| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | $\{C = \widehat{C}\}$   |
| 4    | $A \to \left(\frac{A_T}{A_B}\right), B \to \left(\frac{B_T}{B_B}\right), C \to \left(\frac{C_{TL}}{C_{BL}}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$   |
| 2    | $ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $   |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land m(C_{TL}) < m(C) $   |
| 5a   | $ \left(\frac{A_T}{A_B}\right) \to \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}}{C_{BL}} \begin{vmatrix} C_{TR} \\ C_{BL} \end{vmatrix} \begin{vmatrix} C_{TR} \\ C_{BR} \end{vmatrix}\right) \to \left(\frac{C_{00}}{c_{01}} \begin{vmatrix} c_{01} & C_{02} \\ C_{10} & c_{11} & c_{12} \\ C_{20} & c_{21} & C_{22} \end{vmatrix}\right) $ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$   |
| 6    | $ \left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{c}_{01} & \hat{C}_{02} \\ a_1^T B_0^T + \hat{c}_{10}^T & \hat{\gamma}_{11} & \hat{c}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{c}_{21} & \hat{C}_{22} \end{pmatrix} $   |
| 8    | $c_{01} = A_0(b_1^T)^T + B_0(a_1^T)^T + c_{01}^T$ $\gamma_{11} = a_1^T(b_1^T)^T + b_1^T(a_1^T)^T + \gamma_{11}$ $c_{10}^T = b_1^T A_0^T + c_{10}^T$ $C_{20} = A_2(b_1^T)^T + C_{20}^T$ $c_{21} = A_2 B_0^T + a_1^T B_0^T + C_{20}^T$  |
| 7    | $ \left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ A_2 (b_1^T)^T + A_2 B_0^T + \widehat{C}_{20} & A_2 B_0^T + a_1^T B_0^T + \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} $ |
| 5b   | $\left(\frac{A_{T}}{A_{B}}\right) \leftarrow \left(\frac{A_{0}}{a_{1}^{T}}\right), \left(\frac{B_{T}}{B_{B}}\right) \leftarrow \left(\frac{B_{0}}{b_{1}^{T}}\right), \left(\frac{C_{TL} C_{TR}}{C_{BL} C_{BR}}\right) \leftarrow \left(\frac{C_{00} c_{01} C_{02}}{c_{10}^{T} \gamma_{11} c_{12}^{T}}\right)$   |
| 2    | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$   |
| 1b   | $\{C = AB^T + BA^T + \widehat{C} $ }  |

| Step | Algorithm: $C = AB^T + BA^T + C$                                   |
|------|--|
| 1a   | {  |
| 4    | where  |
| 2    |  |
| 3    | while do   |
| 2,3  |  |
| 5a   | where  |
| 6    |  |
| 8    |  |
| 7    |  |
| 5b   |  |
| 2    |  |
|      | endwhile   |
| 2,3  | $ \left\{ \begin{array}{c} \wedge \neg ( \\ \end{array} \right. )$ |
| 1b   | {  |

| Step | Algorithm: $C = AB^T + BA^T + C$  |          |   |
|------|-----------------------------------|----------|---|
| 1a   | $\{C=\widehat{C}$                 |          | } |
| 4    | where                             |          |   |
| 2    |                                   |          |   |
| 3    | while do                          |          |   |
| 2,3  |                                   | $\wedge$ |   |
| 5a   | where                             |          |   |
| 6    |                                   |          |   |
| 8    |                                   |          |   |
| 7    |                                   |          |   |
| 5b   |                                   |          |   |
| 2    |                                   |          |   |
|      | endwhile                          |          |   |
| 2,3  |                                   | ∧¬(      |   |
| 1b   | $\{C = AB^T + BA^T + \widehat{C}$ |          | } |

| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | ${C = \widehat{C}}$   |
| 4    | where   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$                   |
| 3    | while do  |
| 2,3  | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \right.$           |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 7    |   |
| 5b   |   |
| 2    | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$                          |
|      | endwhile  |
| 2,3  | $ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg ( )  $ |
| 1b   | $\left\{ C = AB^T + BA^T + \widehat{C} \right\}$  |

| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | $\{C = \widehat{C}$   |
| 4    | where   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$   |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \end{array} \right\}$ |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 7    |   |
| 5b   |   |
| 2    | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg (m(C_{TL}) < m(C)) \right\}$                         |
| 1b   | $\{C = AB^T + BA^T + \widehat{C} $ }  |

| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | $\{C = \widehat{C}\}$   |
| 4    | $A 	o \left(\frac{A_T}{A_B}\right), B 	o \left(\frac{B_T}{B_B}\right), C 	o \left(\frac{C_{TL} C_{TR}}{C_{BL} C_{BR}}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 	imes 0$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$   |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \end{array} \right\}$ |
| 5a   | where   |
| 6    |   |
| 8    |   |
| 7    |   |
| 5b   |   |
| 2    | $\left\{ \begin{array}{c c} \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right.$                                      |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$                                   |
| 1b   | $\{C = AB^T + BA^T + \widehat{C} $  |

| Step | Algorithm: $C = AB^T + BA^T + C$  |   |
|------|---|---|
| 1a   | $\{C=\widehat{C}$   | }   |
| 4    | $A 	o \left(\frac{A_T}{A_B}\right), B 	o \left(\frac{B_T}{B_B}\right), C 	o \left(\frac{C_{TL} C_{TR}}{C_{BL} C_{BR}}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 	imes 0$   |   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$   |   |
| 3    | while $m(C_{TL}) < m(C)$ do   |   |
| 2,3  | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C)$  |   |
| 5a   | $ \begin{pmatrix} A_T \\ A_B \end{pmatrix} \rightarrow \begin{pmatrix} A_0 \\ a_1^T \\ A_2 \end{pmatrix}, \begin{pmatrix} B_T \\ B_B \end{pmatrix} \rightarrow \begin{pmatrix} B_0 \\ b_1^T \\ B_2 \end{pmatrix}, \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix} \rightarrow \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} $ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is 1 × 1 |   |
| 6    |   |   |
| 8    |   |   |
| 7    |   | $\left.\begin{array}{c} \\ \end{array}\right\}$ |
| 5b   | $ \left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}^T   \gamma_{11}   c_{12}^T}\right) $   |   |
| 2    | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $  |   |
|      | endwhile  |   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$   | $\left. \right\}$                               |
| 1b   | $\{C = AB^T + BA^T + \widehat{C}$   | }   |

| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | $\{C = \widehat{C}$   |
| 4    | $A \to \left(\frac{A_T}{A_B}\right), B \to \left(\frac{B_T}{B_B}\right), C \to \left(\frac{C_{TL}}{C_{BL}}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$   |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C)$  |
| 5a   | where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$   |
| 6    | $ \left\{  \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} =  \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{c}_{01} & \hat{C}_{02} \\ a_1^T B_0^T + \hat{c}_{10}^T & \hat{\gamma}_{11} & \hat{c}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{c}_{21} & \hat{C}_{22} \end{pmatrix} $ |
| 8    |   |
| 7    |   |
| 5b   | $ \left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}   \gamma_{11}   c_{12}^T}\right) \\ \frac{c_{10}   c_{12}   c_{12}}{C_{20}   c_{21}   c_{22}}\right) $   |
| 2    | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) $  |
|      | endwhile  |
| 2,3  | $ \left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C)) $   |
| 1b   | $\{C = AB^T + BA^T + \widehat{C} $ }  |

