

Capture-avoiding substitution for System F (plus numbers) terms:

$$\begin{aligned}
n\{e/x\} &= n \\
y\{e/x\} &= \begin{cases} e & \text{if } y = x \\ y & \text{otherwise} \end{cases} \\
(e_1 e_2)\{e/x\} &= (e_1\{e/x\}) (e_2\{e/x\}) \\
(\lambda y:\tau.e_1)\{e/x\} &= \lambda y:\tau.(e_1\{e/x\}) && \text{where } y \neq x \text{ and } y \notin \text{fv}(e) \\
(\Lambda X.e_1)\{e/x\} &= \Lambda X.(e_1\{e/x\}) && \text{where } X \notin \text{fv}(e) \\
(e_1[\tau])\{e/x\} &= (e_1\{e/x\})[\tau]
\end{aligned}$$

where $\text{fv}(e)$ is the set of free type variables in an expression e .

And for substituting types in System F terms:

$$\begin{aligned}
n\{\tau/X\} &= n \\
x\{\tau/X\} &= x \\
(e_1 e_2)\{\tau/X\} &= (e_1\{\tau/X\}) (e_2\{\tau/X\}) \\
(\lambda x:\tau_1.e_1)\{\tau/X\} &= \lambda x:\tau_1\{\tau/X\}.(e_1\{\tau/X\}) \\
(\Lambda Y.e_1)\{\tau/X\} &= \Lambda Y.(e_1\{\tau/X\}) && \text{where } X \neq Y \text{ and } Y \notin \text{fv}(\tau) \\
(e_1[\tau_1])\{\tau/X\} &= (e_1\{\tau/X\})[\tau_1\{\tau/X\}]
\end{aligned}$$

where $\text{fv}(\tau)$ is the set of free type variables in a type τ .

Finally, for substituting types in types:

$$\begin{aligned}
\mathbf{int}\{\tau/X\} &= \mathbf{int} \\
(\tau_1 \rightarrow \tau_2)\{\tau/X\} &= (\tau_1\{\tau/X\}) \rightarrow (\tau_2\{\tau/X\}) \\
Y\{\tau/X\} &= \begin{cases} \tau & \text{if } Y = X \\ Y & \text{otherwise} \end{cases} \\
(\forall Y.\tau_1)\{\tau/X\} &= \forall Y.(\tau_1\{\tau/X\}) && \text{where } X \neq Y \text{ and } Y \notin \text{fv}(\tau)
\end{aligned}$$