

|      |   |
|------|---|
| Step | Algorithm: $C = AB^T + BA^T + C$  |
| 1a   | $\{C = \hat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$  |
| 5a   | Determine block size $b$<br>$\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ \hline A_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline B_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$<br>where $A_1$ has $b$ rows, $B_1$ has $b$ rows, $C_{11}$ is $b \times b$ |
| 6    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{C}_{01} & \hat{C}_{02} \\ \hline A_1^T B_0^T + \hat{C}_{10}^T & \hat{C}_{11} & \hat{C}_{12}^T \\ \hline A_2 B_0^T + \hat{C}_{20} & \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$  |
| 8    | $C_{01} = A_0(B_1^T)^T + B_0(A_1^T)^T + C_{01}^T$<br>$C_{11} = A_1^T(B_1^T)^T + B_1^T(A_1^T)^T + C_{11}$<br>$C_{10}^T = B_1^T A_0^T + C_{10}^T$<br>$C_{20} = A_2(B_1^T)^T + C_{20}^T$<br>$C_{21} = A_2 B_0^T + A_1^T B_0^T + C_{20}^T$  |
| 7    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & A_0(B_1^T)^T + B_0(A_1^T)^T + \hat{C}_{01} & \hat{C}_{02} \\ \hline A_1 B_0^T + B_1^T A_0^T + \hat{C}_{10}^T & A_1^T(b_1^T)^T + B_1^T(A_1^T)^T + \hat{C}_{11} & \hat{C}_{12}^T \\ \hline A_2(b_1^T)^T + A_2 B_0^T + \hat{C}_{20} & A_2 B_0^T + A_1^T B_0^T + \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$   |
| 5b   | $\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ \hline A_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline B_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$  |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$   |

|      |                                  |
|------|----------------------------------|
| Step | Algorithm: $C = AB^T + BA^T + C$ |
| 1a   | {                                |
| 4    | where                            |
| 2    | {                                |
| 3    | while do                         |
| 2,3  | {                                |
| 5a   | Determine block size $b$         |
|      | where                            |
| 6    | {                                |
| 8    |                                  |
| 7    | {                                |
| 5b   |                                  |
| 2    | {                                |
|      | endwhile                         |
| 2,3  | {                                |
| 1b   | {                                |

|      |                                  |
|------|----------------------------------|
| Step | Algorithm: $C = AB^T + BA^T + C$ |
| 1a   | { $C = \hat{C}$ }                |
| 4    | where                            |
| 2    | {                                |
| 3    | while do                         |
| 2,3  | { $\wedge$ }                     |
| 5a   | Determine block size $b$         |
|      | where                            |
| 6    | {                                |
| 8    |                                  |
| 7    | {                                |
| 5b   |                                  |
| 2    | {                                |
|      | endwhile                         |
| 2,3  | { $\wedge \neg($ ) }             |
| 1b   | { $C = AB^T + BA^T + \hat{C}$ }  |

|      |  |
|------|--|
| Step | Algorithm: $C = AB^T + BA^T + C$   |
| 1a   | $\{C = \hat{C}$  |
| 4    |  |
|      | where  |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$                      |
| 3    | while do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \right\}$               |
| 5a   | Determine block size $b$   |
|      | where  |
| 6    | $\left\{ \right.$  |
| 8    |  |
| 7    | $\left\{ \right.$  |
| 5b   |  |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$                      |
|      | endwhile   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg( \quad ) \right\}$ |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$  |

|      |   |
|------|---|
| Step | Algorithm: $C = AB^T + BA^T + C$  |
| 1a   | $\{C = \hat{C}$   |
| 4    |   |
|      | where   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$                               |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$       |
| 5a   | Determine block size $b$  |
|      | where   |
| 6    | $\left\{ \right.$   |
| 8    |   |
| 7    | $\left\{ \right.$   |
| 5b   |   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$                               |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_TB_T^T + B_TA_T^T + \hat{C}_{TL} & * \\ \hline A_BB_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$   |

|      |   |
|------|---|
| Step | Algorithm: $C = AB^T + BA^T + C$  |
| 1a   | $\{C = \hat{C}$ }   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ <p style="color: red;">where <math>A_T</math> has 0 rows, <math>B_T</math> has 0 rows, <math>C_{TL}</math> is <math>0 \times 0</math></p> |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$  |
| 5a   | <p><b>Determine block size <math>b</math></b></p> <p>where</p>  |
| 6    | $\left\{ \right.$   |
| 8    |   |
| 7    | $\left\{ \right.$   |
| 5b   |   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$  |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$ }   |

