

CS 350S: Privacy-Preserving Systems

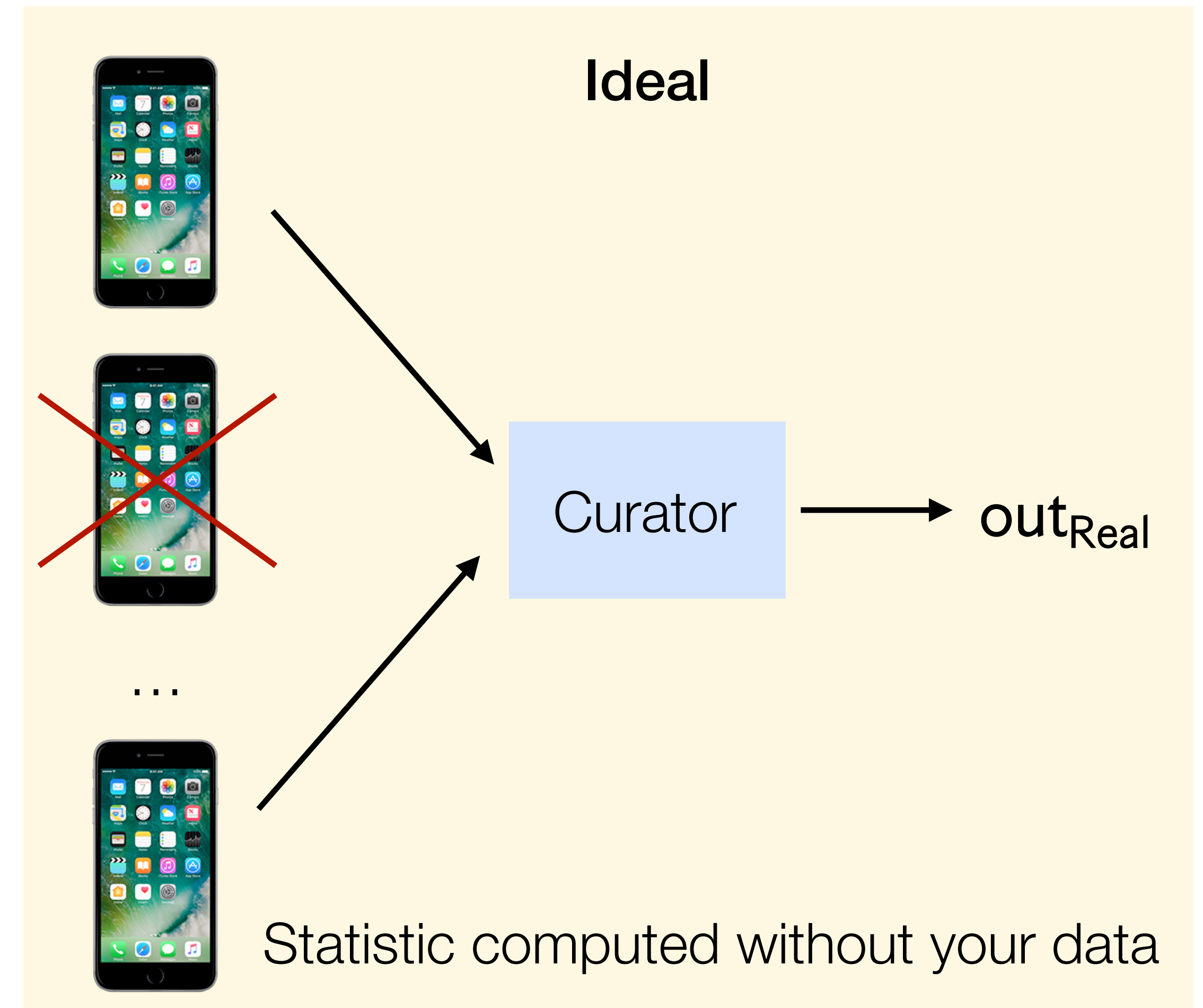
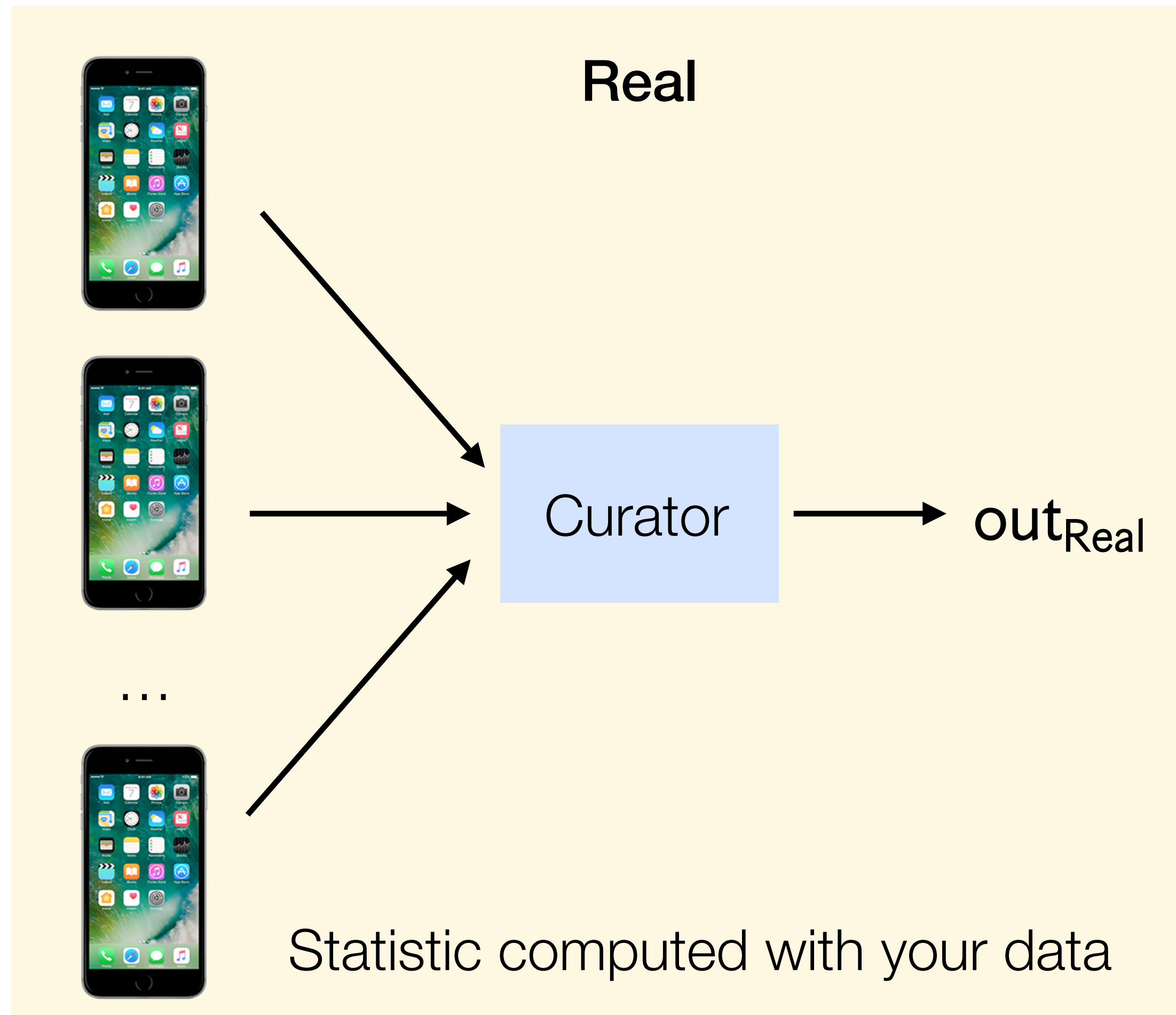
Federated learning

