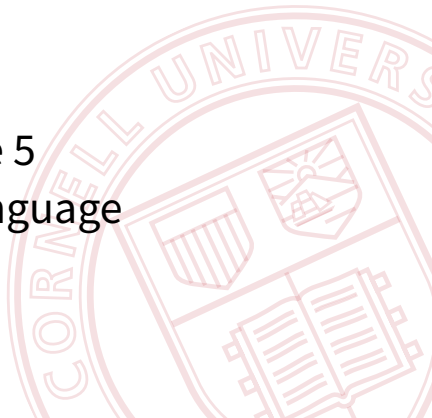


CS 4110

Programming Languages & Logics

Lecture 5 The IMP Language



Simple imperative language

We'll now consider a more realistic programming language...

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arithmetic expressions $a \in \mathbf{Aexp}$ $a ::= x \mid n \mid a_1 + a_2 \mid a_1 \times a_2$
boolean expressions $b \in \mathbf{Bexp}$ $b ::= \mathbf{true} \mid \mathbf{false} \mid a_1 < a_2$

Simple imperative language

We'll now consider a more realistic programming language...

arithmetic expressions	$a \in \mathbf{Aexp}$	$a ::= x \mid n \mid a_1 + a_2 \mid a_1 \times a_2$
boolean expressions	$b \in \mathbf{Bexp}$	$b ::= \mathbf{true} \mid \mathbf{false} \mid a_1 < a_2$
commands	$c \in \mathbf{Com}$	$c ::= \mathbf{skip}$ $\mid x := a$ $\mid c_1; c_2$ $\mid \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2$ $\mid \mathbf{while } b \mathbf{ do } c$

Small-Step Semantics

Three relations, one for each syntactic category:

$$\rightarrow_{\mathbf{Aexp}} \subseteq (\mathbf{Store} \times \mathbf{Aexp}) \times (\mathbf{Store} \times \mathbf{Aexp})$$

$$\rightarrow_{\mathbf{Bexp}} \subseteq (\mathbf{Store} \times \mathbf{Bexp}) \times (\mathbf{Store} \times \mathbf{Bexp})$$

$$\rightarrow_{\mathbf{Com}} \subseteq (\mathbf{Store} \times \mathbf{Com}) \times (\mathbf{Store} \times \mathbf{Com})$$

Small-Step Semantics

$$\frac{n = \sigma(x)}{\langle \sigma, x \rangle \rightarrow \langle \sigma, n \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, e_1 \rangle \rightarrow \langle \sigma, e'_1 \rangle}{\langle \sigma, e_1 + e_2 \rangle \rightarrow \langle \sigma, e'_1 + e_2 \rangle}$$

$$\frac{\langle \sigma, e_2 \rangle \rightarrow \langle \sigma, e'_2 \rangle}{\langle \sigma, n + e_2 \rangle \rightarrow \langle \sigma, n + e'_2 \rangle}$$

$$\frac{p = n + m}{\langle \sigma, n + m \rangle \rightarrow \langle \sigma, p \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, e_1 \rangle \rightarrow \langle \sigma, e'_1 \rangle}{\langle \sigma, e_1 \times e_2 \rangle \rightarrow \langle \sigma, e'_1 \times e_2 \rangle}$$

$$\frac{\langle \sigma, e_2 \rangle \rightarrow \langle \sigma, e'_2 \rangle}{\langle \sigma, n \times e_2 \rangle \rightarrow \langle \sigma, n \times e'_2 \rangle}$$

$$\frac{p = n \times m}{\langle \sigma, n \times m \rangle \rightarrow \langle \sigma, p \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, a_1 \rangle \rightarrow \langle \sigma, a'_1 \rangle}{\langle \sigma, a_1 < a_2 \rangle \rightarrow \langle \sigma, a'_1 < a_2 \rangle}$$

$$\frac{\langle \sigma, a_2 \rangle \rightarrow \langle \sigma, a'_2 \rangle}{\langle \sigma, n < a_2 \rangle \rightarrow \langle \sigma, n < a'_2 \rangle}$$

$$\frac{n < m}{\langle \sigma, n < m \rangle \rightarrow \langle \sigma, \mathbf{true} \rangle}$$

$$\frac{n \geq m}{\langle \sigma, n < m \rangle \rightarrow \langle \sigma, \mathbf{false} \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, e \rangle \rightarrow \langle \sigma, e' \rangle}{\langle \sigma, x := e \rangle \rightarrow \langle \sigma, x := e' \rangle}$$

$$\frac{}{\langle \sigma, x := n \rangle \rightarrow \langle \sigma[x := n], \mathbf{skip} \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, c_1 \rangle \rightarrow \langle \sigma', c'_1 \rangle}{\langle \sigma, c_1; c_2 \rangle \rightarrow \langle \sigma', c'_1; c_2 \rangle}$$

$$\frac{}{\langle \sigma, \mathbf{skip}; c_2 \rangle \rightarrow \langle \sigma, c_2 \rangle}$$

