| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$ |
| 6 | $\left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} \right\}$ |
| 8 | $c_{01} = A_0 (b_1^T)^T + B_0 (a_1^T)^T + c_{01}$<br>$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$<br>$c_{10}^T = a_1 B_0^T + b_1^T A_0^T + c_{10}^T$ |
| 7 | $\left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} \right\}$ |
| 5b | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{ \qquad \}$ |
| 4 | where |
| 2 | $\left\{ \qquad \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \qquad \wedge \qquad \right\}$ |
| 5a | where |
| 6 | $\left\{ \qquad \right\}$ |
| 8 | |
| 7 | $\left\{ \qquad \right\}$ |
| 5b | |
| 2 | $\left\{ \qquad \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \qquad \wedge \neg( \qquad ) \right\}$ |
| 1b | $\{ \qquad \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \wedge \right\}$ |
| 5a | where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \wedge \neg( \quad ) \right\}$ |
| 1b | $\{C = AB^T + BA^T + \widehat{C}$ $\}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | where |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \right\}$ |
| 5a | where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg( \quad ) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | where |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
| --- | --- |
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | $A \rightarrow \left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right)$ , $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C \rightarrow \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ <br> where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | where |
| 6 | $\{ \}$ |
| 8 | |
| 7 | $\{ \}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg (m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
|---|---|
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | $A \rightarrow \left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right)$ , $B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right)$ , $C \rightarrow \left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array} \right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$ |
| 6 | $\{ \}$ |
| 8 | |
| 7 | $\{ \}$ |
| 5b | $\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array} \right)$ , $\left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array} \right)$ , $\left( \begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$ |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg (m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
| --- | --- |
| 1a | $\{ C = \widehat{C} \}$ |
| 4 | $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$ |
| 6 | $\left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} \right\}$ |
| 8 | |
| 7 | $\{ \quad \}$ |
| 5b | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{ C = AB^T + BA^T + \widehat{C} \}$ |

| Step | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
| --- | --- |
| 1a | $\{C = \widehat{C} \qquad \}$ |
| 4 | $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$ , $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ <br> where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$ , $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ <br> where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$ |
| 6 | $\left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} \right\}$ |
| 8 | |
| 7 | $\left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} \right\}$ |
| 5b | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$ , $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{C = AB^T + BA^T + \widehat{C} \qquad \}$ |

<table>
<tr><th>Step</th><th>Algorithm: $C = AB^T + BA^T + \widehat{C}$</th></tr>
<tr><td>1a</td><td>$\{ C = \widehat{C} \qquad \}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), \; B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), \; C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$</td></tr>
<tr><td>3</td><td>while $m(C_{TL}) < m(C)$ do</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$</td></tr>
<tr><td>5a</td><td>$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \; \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$<br>where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$</td></tr>
<tr><td>6</td><td>$\left\{ \left(\begin{array}{ccc} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & \widehat{c}_{01} & \widehat{C}_{02} \\ \widehat{c}_{10}^T & \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$</td></tr>
<tr><td>8</td><td>$c_{01} = A_0 (b_1^T)^T + B_0 (a_1^T)^T + c_{01}$<br>$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$<br>$c_{10}^T = a_1 B_0^T + b_1^T A_0^T + c_{10}^T$</td></tr>
<tr><td>7</td><td>$\left\{ \left(\begin{array}{ccc} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \; \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg (m(C_{TL}) < m(C)) \right\}$</td></tr>
<tr><td>1b</td><td>$\{ C = AB^T + BA^T + \widehat{C} \qquad \}$</td></tr>
</table>

| | |
|---|---|
| | Algorithm: $C = AB^T + BA^T + \widehat{C}$ |
| | |
| | $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$ , $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ , $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ <br> where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$ |
| | |
| | while $m(C_{TL}) < m(C)$ do |
| | |
| | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$ , $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$ <br> where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$ |
| | |
| | $c_{01} = A_0 (b_1^T)^T + B_0 (a_1^T)^T + c_{01}$ <br> $\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$ <br> $c_{10}^T = a_1 B_0^T + b_1^T A_0^T + c_{10}^T$ |
| | |
| | $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$ , $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ |
| | |
| | endwhile |
| | |
| | |

**Algorithm:** $C = AB^T + BA^T + \widehat{C}$

$$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$$

where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$

**while** $m(C_{TL}) < m(C)$ **do**

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$

$$c_{01} = A_0(b_1^T)^T + B_0(a_1^T)^T + c_{01}$$

$$\gamma_{11} = a_1^T(b_1^T)^T + b_1^T(a_1^T)^T + \gamma_{11}$$

$$c_{10}^T = a_1 B_0^T + b_1^T A_0^T + c_{10}^T$$

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

**endwhile**