Recap: AOL query deanonymization attack

- AOL query dataset had >20M anonymized search queries from 650,000 AOL users over 3 months
- Dataset released where each username was replaced with a random identifier
- Queries for
 - “Landscapers in Lilburn, Ga”
 - Several people with last name Arnold
 - “Homes sold in shadow lake subdivision Gwinnett county Georgia”
 - ... other sensitive queries
- Only 14 citizens with last name Arnold in Gwinnett County
- Found that user was Thelma Arnold, 62-year old woman in Georgia



Recap: Differential privacy



$$\text{out}_{\text{Real}} \approx \text{out}_{\text{Ideal}} \quad (\text{Not cryptographic indistinguishability})$$

Recap: Differential privacy

[Dwork, Sherry, Nissim, Smith]

Mechanism $\mathcal{M} : \mathcal{X}^n \rightarrow \mathcal{Y}$

For database with n rows of type \mathcal{X} and output statistic \mathcal{Y}

Two databases are “neighboring” if they differ in at most one row

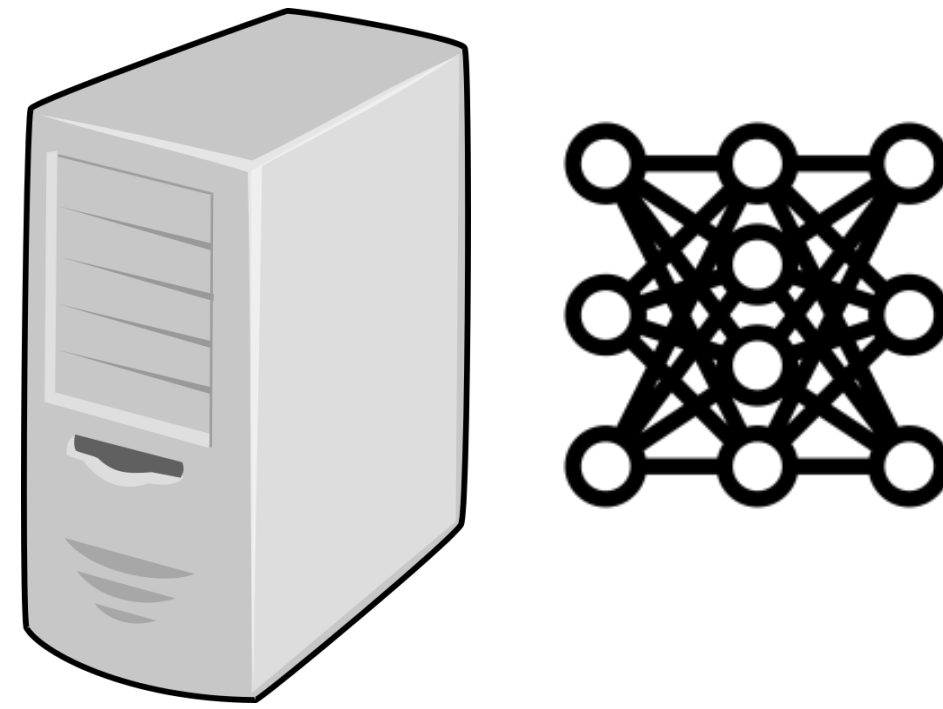
A mechanism \mathcal{M} is ε differentially private if for all pairs of “neighboring databases” D, D' and every set of values $S \in \mathcal{Y}$:

$$\Pr[\mathcal{M}(D) \in S] \leq e^\varepsilon \cdot \Pr[\mathcal{M}(D') \in S]$$

Outline

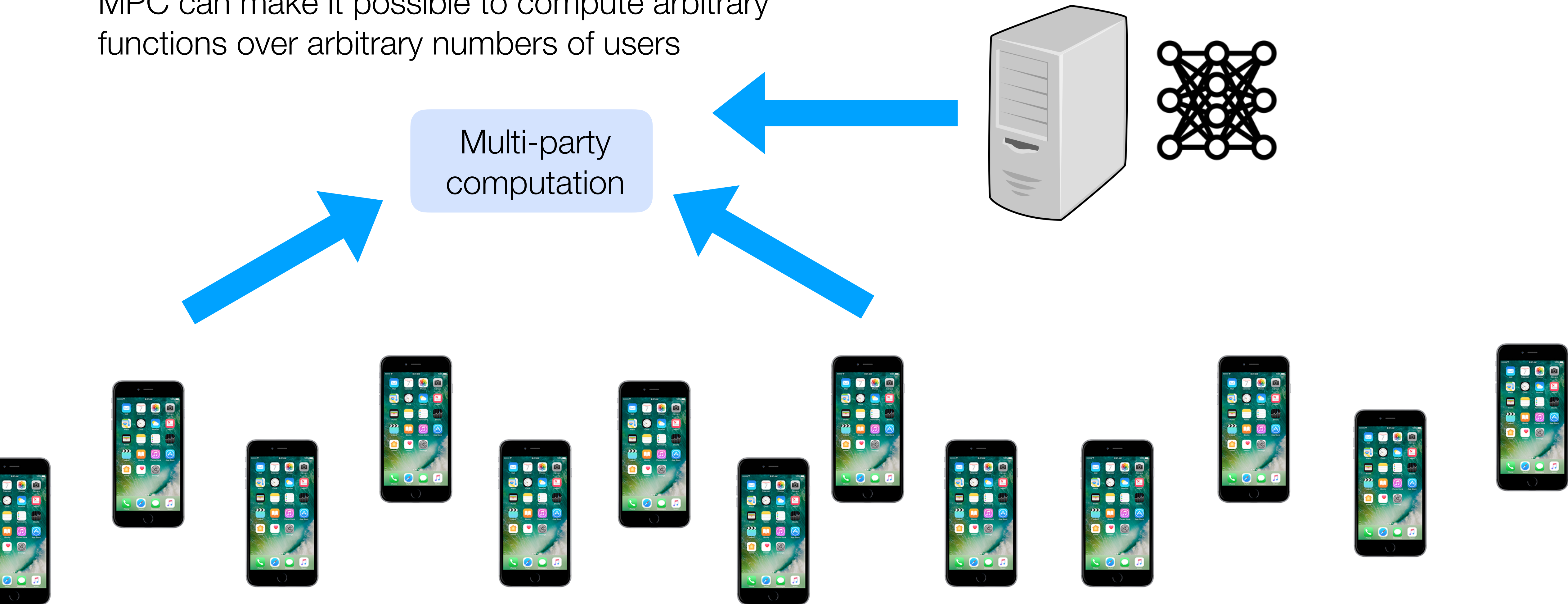
1. Federated learning
2. Flamingo
3. Other research topics
4. Student presentation

