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| Step | Algorithm: $C = AB^T + BA^T + C$ |
|---|---|
| 1a | $\{ \quad \}$ |
| 4 | where |
| 2 | $\left\{ \quad \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \wedge \right\}$ |
| 5a | where |
| 6 | $\left\{ \quad \right\}$ |
| 8 | |
| 7 | $\left\{ \quad \right\}$ |
| 5b | |
| 2 | $\left\{ \quad \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \wedge \neg( \quad ) \right\}$ |
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<tr><td>7</td><td>$\left\{ \left(\begin{array}{ccc} C_{00} & c_{01} & C_{02} \\ c_{10}^T & \gamma_{11} & c_{12}^T \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{ccc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & \widehat{*} \\ a_1 B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \widehat{\gamma}_{11} & * \\ A_2 B_0^T + \widehat{C}_{20} & A_2 (b_1^T)^T + \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right) \right\}$</td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right)$ , $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right)$ , $\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$</td></tr>
<tr><td>2</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$</td></tr>
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<tr><td>2,3</td><td>$\left\{ \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline A_B B_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg (m(C_{TL}) < m(C)) \right\}$</td></tr>
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</table>

<table>
<tr><th>Step</th><th>Algorithm: $C = AB^T + BA^T + C$</th></tr>
<tr><td>1a</td><td>$\{C = \widehat{C}$ $\}$</td></tr>
<tr><td>4</td><td>$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$<br>where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$</td></tr>
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<tr><td>3</td><td>while $m(C_{TL}) < m(C)$ do</td></tr>
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<tr><td>8</td><td>$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$<br>$c_{10}^T = b_1^T A_0^T + c_{10}^T$<br>$c_{21} = A_2 (b_1^T)^T + c_{21}$</td></tr>
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Algorithm: $C = AB^T + BA^T + C$

$$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), \; B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), \; C \rightarrow \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$$

where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$

while $m(C_{TL}) < m(C)$ do

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \; \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$

$$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$$

$$c_{10}^T = b_1^T A_0^T + c_{10}^T$$

$$c_{21} = A_2 (b_1^T)^T + c_{21}$$

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \; \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \; \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile

Algorithm: $C = AB^T + BA^T + C$

$$A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right), C \rightarrow \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$$

where $A_T$ has 0 rows, $B_T$ has 0 rows, $C_{TL}$ is $0 \times 0$

while $m(C_{TL}) < m(C)$ do

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$$

where $a_1$ has 1 row, $b_1$ has 1 row, $\gamma_{11}$ is $1 \times 1$

$\gamma_{11} = a_1^T (b_1^T)^T + b_1^T (a_1^T)^T + \gamma_{11}$

$c_{10}^T = b_1^T A_0^T + c_{10}^T$

$c_{21} = A_2 (b_1^T)^T + c_{21}$

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ a_1^T \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ b_1^T \\ \hline B_2 \end{array}\right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$$

endwhile