Step	Algorithm: $[C] := \text{SYR}2\text{K}_{\text{LN}}\text{UNB}_{\text{VAR}}4(A, B, C)$
1a	$\{C = \widehat{C} \}$
	$C \to \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ where $C_{TL}$ is $0 \times 0$ , $A_T$ has 0 rows, $B_T$ has 0 rows
2	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
3	while $m(C_{TL}) < m(C)$ do
2,3	$\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C)$
5a	$ \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \to \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \to \left(\begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right) $ where $\gamma_{11}$ is $1 \times 1$ , $a_1$ has 1 row, $b_1$ has 1 row
6	$ \left\{ \begin{array}{c c} \frac{C_{00}}{c_{10}} & * & * \\ \hline c_{10}^{T} & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right\} = \left(\begin{array}{c c} \frac{A_{0}B_{0}^{T} + B_{0}A_{0}^{T} + \widehat{C}_{00}}{b_{1}^{T}A_{0}^{T} + \widehat{C}_{10}^{T}} & * & * \\ \hline b_{1}^{T}A_{0}^{T} + \widehat{C}_{10}^{T} & \gamma_{11} & * \\ B_{2}A_{0}^{T} + \widehat{C}_{20} & c_{21} & C_{22} \end{array}\right) $
8	$ \gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}  c_{10}^T := a_1^T B_0^T + c_{10}^T  c_{21} := B_2 a_1 + c_{21} $
7	$ \left\{ \begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right) = \left( \begin{array}{c c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ \hline a_1^T B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & * \\ \hline B_2 A_0^T + \widehat{C}_{20} & B_2 a_1 + \widehat{c}_{21} & C_{22} \end{array} \right) $
5b	$ \left(\begin{array}{c c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) , \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right) , \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1^T \\ \hline B_2 \end{array}\right) $
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
	endwhile
2,3	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))  $
1b	$\left\{ [C] = \operatorname{syr}2k \ln(A, B, \widehat{C}) \right\}$

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	{
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$\left\{ \qquad \qquad \land \neg ( \qquad ) \right\}$
1b	{

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	$\{C = \widehat{C}$
4	where
2	
3	while do
2,3	
5a	where
6	
8	
7	
5b	
2	
	endwhile
2,3	$ \left\{ \qquad \qquad \wedge \neg ( \qquad \qquad ) \right. $
1b	$\left\{ [C] = \operatorname{syr}2\mathrm{k.ln}(A, B, \widehat{C}) \right\}$

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	$\{C=\widehat{C}$
4	where
2	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
3	while do
2,3	$\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \right.$
5a	where
6	
8	
7	
5b	
2	$\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
	endwhile
2,3	$\left\{ \begin{pmatrix} C_{TL} & * \\ C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} A_T B_T^T + B_T A_T^T + \hat{C}_{TL} & * \\ B_B A_T^T + \hat{C}_{BL} & \hat{C}_{BR} \end{pmatrix} \land \neg ( ) \right\}$
1b	$\{ [C] = \operatorname{syr}2k \ln(A, B, \widehat{C}) $

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	$\left\{ C = \widehat{C} \right\}$
4	where
2	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$
3	while $m(C_{TL}) < m(C)$ do
2,3	$\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C)$
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$
1b	$\{[C] = \operatorname{syr}2k \ln(A, B, \widehat{C}) $

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	$\{C = \widehat{C}$
4	$C \to \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ where $C_{TL}$ is $0 \times 0$ , $A_T$ has 0 rows, $B_T$ has 0 rows
2	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
3	while $m(C_{TL}) < m(C)$ do
2,3	$\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) $
5a	where
6	
8	
7	
5b	
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C)) $
1b	$\left\{ [C] = \operatorname{syr}2\mathrm{k.ln}(A, B, \widehat{C}) \right\}$