Goal: privately train a model across many users



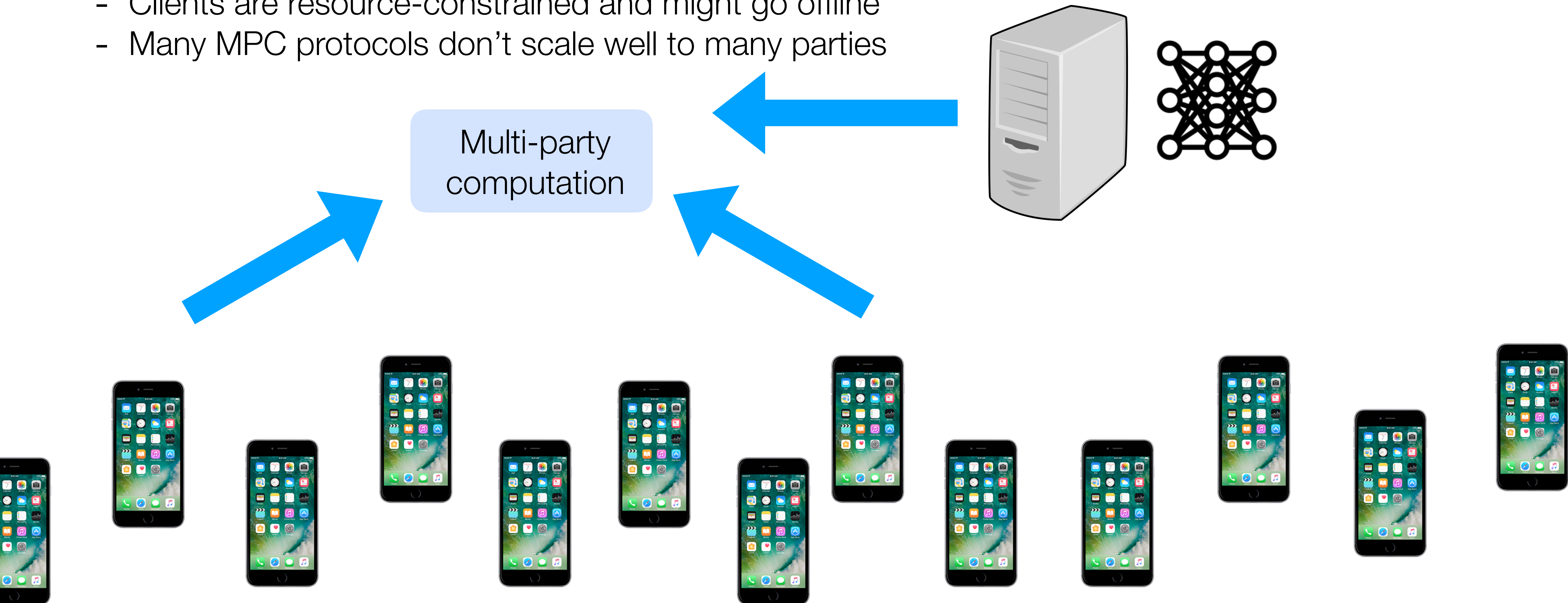
One approach: generic multi-party computation

MPC can make it possible to compute arbitrary functions over arbitrary numbers of users



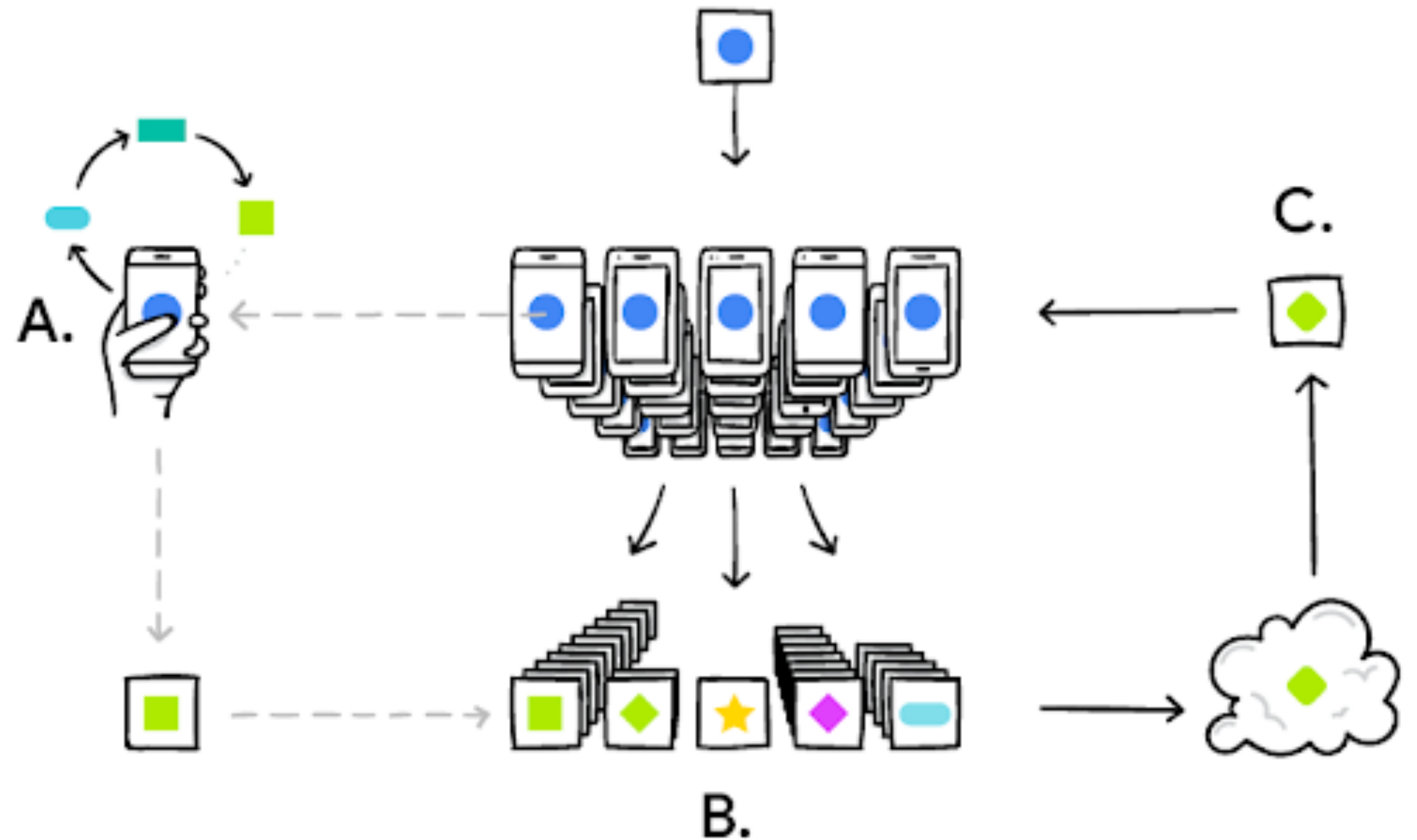
Why not run a MPC across all users?

