✩ TYPES! ✩

STLC a.k.a. $\vec{\lambda}$

$n \in$ Int

$e ::= x$
$\quad | \ e_1 \ e_2$
$\quad | \ \lambda x : \tau . \ e$
$\quad | \ n$
$\quad | \ ()$
$\quad | \ e_1 + e_2$

$\tau ::= int$
$\quad | \ unit$
$\quad | \ \tau_1 \to \tau_2$

$(\lambda x : int. \ 5) \ ()$

$\underbrace{}_{int \to int}$ $\underbrace{}_{unit}$

$\underbrace{(int \to int) \to (int \to int)}_{int \to int}$ $\underbrace{int \to int}_{int}$

$\Bigl(\underbrace{(\lambda x : int \to int . \ x) \ (\lambda x : int . \ 5)}_{int \to int} \Bigr) \ \underbrace{()}_{unit}$

DYNAMIC SEMANTICS

ARE BASICALLY THE SAME

$$\frac{e \to e'}{E[e] \to E[e']}$$

$$\overline{(\lambda x : \tau . e)\, v \to e\{v/x\}}$$

(IGNORED)

$$\frac{m = n_1 + n_2}{n_1 + n_2 \to m}$$

WELL-TYPED PROGRAMS DO NOT GET STUCK.

$$42 + \lambda x : \text{int} . x \not\to$$

$$()\; 7 \not\to$$

$$(\lambda x : \text{int} . x)\; () \to ()$$

$$\Gamma \vdash e : \tau$$

$\uparrow$ TYPE CONTEXT $\qquad$ $\uparrow$ TYPE

$$\Gamma : Var \rightarrow Type$$

$$(x \mapsto int) \vdash x : int$$

$$\phi \vdash \lambda x : int. \; x : int \rightarrow int$$

$$\Gamma[x \mapsto \tau]$$
$$\Gamma, x : \tau$$

$$\boxed{\Gamma \vdash e : \tau}$$

$$\frac{}{\Gamma \vdash () : unit}$$

$$\frac{}{\Gamma \vdash n : int}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau} \qquad \xcancel{(y : int) \vdash x : \_}$$

$$\frac{\Gamma \vdash e_1 : \text{int} \qquad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1 + e_2 : \text{int}}$$

$(2 + 4) + 2$

$$\frac{\Gamma[x \mapsto \tau] \vdash e : \tau'}{\Gamma \vdash \lambda x : \tau. e : \tau \to \tau'}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \to \tau_2 \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 \, e_2 : \tau_2}$$

$$e = (\lambda x : int.\ x + 40)\ 2$$

$\vdash e : int$

$\vdash e : \tau$

$\emptyset \vdash e : \tau$

"e IS WELL-TYPED"

$\exists \tau.\ \vdash e : \tau$

$$\dfrac{\text{T-VAR}}{x : int \vdash x : int} \qquad \dfrac{\text{T-INT}}{x : int \vdash 40 : int}$$

$$\dfrac{\text{T-ADD}}{x : int \vdash x + 40 : int}$$

$$\dfrac{\text{ABS}}{\vdash \lambda x : int.\ x + 40 : int \to int} \qquad \dfrac{\text{T-INT}}{\vdash 2 : int}$$

$$\dfrac{\text{T-APP}}{\vdash e : int}$$