Small-Step Semantics

$$\frac{\langle \sigma, b \rangle \rightarrow \langle \sigma, b' \rangle}{\langle \sigma, \text{if } b \text{ then } c_1 \text{ else } c_2 \rangle \rightarrow \langle \sigma, \text{if } b' \text{ then } c_1 \text{ else } c_2 \rangle}$$

$$\frac{}{\langle \sigma, \text{if true then } c_1 \text{ else } c_2 \rangle \rightarrow \langle \sigma, c_1 \rangle}$$

$$\frac{}{\langle \sigma, \text{if false then } c_1 \text{ else } c_2 \rangle \rightarrow \langle \sigma, c_2 \rangle}$$

$$\frac{}{\langle \sigma, \text{while } b \text{ do } c \rangle \rightarrow \langle \sigma, \text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip} \rangle}$$

Small-Step Example

$\langle \sigma, \text{foo} := 3; \textbf{while } \text{foo} < 4 \textbf{ do } \text{foo} := \text{foo} + 5 \rangle$

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Small-Step Example

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→ $\langle \sigma'', \text{skip} \rangle$

Large-Step Semantics

Again, three relations, one for each syntactic category:

$$\Downarrow_{\mathbf{Aexp}} \subseteq \mathbf{Store} \times \mathbf{Aexp} \times \mathbf{Store}$$

$$\Downarrow_{\mathbf{Bexp}} \subseteq \mathbf{Store} \times \mathbf{Bexp} \times \mathbf{Store}$$

$$\Downarrow_{\mathbf{Com}} \subseteq \mathbf{Store} \times \mathbf{Com} \times \mathbf{Store}$$

Large-Step Semantics

$$\frac{}{\langle \sigma, n \rangle \Downarrow n} \qquad \frac{\sigma(x) = n}{\langle \sigma, x \rangle \Downarrow n}$$

$$\frac{\langle \sigma, e_1 \rangle \Downarrow n_1 \quad \langle \sigma, e_2 \rangle \Downarrow n_2 \quad n = n_1 + n_2}{\langle \sigma, e_1 + e_2 \rangle \Downarrow n}$$

$$\frac{\langle \sigma, e_1 \rangle \Downarrow n_1 \quad \langle \sigma, e_2 \rangle \Downarrow n_2 \quad n = n_1 \times n_2}{\langle \sigma, e_1 \times e_2 \rangle \Downarrow n}$$

Large-Step Semantics

$$\overline{\langle \sigma, \mathbf{true} \rangle \Downarrow \mathbf{true}}$$

$$\overline{\langle \sigma, \mathbf{false} \rangle \Downarrow \mathbf{false}}$$

$$\frac{\langle \sigma, a_1 \rangle \Downarrow n_1 \quad \langle \sigma, a_2 \rangle \Downarrow n_2 \quad n_1 < n_2}{\langle \sigma, a_1 < a_2 \rangle \Downarrow \mathbf{true}}$$

$$\frac{\langle \sigma, a_1 \rangle \Downarrow n_1 \quad \langle \sigma, a_2 \rangle \Downarrow n_2 \quad n_1 \geq n_2}{\langle \sigma, a_1 < a_2 \rangle \Downarrow \mathbf{false}}$$

Large-Step Semantics

SKIP

$$\frac{}{\langle \sigma, \mathbf{skip} \rangle \Downarrow \sigma}$$

Large-Step Semantics

$$\text{ASSGN} \quad \frac{\langle \sigma, e \rangle \Downarrow n}{\langle \sigma, x := e \rangle \Downarrow \sigma[x \mapsto n]}$$

Large-Step Semantics

SEQ

$$\frac{\langle \sigma, c_1 \rangle \Downarrow \sigma' \quad \langle \sigma', c_2 \rangle \Downarrow \sigma''}{\langle \sigma, c_1; c_2 \rangle \Downarrow \sigma''}$$

Large-Step Semantics

IF-T

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{true} \quad \langle \sigma, c_1 \rangle \Downarrow \sigma'}{\langle \sigma, \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \rangle \Downarrow \sigma'}$$

Large-Step Semantics

$$\frac{\text{IF-F} \quad \langle \sigma, b \rangle \Downarrow \mathbf{false} \quad \langle \sigma, c_2 \rangle \Downarrow \sigma'}{\langle \sigma, \mathbf{if } b \mathbf{ then } c_1 \mathbf{ else } c_2 \rangle \Downarrow \sigma'}$$

Large-Step Semantics

WHILE-F

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{false}}{\langle \sigma, \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma}$$

WHILE-T

$$\frac{\langle \sigma, b \rangle \Downarrow \mathbf{true} \quad \langle \sigma, c \rangle \Downarrow \sigma' \quad \langle \sigma', \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma''}{\langle \sigma, \mathbf{while } b \mathbf{ do } c \rangle \Downarrow \sigma''}$$

Command Equivalence

Intuitively, two commands are equivalent if they produce the same result under any store...

Definition (Equivalence of commands)

Two commands c and c' are equivalent (written $c \sim c'$) if, for any stores σ and σ' , we have

$$\langle \sigma, c \rangle \Downarrow \sigma' \iff \langle \sigma, c' \rangle \Downarrow \sigma'.$$

Command Equivalence

For example, we can prove that every **while** command is equivalent to its unfolding:

Theorem

For all $b \in \mathbf{Bexp}$ and $c \in \mathbf{Com}$ we have

$\text{while } b \text{ do } c \sim \text{if } b \text{ then } (c; \text{while } b \text{ do } c) \text{ else skip.}$

Proof.

We show each implication separately...