- Clients are resource-constrained and might go offline
- Many MPC protocols don't scale well to many parties



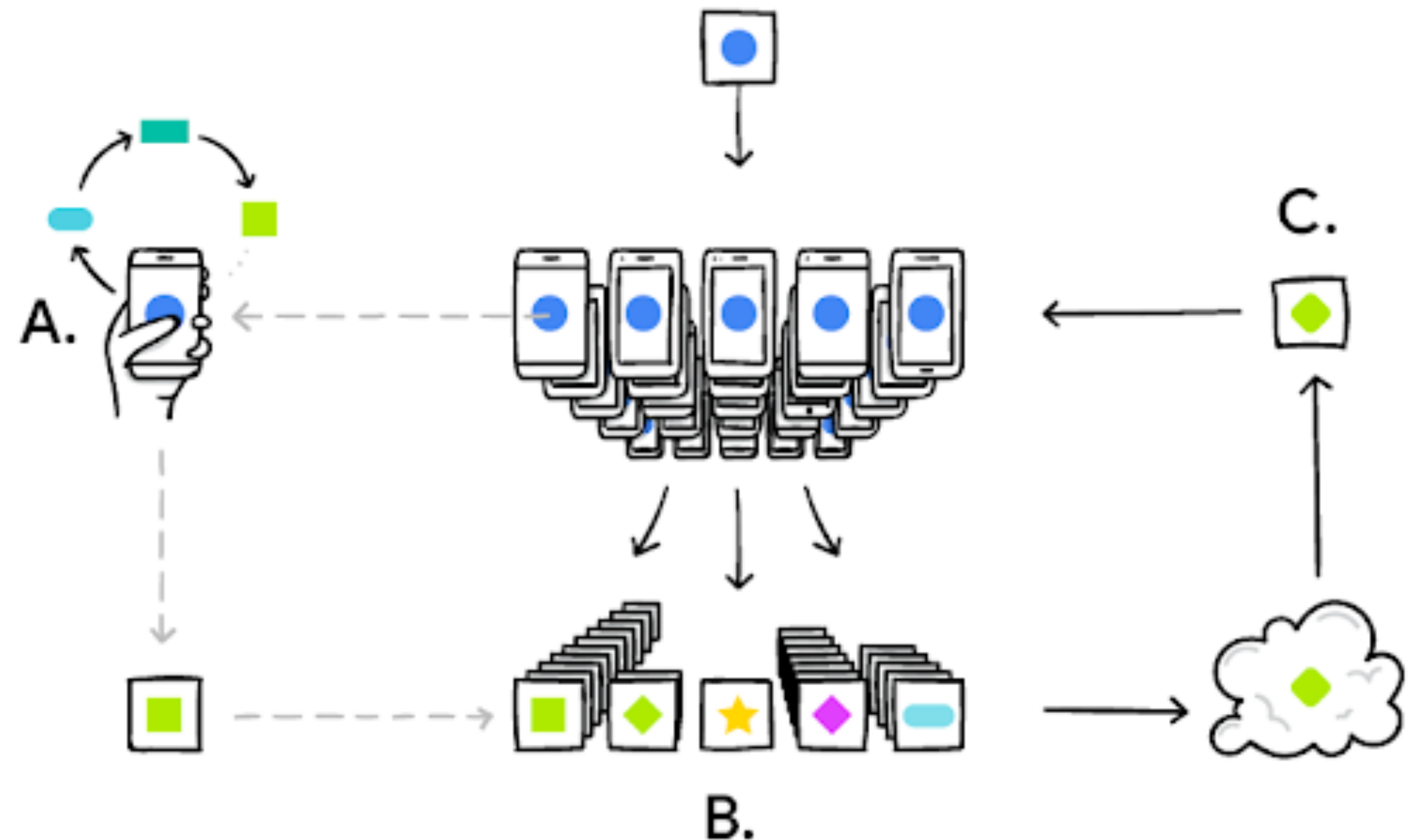
Federated learning

- Training data remains on the device
- Clients send updates to the server
- The server aggregates many user updates to produce a model update
- The server pushes the model update to the clients
- Repeat



Challenge: resource-constrained, heterogeneous clients

- Mobile devices have powerful processors, but limited and intermittent connectivity (in contrast to datacenters, where compute is dominant factor)
- Data is not IID (user's dataset is not representative of population) and unbalanced (some users use the app more than others)
- Naively distributing a training algorithm like SGD does not satisfy these constraints



Idea: more compute for higher-quality updates

[McMahan, Moore, Ramage, Hampson, Arcas]

- For each round, choose a set of clients
- Each chosen client runs some number of SGD updates locally
- Each chosen client sends its update to the aggregator

Algorithm 1 FederatedAveraging. The K clients are indexed by k ; B is the local minibatch size, E is the number of local epochs, and η is the learning rate.

Server executes:

```
initialize  $w_0$ 
for each round  $t = 1, 2, \dots$  do
   $m \leftarrow \max(C \cdot K, 1)$ 
   $S_t \leftarrow$  (random set of  $m$  clients)
  for each client  $k \in S_t$  in parallel do
     $w_{t+1}^k \leftarrow \text{ClientUpdate}(k, w_t)$ 
   $m_t \leftarrow \sum_{k \in S_t} n_k$ 
   $w_{t+1} \leftarrow \sum_{k \in S_t} \frac{n_k}{m_t} w_{t+1}^k$  // Erratum4
```

ClientUpdate(k, w): // Run on client k

```
 $\mathcal{B} \leftarrow$  (split  $\mathcal{P}_k$  into batches of size  $B$ )
for each local epoch  $i$  from 1 to  $E$  do
  for batch  $b \in \mathcal{B}$  do
     $w \leftarrow w - \eta \nabla \ell(w; b)$ 
  return  $w$  to server
```

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[McMahan, Moore, Ramage, Hampson, Arcas]

