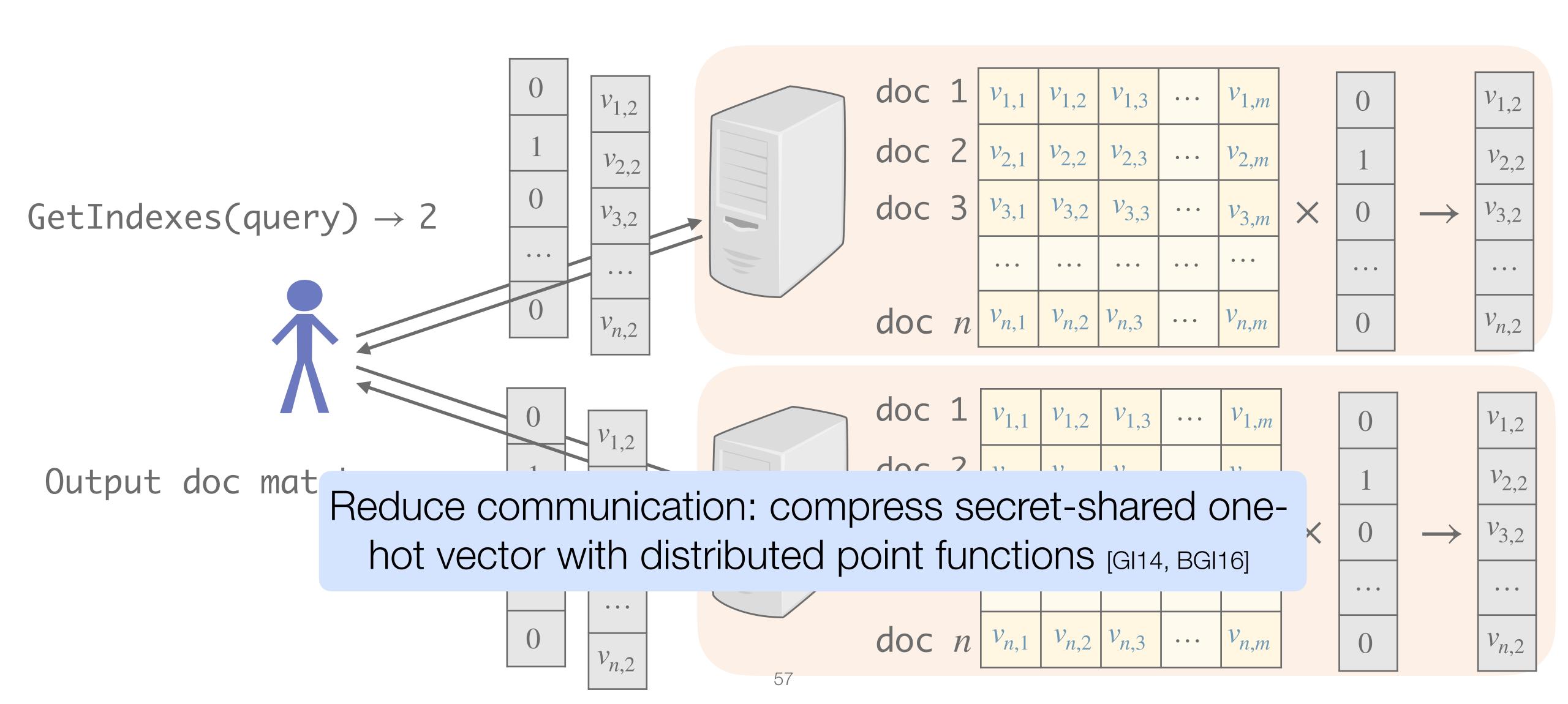
doc	1	$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	• • •	$v_{1,m}$
doc	2	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	• • •	$v_{2,m}$
doc	3	$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	• • •	$v_{3,m}$
		• • •	• • •	• • •	• • •	• • •
doc	n	$v_{n,1}$	$v_{n,2}$	$v_{n,3}$	• • •	$v_{n,m}$











Encrypting the search index

Challenge: Client uploads rows of bits, but retrieves columns of bits

- Don't want to increase communication by a factor of $\lambda \approx 128$.

Idea: Generate a unique mask using document version number

doc 1		$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	• • •	$v_{1,m}$		$PRF_k(0 version_0)$	doc	1	$v'_{1,1}$	$v'_{1,2}$	$v'_{1,3}$	• • •	$v'_{1,m}$
doc 2	2	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	•••	$v_{2,m}$		$PRF_k(1 version_1)$	doc	2	$v'_{2,1}$	$v_{2,2}'$	$v_{2,3}'$	• • •	$v_{2,m}'$
doc 3	3	$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	• • •	$v_{3,m}$	\oplus	$PRF_k(2 version_2)$	doc	3	$v_{3,1}'$	$v_{3,2}'$	$v_{3,3}'$	• • •	$v_{3,m}'$
		• • •	• • •	• • •	• • •	• • •		• • •			• • •	• • •	• • •	• • •	• • •
doc 1		$v_{n,1}$	$v_{n,2}$	$v_{n,3}$	• • •	$V_{n,m}$		$PRF_k(n \mid version_n)$	doc	n	$v'_{n,1}$	$v'_{n,2}$	$v'_{n,3}$	• • •	$v'_{n,m}$

Search result integrity

MAC tags allow client to verify result corresponds to valid update

Aggregate MACs compress all MAC tags in a column into a single tag [KL08]

XOR individual MAC tags into single aggregate tag

Aggregate MACs	t_1	t_2	t_3	• • •	t_m
doc 1	$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	• • •	$v_{1,m}$
doc 2	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	• • •	$v_{2,m}$
doc 3	$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	• • •	$v_{3,m}$
	• • •	• • •	• • •	• • •	• • •
doc n	$v_{n,1}$	$v_{n,2}$	$v_{n,3}$	• • •	$v_{n,m}$

Logistics

Signups for meetings for feedback on project proposals

- Required for all groups

Course feedback form on Ed

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