| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$, $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$<br>where $C_{TL}$ is $0 \times 0$, $A_T$ has 0 rows, $B_T$ has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row |
| 6 | $\left\{ \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|cc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ \hline b_1^T A_0^T + \widehat{c}_{10}^T & \gamma_{11} & * \\ B_2 A_0^T + \widehat{C}_{20} & c_{21} & C_{22} \end{array}\right) \right\}$ |
| 8 | $\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}$<br>$c_{10}^T := a_1^T B_0^T + c_{10}^T$<br>$c_{21} := B_2 a_1 + c_{21}$ |
| 7 | $\left\{ \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc\|c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ a_1^T B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & * \\ \hline B_2 A_0^T + \widehat{C}_{20} & B_2 a_1 + \widehat{c}_{21} & C_{22} \end{array}\right) \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{[C] = \text{syr2k\_ln}(A, B, \widehat{C})$ $\}$ |

| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
|---|---|
| 1a | $\{$ $\}$ |
| 4 | where |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \right.$ $\wedge$ $\left. \right\}$ |
| 5a | where |
| 6 | $\left\{ \right.$ $\left. \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \right.$ $\wedge \neg($ $)$ $\left. \right\}$ |
| 1b | $\{$ $\}$ |

| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
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| 1a | $\{C = \widehat{C}$ $\}$ |
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| 5b | |
| 2 | $\left\{ \right.$ $\left. \right\}$ |
| | endwhile |
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| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
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| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | where |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| 3 | while do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \right\}$ |
| 5a | where |
| 6 | $\left\{ \right\}$ |
| 8 | |
| 7 | $\left\{ \right\}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg( \quad ) \right\}$ |
| 1b | $\{[C] = \text{syr2k\_ln}(A, B, \widehat{C})$ $\}$ |

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| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | where |
| 6 | $\left\{ \right.$ $\left. \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | |
| 2 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left( \begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array} \right) = \left( \begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array} \right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{[C] = \text{syr2k\_ln}(A, B, \widehat{C})$ $\}$ |

| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
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| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$ , $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$ , $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$ <br> where $C_{TL}$ is $0 \times 0$, $A_T$ has 0 rows, $B_T$ has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | where |
| 6 | $\left\{ \right.$ $\left. \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
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| 5a | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row |
| 6 | $\left\{ \right.$ $\left. \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ \hline A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ \hline B_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
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| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row |
| 6 | $\left\{ \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|cc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ \hline b_1^T A_0^T + \widehat{c}_{10}^T & \gamma_{11} & * \\ B_2 A_0^T + \widehat{C}_{20} & c_{21} & C_{22} \end{array}\right) \right\}$ |
| 8 | |
| 7 | $\left\{ \right.$ $\left. \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
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| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row |
| 6 | $\left\{ \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|cc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ \hline b_1^T A_0^T + \widehat{c}_{10}^T & \gamma_{11} & * \\ B_2 A_0^T + \widehat{C}_{20} & c_{21} & C_{22} \end{array}\right) \right\}$ |
| 8 | |
| 7 | $\left\{ \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc\|c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ a_1^T B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & * \\ \hline B_2 A_0^T + \widehat{C}_{20} & B_2 a_1 + \widehat{c}_{21} & C_{22} \end{array}\right) \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{[C] = \text{syr2k\_ln}(A, B, \widehat{C})$ $\}$ |

| Step | Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$ |
|---|---|
| 1a | $\{C = \widehat{C}$ $\}$ |
| 4 | $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right)$, $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$<br>where $C_{TL}$ is $0 \times 0$, $A_T$ has 0 rows, $B_T$ has 0 rows |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| 3 | while $m(C_{TL}) < m(C)$ do |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge m(C_{TL}) < m(C) \right\}$ |
| 5a | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$<br>where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row |
| 6 | $\left\{ \left(\begin{array}{c\|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|cc} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ \hline b_1^T A_0^T + \widehat{c}_{10}^T & \gamma_{11} & * \\ B_2 A_0^T + \widehat{C}_{20} & c_{21} & C_{22} \end{array}\right) \right\}$ |
| 8 | $\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}$<br>$c_{10}^T := a_1^T B_0^T + c_{10}^T$<br>$c_{21} := B_2 a_1 + c_{21}$ |
| 7 | $\left\{ \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{cc\|c} A_0 B_0^T + B_0 A_0^T + \widehat{C}_{00} & * & * \\ a_1^T B_0^T + b_1^T A_0^T + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & * \\ \hline B_2 A_0^T + \widehat{C}_{20} & B_2 a_1 + \widehat{c}_{21} & C_{22} \end{array}\right) \right\}$ |
| 5b | $\left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc\|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$, $\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right)$, $\left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$ |
| 2 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \right\}$ |
| | endwhile |
| 2,3 | $\left\{ \left(\begin{array}{c\|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_T B_T^T + B_T A_T^T + \widehat{C}_{TL} & * \\ \hline B_B A_T^T + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(m(C_{TL}) < m(C)) \right\}$ |
| 1b | $\{[C] = \text{syr2k\_ln}(A, B, \widehat{C})$ $\}$ |

Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$

$$C \rightarrow \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), \quad A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), \quad B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$$

where $C_{TL}$ is $0 \times 0$, $A_T$ has 0 rows, $B_T$ has 0 rows

while $m(C_{TL}) < m(C)$ do

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right), \quad \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \quad \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$$

where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row

$$\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}$$
$$c_{10}^T := a_1^T B_0^T + c_{10}^T$$
$$c_{21} := B_2 a_1 + c_{21}$$

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \quad \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \quad \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$$

endwhile

Algorithm: $[C] := \text{SYR2K\_LN\_UNB\_VAR4}(A, B, C)$

$$C \rightarrow \left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right), A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right), B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right)$$

where $C_{TL}$ is $0 \times 0$, $A_T$ has 0 rows, $B_T$ has 0 rows

while $m(C_{TL}) < m(C)$ do

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|cc} C_{00} & * & * \\ \hline c_{10}^T & \gamma_{11} & * \\ C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$$

where $\gamma_{11}$ is $1 \times 1$, $a_1$ has 1 row, $b_1$ has 1 row

$$\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \gamma_{11}$$
$$c_{10}^T := a_1^T B_0^T + c_{10}^T$$
$$c_{21} := B_2 a_1 + c_{21}$$

$$\left(\begin{array}{c|c} C_{TL} & * \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} C_{00} & * & * \\ c_{10}^T & \gamma_{11} & * \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ \hline a_1^T \\ A_2 \end{array}\right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array}\right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline b_1^T \\ B_2 \end{array}\right)$$

endwhile