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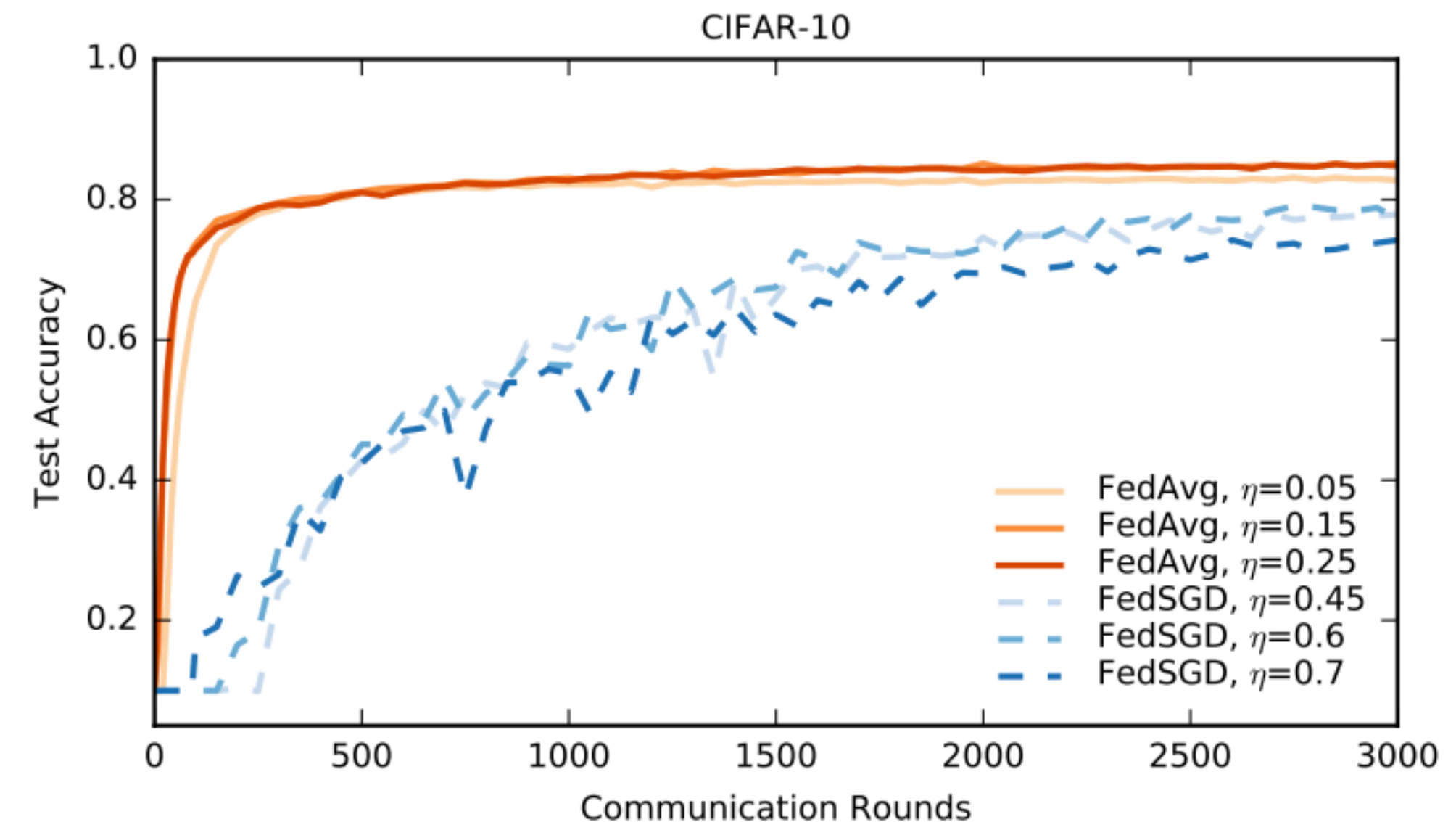


Figure 4: Test accuracy versus communication for the CIFAR10 experiments. FedSGD uses a learning-rate decay of 0.9934 per round; FedAvg uses $B = 50$, learning-rate decay of 0.99 per round, and $E = 5$.

Use-cases of federated learning at Google

- Gboard: Next-word prediction, emoji usage, out-of-vocabulary word discovery
- “Hey Google” detection models in Assistant
- Suggesting replies in Google messages
- Predicting text selections

Federated Learning

[McMahan, Moore, Ramage, Hampson, Arcas]

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Server executes:

initialize w_0

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for each client $k \in S_t$ **in parallel do**

$w_{t+1}^k \leftarrow \text{ClientUpdate}(k, w_t)$

$m_t \leftarrow \sum_{k \in S_t} n_k$

$w_{t+1} \leftarrow \sum_{k \in S_t} \frac{n_k}{m_t} w_{t+1}^k$ // *Erratum*⁴

ClientUpdate(k, w): // *Run on client k*

$\mathcal{B} \leftarrow$ (split \mathcal{P}_k into batches of size B)