| Step | Algorithm: $C = AB^T + BA^T + C$   |                   |
|------|--|-------------------|
| 1a   | $\{C=\widehat{C}$  | }                 |
| 4    | $A 	o \left(\frac{A_T}{A_B}\right), B 	o \left(\frac{B_T}{B_B}\right), C 	o \left(\frac{C_{TL}}{C_{BL}}\right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is 0 × 0  |                   |
| 2    | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$  | $\left. \right\}$ |
| 3    | while $m(C_{TL}) < m(C)$ do  |                   |
| 2,3  | $ \left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) $   | $\left. ight\}$   |
| 5a   | $ \left(\begin{array}{c} A_T \\ \overline{A_B} \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \overline{a_1^T} \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \overline{B_B} \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \overline{b_1^T} \\ B_2 \end{array}\right), \left(\begin{array}{c} C_{TL} & C_{TR} \\ \overline{C_{BL}} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c} C_{00} & c_{01} & C_{02} \\ \overline{c_{10}} & \gamma_{11} & c_{12}^T \\ \overline{C_{20}} & c_{21} & C_{22} \end{array}\right) $ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is 1 × 1 |                   |
| 6    | $ \begin{cases} \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \hat{C}_{00} & \hat{c}_{01} & \hat{C}_{02} \\ a_1^T B_0^T + \hat{c}_{10}^T & \hat{\gamma}_{11} & \hat{c}_{12}^T \\ A_2 B_0^T + \hat{C}_{20} & \hat{c}_{21} & \hat{C}_{22} \end{pmatrix} $  | $\left. \right\}$ |
| 8    |  |                   |
| 7    | $ \begin{cases} \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ A_2 (b_1^T)^T + A_2 B_0^T + \widehat{C}_{20} & A_2 B_0^T + a_1^T B_0^T + \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} $  | $\left. \right\}$ |
| 5b   | $\left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}^T   \gamma_{11}   c_{12}^T}\right)$  |                   |
| 2    | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$   | igg               |
|      | endwhile   |                   |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C))$  | $\left. \right\}$ |
| 1b   | $\{C = AB^T + BA^T + \widehat{C}$  | }                 |

| Step | Algorithm: $C = AB^T + BA^T + C$  |
|------|---|
| 1a   | $\{C = \widehat{C}$   |
| 4    | $A \to \begin{pmatrix} A_T \\ A_B \end{pmatrix}, B \to \begin{pmatrix} B_T \\ B_B \end{pmatrix}, C \to \begin{pmatrix} C_{TL} & C_{TR} \\ C_{BL} & C_{BR} \end{pmatrix}$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$   |
| 2    | $\left\{ \begin{pmatrix} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{pmatrix} = \begin{pmatrix} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{pmatrix} \right\}$  |
| 3    | while $m(C_{TL}) < m(C)$ do   |
| 2,3  | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) $   |
| 5a   | $ \left(\frac{A_T}{A_B}\right) \to \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL} C_{TR}}{C_{BL} C_{BR}}\right) \to \left(\frac{C_{00} c_{01} C_{02}}{c_{10}^T \gamma_{11} c_{12}^T}\right) $  |
| 6    | $ \begin{cases}                                    $  |
| 8    | $c_{01} = A_0(b_1^T)^T + B_0(a_1^T)^T + c_{01}^T$ $\gamma_{11} = a_1^T(b_1^T)^T + b_1^T(a_1^T)^T + \gamma_{11}$ $c_{10}^T = b_1^T A_0^T + c_{10}^T$ $C_{20} = A_2(b_1^T)^T + C_{20}^T$ $c_{21} = A_2 B_0^T + a_1^T B_0^T + C_{20}^T$  |
| 7    | $ \left\{ \begin{pmatrix} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & A_0 (b_1^T)^T + B_0 (a_1^T)^T + \widehat{c}_{01} & \widehat{C}_{02} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & \widehat{c}_{12}^T \\ A_2 (b_1^T)^T + A_2 B_0^T + \widehat{C}_{20} & A_2 B_0^T + a_1^T B_0^T + \widehat{c}_{21} & \widehat{C}_{22} \end{pmatrix} $ |
|      | $ \left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}^T   \gamma_{11}   c_{12}^T}\right) \\ \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{C_{20}   c_{21}   C_{22}}\right) $   |
| 2    | $\left\{ \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right)$  |
|      | endwhile  |
| 2,3  | $\left\{ \left( \begin{array}{c c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \land \neg (m(C_{TL}) < m(C)) \right\}$  |
| 1b   | $\{C = AB^T + BA^T + \widehat{C} $  |

