$g(n) = 1 + c + c^2 + \ldots + c^n$

3) a) if $c<1$ then all $c^k<1$ $1 \le k \le n$, then look at the largest factor which is the first term, 1, which dominates all c^k. So it will simplify to 1, thus $\Theta(1)$

b) if $c=1$ then we must add 1 n number of times thus giving us $\Theta(n)$.

c) if $c>1$ then we must look at the greatest term which will be c^n, which will dominate all c^k $k<n$. So $g(n)$ is dominated by c^n so $\Theta(c^n)$

4) Show $\log(n!) = \Theta(n \log n)$

$$\lim_{n\to\infty} \frac{\log(n!)}{n \log n} = \lim_{n\to\infty} \frac{\log(n!)}{\log(n^n)} = \lim_{n\to\infty} \frac{n!}{n^n}$$

$n! = n \cdot (n-1) \cdot \ldots \cdot 2 \cdot 1 \le n^n$ $\quad n! \le n^n$ $\quad \log(n!) \le \log(n^n)$

$\log(n!) \le n \log n \Rightarrow \log(n!) = O(n \log n)$

$n! = \underbrace{n \cdot (n-1) \cdot \ldots}_{\text{take first } n/2 \text{ terms}} \cdot 2 \cdot 1 \ge (n/2)^{n/2}$ $\quad n! \ge (n/2)^{n/2}$

$$\log(n!) \ge \log(n/2)^{n/2} = \tfrac{n}{2} \log(n/2) = n \log(n/2) = n(\log n - \log 2)$$
$$= n \log n$$

$\log(n!) \ge n \log n \Rightarrow \log(n!) = \Omega(n \log n)$

Thus $\log(n!) = \Theta(n \log n)$. ∎