for each local epoch i from 1 to E **do**

for batch $b \in \mathcal{B}$ **do**

$w \leftarrow w - \eta \nabla \ell(w; b)$

 return w to server

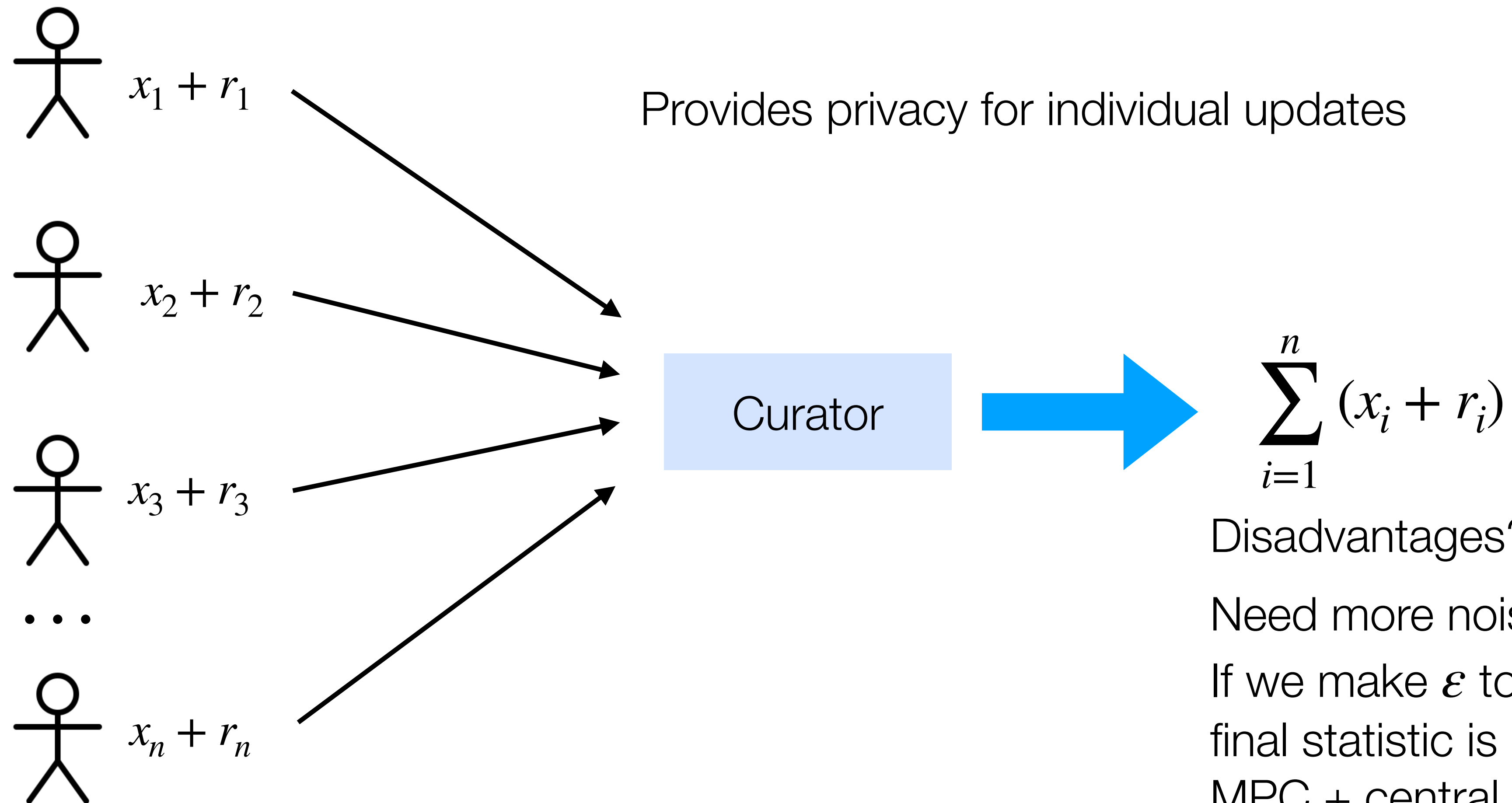
The training data never leaves the client device

Is this enough for privacy?

No — model updates can still reveal sensitive information (student presentation)

How can we ensure the aggregator does not learn sensitive user information?

One approach: Local differential privacy



Disadvantages?

Need more noise for privacy.

If we make ϵ too small, then the final statistic is not very useful
MPC + central DP is better utility vs privacy tradeoff

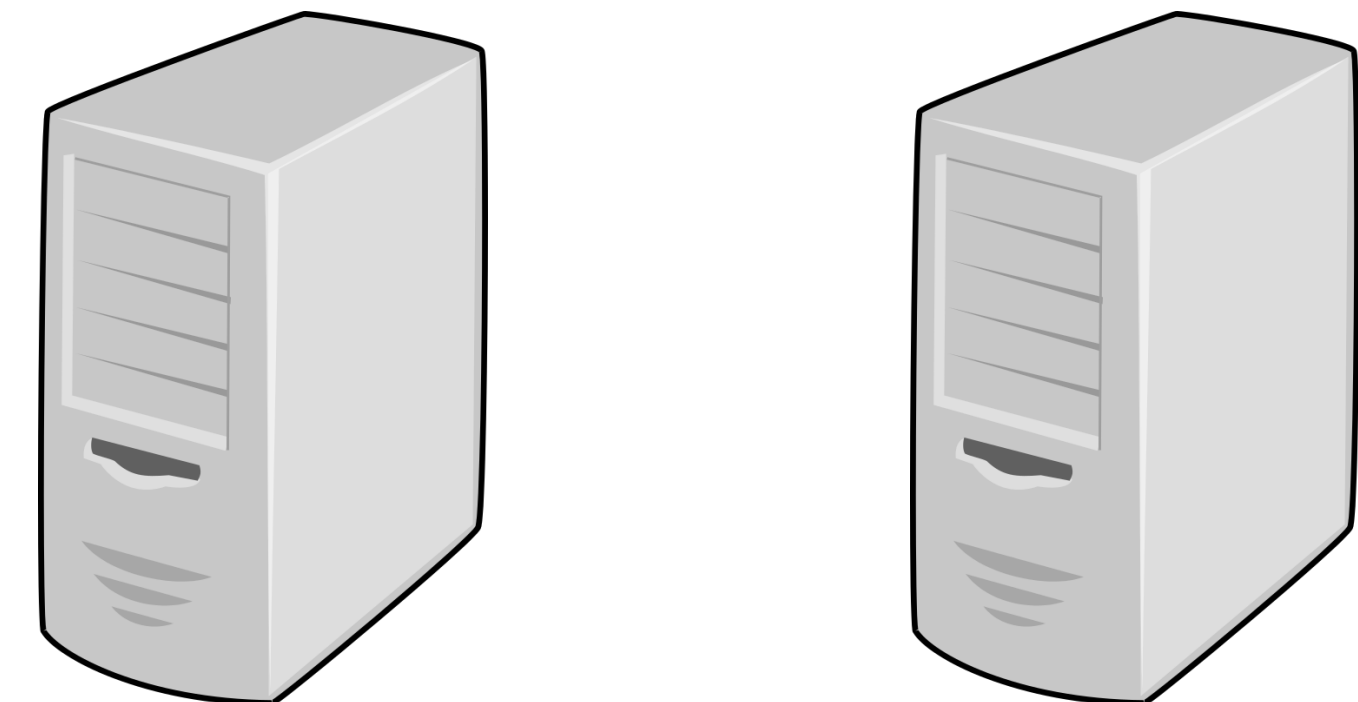
Another approach: two-server private aggregate statistics

- Compute a sum over client inputs
- Robustness: Malicious clients cannot overly influence sum

What does it mean that one user cannot have an outsize influence on the trained model?

- Popular approach: bound the L_2 norm of the update

$$\sum_{i=1}^n x_i$$



Today: Single-server federated learning

- Goal: Server only learns the sum of all client updates, but not individual client updates

$$\sum_{i=1}^n x_i$$



...



Outline

1. Federated learning
- 2. Flamingo**
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Flamingo security properties

[Ma, Woods, Angel, Polychroniadou, Rabin]

Privacy: A malicious adversary that compromises the server and some fraction of the clients only learns the sum of client inputs over at least some fraction of clients

- Individual inputs are hidden

Dropout resilience: When all parties follow the protocol, the server gets a sum of inputs from all online clients

Similarities and differences with Prio?