|      |  |
|------|--|
| Step | Algorithm: $C = AB^T + BA^T + C$   |
| 1a   | $\{C = \hat{C}$  |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$  |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$   |
| 3    | while $m(C_{TL}) < m(C)$ do  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$   |
| 5a   | Determine block size $b$<br>$\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \rightarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & \\ A_2 & \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \rightarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & \\ B_2 & \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right)$<br>where $A_1$ has $b$ rows, $B_1$ has $b$ rows, $C_{11}$ is $b \times b$ |
| 6    | $\left\{ \right.$  |
| 8    |  |
| 7    | $\left\{ \right.$  |
| 5b   | $\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \leftarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & \\ A_2 & \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \leftarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & \\ B_2 & \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right)$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$   |
|      | endwhile   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$   |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$  |

|      |  |
|------|--|
| Step | Algorithm: $C = AB^T + BA^T + C$   |
| 1a   | $\{C = \hat{C}$ }  |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$  |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$   |
| 3    | while $m(C_{TL}) < m(C)$ do  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$   |
| 5a   | Determine block size $b$<br>$\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \rightarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & A_2 \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \rightarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right)$<br>where $A_1$ has $b$ rows, $B_1$ has $b$ rows, $C_{11}$ is $b \times b$ |
| 6    | $\left\{ \begin{pmatrix} C_{00} & C_{01} & C_{02} \\ C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{C}_{01} & \hat{C}_{02} \\ A_1^T B_0^T + \hat{C}_{10}^T & \hat{C}_{11} & \hat{C}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{C}_{21} & \hat{C}_{22} \end{pmatrix} \right\}$   |
| 8    |  |
| 7    | $\left\{ \right\}$   |
| 5b   | $\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \leftarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & A_2 \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \leftarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right)$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$   |
|      | endwhile   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$   |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$ }  |



|      |   |
|------|---|
| Step | Algorithm: $C = AB^T + BA^T + C$  |
| 1a   | $\{C = \hat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c} A_T \\ A_B \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$  |
| 5a   | Determine block size $b$<br>$\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ A_1^T \\ A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ B_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$<br>where $A_1$ has $b$ rows, $B_1$ has $b$ rows, $C_{11}$ is $b \times b$ |
| 6    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{C}_{01} & \hat{C}_{02} \\ A_1^T B_0^T + \hat{C}_{10}^T & \hat{C}_{11} & \hat{C}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$  |
| 8    |   |
| 7    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & A_0 (B_1^T)^T + B_0 (A_1^T)^T + \hat{C}_{01} & \hat{C}_{02} \\ A_1 B_0^T + B_1^T A_0^T + \hat{C}_{10}^T & A_1^T (b_1^T)^T + B_1^T (A_1^T)^T + \hat{C}_{11} & \hat{C}_{12}^T \\ A_2 (b_1^T)^T + A_2 B_0^T + \hat{C}_{20} & A_2 B_0^T + A_1^T B_0^T + \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$  |
| 5b   | $\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ A_1^T \\ A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ B_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$  |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$   |

|      |   |
|------|---|
| Step | Algorithm: $C = AB^T + BA^T + C$  |
| 1a   | $\{C = \hat{C}$   |
| 4    | $A \rightarrow \left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right), B \rightarrow \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ <p>where <math>A_T</math> has 0 rows, <math>B_T</math> has 0 rows, <math>C_{TL}</math> is <math>0 \times 0</math></p>   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$  |
| 5a   | <p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \rightarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & A_2 \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \rightarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$ <p>where <math>A_1</math> has <math>b</math> rows, <math>B_1</math> has <math>b</math> rows, <math>C_{11}</math> is <math>b \times b</math></p> |
| 6    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{C}_{01} & \hat{C}_{02} \\ A_1^T B_0^T + \hat{C}_{10}^T & \hat{C}_{11} & \hat{C}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$  |
| 8    | $C_{01} = A_0(B_1^T)^T + B_0(A_1^T)^T + C_{01}^T$ $C_{11} = A_1^T(B_1^T)^T + B_1^T(A_1^T)^T + C_{11}$ $C_{10}^T = B_1^T A_0^T + C_{10}^T$ $C_{20} = A_2(B_1^T)^T + C_{20}^T$ $C_{21} = A_2 B_0^T + A_1^T B_0^T + C_{20}^T$  |
| 7    | $\left\{ \left( \begin{array}{ccc} C_{00} & C_{01} & C_{02} \\ C_{10}^T & C_{11} & C_{12}^T \\ C_{20} & C_{21} & C_{22} \end{array} \right) = \left( \begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & A_0(B_1^T)^T + B_0(A_1^T)^T + \hat{C}_{01} & \hat{C}_{02} \\ A_1 B_0^T + B_1^T A_0^T + \hat{C}_{10}^T & A_1^T(b_1^T)^T + B_1^T(A_1^T)^T + \hat{C}_{11} & \hat{C}_{12}^T \\ A_2(b_1^T)^T + A_2 B_0^T + \hat{C}_{20} & A_2 B_0^T + A_1^T B_0^T + \hat{C}_{21} & \hat{C}_{22} \end{array} \right) \right\}$   |
| 5b   | $\left( \begin{array}{c c} A_T & \\ \hline & A_B \end{array} \right) \leftarrow \left( \begin{array}{c c} A_0 & \\ \hline A_1^T & A_2 \end{array} \right), \left( \begin{array}{c c} B_T & \\ \hline & B_B \end{array} \right) \leftarrow \left( \begin{array}{c c} B_0 & \\ \hline B_1^T & B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \right\}$  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ \hline A_B B_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$  |
| 1b   | $\{C = AB^T + BA^T + \hat{C}$   |