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$
1a	$\{C = \widehat{C}$
4	$C \to \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ where $C_{TL}$ is $0 \times 0$ , $A_T$ has 0 rows, $B_T$ has 0 rows
2	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
3	while $m(C_{TL}) < m(C)$ do
2,3	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) $
5a	$ \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) $ where $\gamma_{11}$ is $1 \times 1$ , $a_1$ has 1 row, $b_1$ has 1 row
6	
8	
7	
5b	$\left( \begin{array}{c c} C_{BL} & C_{BR} \end{array} \right)  \left( \begin{array}{c c} C_{20} & c_{21} & C_{22} \end{array} \right)  \left( \begin{array}{c c} A_B \end{array} \right)  \left( \begin{array}{c c} A_2 \end{array} \right)  \left( \begin{array}{c c} B_B \end{array} \right)  \left( \begin{array}{c c} B_2 \end{array} \right)$
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $
	endwhile
2,3	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$
1b	$\{[C] = \operatorname{syr}2k \ln(A, B, \widehat{C}) $

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$	
1a	$\{C=\widehat{C}$	}
4	$C \to \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ where $C_{TL}$ is $0 \times 0$ , $A_T$ has 0 rows, $B_T$ has 0 rows	
2	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $	$\left. \begin{array}{c} \\ \end{array} \right\}$
3	while $m(C_{TL}) < m(C)$ do	
2,3	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land m(C_{TL}) < m(C) $	$oxed{\ }$
5a	$ \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right) $ where $\gamma_{11}$ is $1 \times 1$ , $a_1$ has 1 row, $b_1$ has 1 row	
6	$ \begin{cases} \frac{C_{00}}{c_{10}^{T}} & * & * \\ \frac{C_{10}}{c_{10}^{T}} & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{cases} = \begin{pmatrix} \frac{A_{0}B_{0}^{T} + B_{0}A_{0}^{T} + \widehat{C}_{00}}{b_{1}^{T}A_{0}^{T} + \widehat{C}_{10}} & * & * \\ \frac{b_{1}^{T}A_{0}^{T} + \widehat{C}_{10}^{T}}{B_{2}A_{0}^{T} + \widehat{C}_{20}} & c_{21} & C_{22} \end{cases} $	$\left. \begin{array}{c} \\ \end{array} \right\}$
8		
7		$\left \right\}$
5b	$ \left(\begin{array}{c c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) $	
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $	igg
	endwhile	
2,3	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$	$\bigg\}$
1b	$\{[C] = \operatorname{syr}2k \ln(A, B, \widehat{C})$	}