Background: Threshold secret sharing

Threshold secret sharing scheme across n parties where only t shares are necessary to reconstruct the original value

- $\text{Share}(\alpha) \rightarrow (\alpha_1, \alpha_2, \dots, \alpha_n)$:
- $\text{Reconstruct}(\alpha_1, \alpha_2, \dots, \alpha_t) \rightarrow \alpha$

Properties:

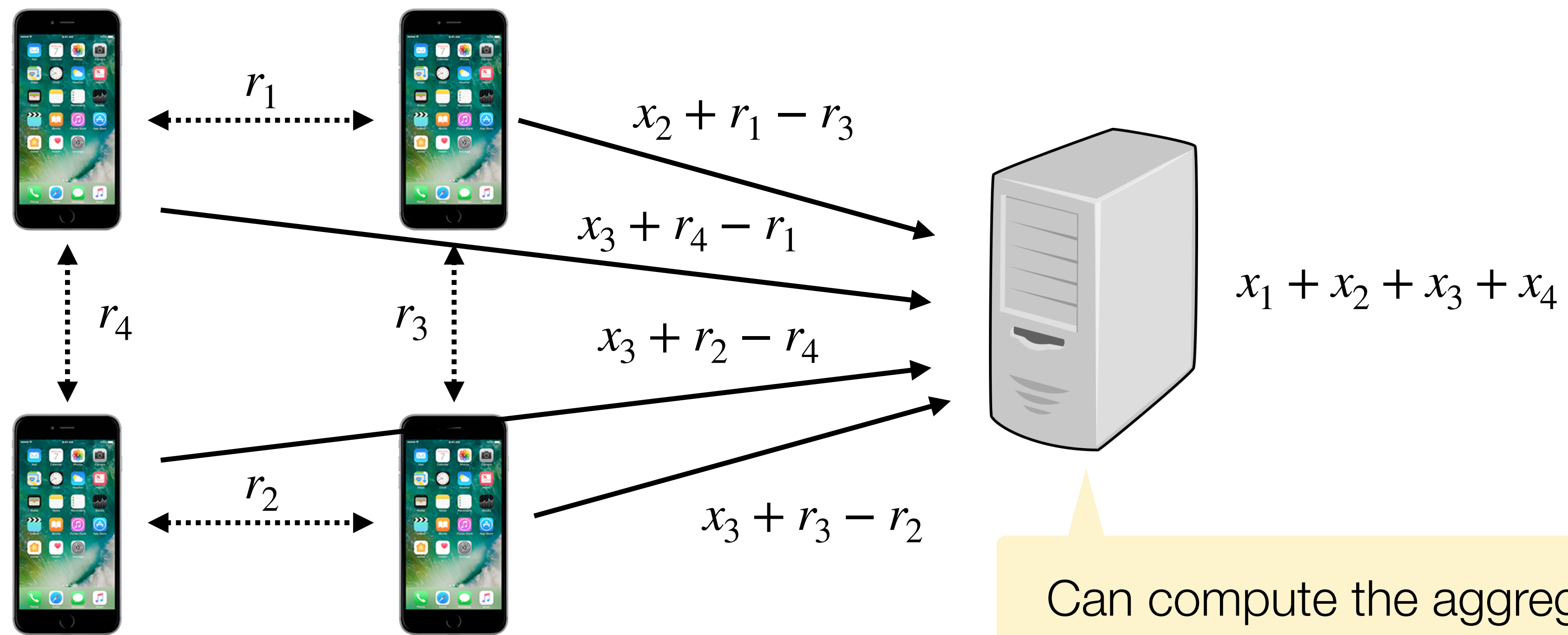
- Any group of t of the n shares can be used to reconstruct x
- Every set of $t - 1$ shares reveal nothing about x
- Can add shares locally, can multiply with communication between parties (similar to additive shares)

Background: Shamir secret sharing

Threshold secret sharing scheme across n parties where only t shares are necessary to reconstruct the original value

- **Share(α)** $\rightarrow (\alpha_1, \alpha_2, \dots, \alpha_n)$:
 1. Sample values a_1, a_2, \dots, a_t
 2. Compute polynomial $f(x) = a_1x + a_2x^2 + \dots + a_tx^t + \alpha$
 3. Output $f(i)$ for $i \in \{1, 2, \dots, n\}$
- **Reconstruct($\alpha_1, \alpha_2, \dots, \alpha_t$)** $\rightarrow \alpha$
 1. Interpolate the polynomial f
 2. Output $f(0)$

Idea: pairwise secrets



Can compute the aggregate, but cannot see individual contributions

Starting point: BBGLR protocol

Step 1: Setup

- Each client samples a keypair
- Each client i samples a set of “neighboring” clients A_i and sends the neighboring clients its public key, forming a graph
- Each client i uses Shamir secret sharing to split its secret key and a random value m_i across neighboring clients

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Step 2: Collection

- Client sends masked vector to server: $\mathbf{v}_i = \mathbf{x}_i + \sum_{j \in A(i), i < j} \text{PRG}(r_{i,j}) - \sum_{j \in A(i), i > j} \text{PRG}(r_{i,j}) + \text{PRG}(m_i)$ where $r_{i,j}$ is a shared secret between client i and j computed using one client’s secret key and the other client’s public key (Diffie-Hellman key exchange)
- For each offline client: the server request shares of the secret key from the client’s neighbors
For each online client: the server requests shares of m_i from the client’s neighbors

Idea: pairwise masks cancel out if all clients are online (need to reconstruct pairwise masks from offline clients)

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Question: Why both the individual and the pairwise masks?

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Question: Why both the individual and the pairwise masks?

- Pairwise mask ensures that the server aggregates enough clients
- Individual mask ensures that if a message is received after the server has reconstructed the pairwise mask, it can't learn the user's input

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Question: Is this protocol secure against a malicious adversary?

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Question: Is this protocol secure against a malicious adversary?

- Server may give an inconsistent view of which clients are online/offline
- Malicious clients can return the wrong values, leading to reconstruction failure

Flamingo

Extend BBGLR protocol to:

- Provide security against a malicious adversary
- Support aggregation across multiple rounds without per-round setup

Three ideas:

1. Dropout resilience by encrypting to a small group of decryptors
2. Reusable secrets across rounds
3. Per-round graphs to handle changing client sets

Flamingo

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Dropout resilience by encrypting to a small group of decryptors

- Group of clients (decryptors) keep secret shares of a secret decryption key
- Clients encrypt their pairwise and individual masks under the corresponding public key
- To recover the original result, they run a MPC to use their shares to decrypt ciphertext
 - Efficient, non-interactive MPC protocol for threshold decryption
- If enough of the decryptors are honest, for each client, the decryptors will decrypt one, but not both, of the two masks

$$\mathbf{v}_i = \mathbf{x}_i + \underbrace{\sum_{j \in A(i), i < j} \text{PRG}(r_{i,j}) - \sum_{j \in A(i), i > j} \text{PRG}(r_{i,j})}_{\text{Pairwise mask}} + \underbrace{\text{PRG}(m_i)}_{\text{Individual mask}}$$

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Question: What goes wrong if you re-use the same secret across rounds?