|  |   |
|--|---|
|  | Algorithm: $C = AB^T + BA^T + C$  |
|  |   |
|  | $A \rightarrow \left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right), C \rightarrow \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$ <p>where <math>A_T</math> has 0 rows, <math>B_T</math> has 0 rows, <math>C_{TL}</math> is <math>0 \times 0</math></p>   |
|  |   |
|  | while $m(C_{TL}) < m(C)$ do   |
|  |   |
|  | <p><b>Determine block size <math>b</math></b></p> $\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ \hline A_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ \hline B_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$ <p>where <math>A_1</math> has <math>b</math> rows, <math>B_1</math> has <math>b</math> rows, <math>C_{11}</math> is <math>b \times b</math></p> |
|  |   |
|  | $C_{01} = A_0(B_1^T)^T + B_0(A_1^T)^T + C_{01}^T$ $C_{11} = A_1^T(B_1^T)^T + B_1^T(A_1^T)^T + C_{11}$ $C_{10}^T = B_1^T A_0^T + C_{10}^T$ $C_{20} = A_2(B_1^T)^T + C_{20}^T$ $C_{21} = A_2 B_0^T + A_1^T B_0^T + C_{20}^T$  |
|  |   |
|  | $\left( \begin{array}{c} A_T \\ \hline A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ \hline A_1^T \\ \hline A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ \hline B_1^T \\ \hline B_2 \end{array} \right), \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$   |
|  |   |
|  | endwhile  |
|  |   |
|  |   |

**Algorithm:**  $C = AB^T + BA^T + C$

$$A \rightarrow \left( \begin{array}{c} A_T \\ A_B \end{array} \right), B \rightarrow \left( \begin{array}{c} B_T \\ B_B \end{array} \right), C \rightarrow \left( \begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$$

**where**  $A_T$  has 0 rows,  $B_T$  has 0 rows,  $C_{TL}$  is  $0 \times 0$

**while**  $m(C_{TL}) < m(C)$  **do**

**Determine block size**  $b$

$$\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \rightarrow \left( \begin{array}{c} A_0 \\ A_1^T \\ A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \rightarrow \left( \begin{array}{c} B_0 \\ B_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left( \begin{array}{c|cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$$

**where**  $A_1$  has  $b$  rows,  $B_1$  has  $b$  rows,  $C_{11}$  is  $b \times b$

$$C_{01} = A_0(B_1^T)^T + B_0(A_1^T)^T + C_{01}^T$$

$$C_{11} = A_1^T(B_1^T)^T + B_1^T(A_1^T)^T + C_{11}$$

$$C_{10}^T = B_1^T A_0^T + C_{10}^T$$

$$C_{20} = A_2(B_1^T)^T + C_{20}^T$$

$$C_{21} = A_2 B_0^T + A_1^T B_0^T + C_{20}^T$$

$$\left( \begin{array}{c} A_T \\ A_B \end{array} \right) \leftarrow \left( \begin{array}{c} A_0 \\ A_1^T \\ A_2 \end{array} \right), \left( \begin{array}{c} B_T \\ B_B \end{array} \right) \leftarrow \left( \begin{array}{c} B_0 \\ B_1^T \\ B_2 \end{array} \right), \left( \begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left( \begin{array}{c|cc} C_{00} & C_{01} & C_{02} \\ \hline C_{10}^T & C_{11} & C_{12}^T \\ \hline C_{20} & C_{21} & C_{22} \end{array} \right)$$

**endwhile**