| Algorithm: $C = AB^T + BA^T + C$  |
|---|
|   |
| $A \to \left(\frac{A_T}{A_B}\right), B \to \left(\frac{B_T}{B_B}\right), C \to \left(\frac{C_{TL}}{C_{BL}} \begin{vmatrix} C_{TR} \\ C_{BL} \end{vmatrix} \begin{vmatrix} C_{BR} \\ C_{BL} \end{vmatrix} \right)$ where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$  |
|   |
| while $m(C_{TL}) < m(C)$ do   |
|   |
| $ \left(\frac{A_T}{A_B}\right) \to \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}}{C_{BL}} \begin{vmatrix} C_{TR} \\ C_{BL} \end{vmatrix} \begin{vmatrix} C_{BR} \\ C_{BR} \end{vmatrix}\right) \to \left(\frac{C_{00}}{c_{01}} \begin{vmatrix} c_{01} & C_{02} \\ C_{10} & c_{11} & c_{12} \\ C_{20} & c_{21} & C_{22} \end{vmatrix}\right) $ where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is 1 × 1  |
|   |
| $c_{01} = A_0(b_1^T)^T + B_0(a_1^T)^T + c_{01}^T$ $\gamma_{11} = a_1^T(b_1^T)^T + b_1^T(a_1^T)^T + \gamma_{11}$ $c_{10}^T = b_1^T A_0^T + c_{10}^T$ $C_{20} = A_2(b_1^T)^T + C_{20}^T$ $c_{21} = A_2 B_0^T + a_1^T B_0^T + C_{20}^T$  |
|   |
| $ \left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}^T   \gamma_{11}   c_{12}^T}\right) \\ \frac{c_{TL}   C_{TR}}{C_{BL}   C_{BR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   C_{02}}{c_{10}   \gamma_{11}   c_{12}^T}\right) \\ \frac{c_{TL}   c_{TR}}{C_{DL}   c_{DR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   c_{02}}{c_{10}   c_{12}}\right) \\ \frac{c_{TL}   c_{TR}}{C_{DL}   c_{DR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   c_{02}}{c_{10}   c_{12}}\right) \\ \frac{c_{TL}   c_{TR}}{C_{DL}   c_{DR}}\right) \leftarrow \left(\frac{C_{00}   c_{01}   c_{02}}{c_{10}   c_{12}}\right) \\ \frac{c_{TL}   c_{TR}}{C_{DL}   c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \\ \frac{c_{TL}   c_{TR}}{C_{DL}   c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \leftarrow \left(\frac{C_{DL}   c_{DR}}{c_{DR}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{TL}}\right) \leftarrow \left(\frac{C_{DL}   c_{TR}}{c_{TL}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{TL}}\right) \leftarrow \left(\frac{C_{DL}   c_{TR}}{c_{TL}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{TL}}\right) \leftarrow \left(\frac{C_{DL}   c_{TR}}{c_{TL}}\right) \\ \frac{c_{TL}   c_{TR}}{c_{TL}}$ |
|   |
| endwhile  |
|   |
|   |

Algorithm:  $C = AB^T + BA^T + C$ 

$$A o \left(\frac{A_T}{A_B}\right), B o \left(\frac{B_T}{B_B}\right), C o \left(\frac{C_{TL} | C_{TR}}{C_{BL} | C_{BR}}\right)$$

where  $A_T$  has 0 rows,  $B_T$  has 0 rows,  $C_{TL}$  is  $0 \times 0$ 

while  $m(C_{TL}) < m(C)$  do

$$\left(\frac{A_T}{A_B}\right) \to \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \to \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL} | C_{TR}}{C_{BL} | C_{BR}}\right) \to \left(\frac{C_{00} | c_{01} | C_{02}}{c_{10}^T | c_{12}^T}\right)$$

where  $a_1$  has 1 row,  $b_1$  has 1 row,  $\gamma_{11}$  is  $1 \times 1$ 

$$c_{01} = A_0(b_1^T)^T + B_0(a_1^T)^T + c_{01}^T$$

$$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$$

$$c_{10}^T = b_1^T A_0^T + c_{10}^T$$

$$C_{20} = A_2(b_1^T)^T + C_{20}^T$$

$$c_{21} = A_2 B_0^T + a_1^T B_0^T + C_{20}^T$$

$$\left(\frac{A_T}{A_B}\right) \leftarrow \left(\frac{A_0}{a_1^T}\right), \left(\frac{B_T}{B_B}\right) \leftarrow \left(\frac{B_0}{b_1^T}\right), \left(\frac{C_{TL} | C_{TR}}{C_{BL} | C_{BR}}\right) \leftarrow \left(\frac{C_{00} | c_{01} | C_{02}}{c_{10}^T | \gamma_{11} | c_{12}^T}\right)$$

endwhile