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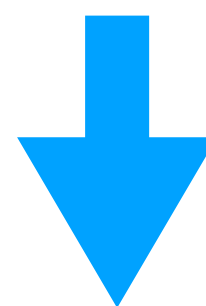
- If a client is online one round and offline the next, the server can learn both the pairwise and individual mask

Reusable secrets across rounds

The client uses the shared secret $r_{i,j}$ as a PRG to compute a per-round pairwise mask

BBGLR

$$\mathbf{v}_i = \mathbf{x}_i + \underbrace{\sum_{j \in A(i), i < j} \text{PRG}(r_{i,j}) - \sum_{j \in A(i), i > j} \text{PRG}(r_{i,j})}_{\text{Pairwise mask}} + \underbrace{\text{PRG}(m_i)}_{\text{Individual mask}}$$



Flamingo

$$\mathbf{v}_i = \mathbf{x}_i + \underbrace{\sum_{j \in A(i), i < j} \text{PRG}(r_{i,j}, t) - \sum_{j \in A(i), i > j} \text{PRG}(r_{i,j}, t)}_{\text{Pairwise mask}} + \underbrace{\text{PRG}(m_i)}_{\text{Individual mask}}$$

Round t

Flamingo

Extend BBGLR protocol to:

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Three ideas:

1. Dropout resilience by encrypting to a small group of decryptors
2. Reusable secrets across rounds
3. **Per-round graphs to handle changing client sets**

Reusable secrets across rounds

- BBGLR uses a sparse graph (for pairwise secrets) to minimize communication between clients
- But across rounds, clients may go offline —> don't want to set up a new graph every time
- One approach: Make one graph at setup time, and then use the subgraph for the participating clients in each subsequent round

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 - Problem: If the graph is more dense, then the communication overheads are high

Reusable secrets across rounds

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- One approach: Make one graph at setup time, and then use the subgraph for the participating clients in each subsequent round
 - Problem: If the graph is fairly sparse, and so if clients go online it might not be connected and may have isolated nodes (hurting privacy)
 - Problem: If the graph is more dense, then the communication overheads are high
- Idea: client can use random seed along with knowledge of which clients are participating in the round to compute its set of neighbors
 - Does not have to materialize the entire graph

Outline

1. Federated learning
2. Flamingo
- 3. Other research topics**
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Zero-knowledge proofs

Informally: Prove that some statement is true without revealing the evidence

More formally: For languages \mathcal{L} in NP with instance x and witness w , prove that $x \in \mathcal{L}$ without revealing w

Applications:

- Cryptocurrencies and blockchains
- Anonymous credentials (age verification, CAPTCHA alternative)
- Middleboxes that enforce properties on encrypted traffic [Zero-knowledge middleboxes]
- Edited image is derived from a photo taken by a certified camera [VerITAS]
- Encrypted authentication logging [Larch]
- ... and more

Homomorphic encryption

- Perform arbitrary computation on encrypted data
- Most efficient for circuits with low multiplicative depth
- Possible to perform computations with unbounded depth, but at a high concrete cost
- Applications to:
 - Searching on data (e.g., Tiptoe)
 - Inference where the client's input remains hidden
 - ... and more

Secure inference

- How can you query a model hosted at a server without the server seeing your query?
- Challenge: large scale, operations that are not “crypto-friendly”
- Common approach: use a blend of multi-party computation and homomorphic encryption (Bolt, Delphi, Gazelle)
- Moving forward:
 - Can we build better cryptographic tools tailored to transformers?
 - Are there ways we can better co-design transformer inference and cryptographic tools?
 - Are there opportunities to use hardware to accelerate cryptographic operations?

Next steps for security + cryptography at Stanford

Courses:

- CS 251: Cryptocurrencies and blockchain technologies (fall)
- CS 255: Introduction to Cryptography (winter)
- CS 258: Quantum Cryptography (fall)
- CS 329T: Trustworthy Machine Learning (fall)
- CS 355: Advanced Topics in Cryptography (spring)
- CS 356: Topics in Computer and Network Security (fall)
- CS 357S: Formal Methods for Computer Systems (winter)

Events:

- Security lunch (Wednesday @12PM, CoDa E160): <https://securitylunch.stanford.edu/>
- Security seminar (some Thursdays @4PM): <https://crypto.stanford.edu/seclab/sem.html>

If you're interested in research / continuing your course project, reach out!

Week 10: Final presentations

12/2, 12/4: 9 minutes for presentation, 2 minutes for questions / transitioning to next group

12/2: Final project reports due

Outline

1. Federated learning
2. Flamingo
3. Other research topics
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References

Bell, James Henry, Kallista A. Bonawitz, Adrià Gascón, Tancrede Lepoint, and Mariana Raykova. "Secure single-server aggregation with (poly) logarithmic overhead." In *Proceedings of the 2020 ACM SIGSAC conference on computer and communications security*, pp. 1253-1269. 2020.

Datta, Trisha, Binyi Chen, and Dan Boneh. "VerITAS: Verifying image transformations at scale." In *2025 IEEE Symposium on Security and Privacy (SP)*, pp. 4606-4623. IEEE, 2025.

Dauterman, Emma, Danny Lin, Henry Corrigan-Gibbs, and David Mazières. "Accountable authentication with privacy protection: The Larch system for universal login." In *17th USENIX Symposium on Operating Systems Design and Implementation (OSDI 23)*, pp. 81-98. 2023.

Grubbs, Paul, Arasu Arun, Ye Zhang, Joseph Bonneau, and Michael Walfish. "{Zero-Knowledge} middleboxes." In *31st USENIX Security Symposium (USENIX Security 22)*, pp. 4255-4272. 2022.

Juvekar, Chiraag, Vinod Vaikuntanathan, and Anantha Chandrakasan. "{GAZELLE}: A low latency framework for secure neural network inference." In *27th USENIX security symposium (USENIX security 18)*, pp. 1651-1669. 2018.

Ma, Yiping, Jess Woods, Sebastian Angel, Antigoni Polychroniadou, and Tal Rabin. "Flamingo: Multi-round single-server secure aggregation with applications to private federated learning." In *2023 IEEE Symposium on Security and Privacy (SP)*, pp. 477-496. IEEE, 2023.

McMahan, Brendan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Aguera y Arcas. "Communication-efficient learning of deep networks from decentralized data." In *Artificial intelligence and statistics*, pp. 1273-1282. PMLR, 2017.

Mishra, Pratyush, Ryan Lehmkuhl, Akshayaram Srinivasan, Wenting Zheng, and Raluca Ada Popa. "Delphi: A cryptographic inference system for neural networks." In *Proceedings of the 2020 Workshop on Privacy-Preserving Machine Learning in Practice*, pp. 27-30. 2020.

Pang, Qi, Jinhao Zhu, Helen Möllering, Wenting Zheng, and Thomas Schneider. "Bolt: Privacy-preserving, accurate and efficient inference for transformers." In *2024 IEEE Symposium on Security and Privacy (SP)*, pp. 4753-4771. IEEE, 2024.

<https://research.google/blog/federated-learning-collaborative-machine-learning-without-centralized-training-data/>

<https://research.google/blog/federated-learning-with-formal-differential-privacy-guarantees/>