Step	Algorithm: $[C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)$	
1a	$\{C=\widehat{C}$	}
4	$C \to \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c c} B_T \\ \hline B_B \end{array}\right)$ where $C_{TL}$ is $0 \times 0$ , $A_T$ has 0 rows, $B_T$ has 0 rows	
2	$ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $	$\left. \begin{array}{c} \\ \end{array} \right\}$
3	while $m(C_{TL}) < m(C)$ do	
2,3	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land m(C_{TL}) < m(C) $	oggr e
5a	$ \left(\begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right) $ where $\gamma_{11}$ is $1 \times 1$ , $a_1$ has 1 row, $b_1$ has 1 row	
6	$ \begin{cases} \frac{C_{00} * * *}{c_{10}^{T} \gamma_{11} *} = \begin{pmatrix} A_{0}B_{0}^{T} + B_{0}A_{0}^{T} + \widehat{C}_{00} * * * \\ b_{1}^{T}A_{0}^{T} + \widehat{C}_{10}^{T} & \gamma_{11} * \\ B_{2}A_{0}^{T} + \widehat{C}_{20} & c_{21} C_{22} \end{pmatrix} $	$\left. \begin{array}{c} - \\ \end{array} \right\}$
8		
7	$ \left\{ \begin{array}{c c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right) = \left( \begin{array}{c c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ a_1^T B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & * \\ \hline B_2 A_0^T + \widehat{C}_{20} & B_2 a_1 + \widehat{c}_{21} & C_{22} \end{array} \right) $	$\left. \begin{array}{c} \\ \end{array} \right\}$
5b	$ \left(\begin{array}{c c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right) $	
2	$ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $	igg
	endwhile	
2,3	$\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$	$\bigg\}$
1b	$\{[C] = \operatorname{syr}2k \ln(A, B, \widehat{C})$	}

```
Algorithm: [C] := \text{SYR}2\text{K\_LN\_UNB\_VAR}4(A, B, C)
Step
                        {C = \widehat{C}}
    1a
                      C \to \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) , A \to \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) , B \to \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)
     4
                               where C_{TL} is 0 \times 0, A_T has 0 rows, B_T has 0 rows
     2
     3
                        while m(C_{TL}) < m(C) do
                                                                                          2,3

\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \to \left(\begin{array}{c|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{31} & C_{22} \\ \end{array}\right), \left(\begin{array}{c|c} A_T \\ \hline A_B \end{array}\right) \to \left(\begin{array}{c|c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c|c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c|c} B_T \\ \hline B_B \end{array}\right) \to \left(\begin{array}{c|c} B_T \\ \hline B_B \end{array}\right)

    5a
                                             where \gamma_{11} is 1 \times 1, a_1 has 1 row, b_1 has 1 row

\begin{vmatrix}
C_{00} & * & * \\
C_{10}^T & \gamma_{11} & * \\
C_{20} & c_{21} & C_{22}
\end{vmatrix} = \begin{pmatrix}
A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & * & * \\
b_1^T A_0^T + \hat{c}_{10}^T & \gamma_{11} & * \\
B_2 A_0^T + \hat{C}_{20} & c_{21} & C_{22}
\end{vmatrix}

     6
                                       \gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}
                                      c_{10}^T := a_1^T B_0^T + c_{10}^T
     8
                                       c_{21} := B_2 a_1 + c_{21}

\begin{pmatrix}
C_{00} & * & * \\
c_{10}^{T} & \gamma_{11} & * \\
C_{20} & c_{21} & C_{22}
\end{pmatrix} = \begin{pmatrix}
A_{0}B_{0}^{T} + B_{0}A_{0}^{T} + \widehat{C}_{00} & * & * \\
a_{1}^{T}B_{0}^{T} + b_{1}^{T}A_{0}^{T} + \widehat{c}_{10}^{T} & a_{1}^{T}b_{1} + b_{1}^{T}a_{1} + \widehat{\gamma}_{11} & * \\
B_{2}A_{0}^{T} + \widehat{C}_{20} & B_{2}a_{1} + \widehat{c}_{21} & C_{22}
\end{pmatrix}

                                                                                                                           A_0 B_0^T + \overline{B_0 A_0^T + \widehat{C}_{00}}
     7

\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{C} & c & C \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} A_0 \\ \hline a_1^T \\ \hline A \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} B_T \\ \hline A \end{array}\right)

   5b
                                                                                         \frac{1}{A_T B_T^T + B_T A_T^T + \widehat{C}_{TL}} = \begin{pmatrix} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} 
     2
                        endwhile
                                                                                                 \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg (m(C_{TL}) < m(C))
  2,3
                        \{[C] = \operatorname{syr}2k\ln(A, B, \widehat{C})\}
   1b
```

Algorithm:  $[C] := SYR2K_LN_UNB_VAR4(A, B, C)$ 

$$C \to \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \to \left(\begin{array}{c|c} A_T \\ \hline A_B \end{array}\right), B \to \left(\begin{array}{c|c} B_T \\ \hline B_B \end{array}\right)$$

where  $C_{TL}$  is  $0 \times 0$ ,  $A_T$  has 0 rows,  $B_T$  has 0 rows

while  $m(C_{TL}) < m(C)$  do

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) , \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c|c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right) , \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

where  $\gamma_{11}$  is  $1 \times 1$ ,  $a_1$  has 1 row,  $b_1$  has 1 row

$$\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}$$

$$c_{10}^T := a_1^T B_0^T + c_{10}^T$$

$$c_{21} := B_2 a_1 + c_{21}$$

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) , \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c|c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right) , \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$$

endwhile