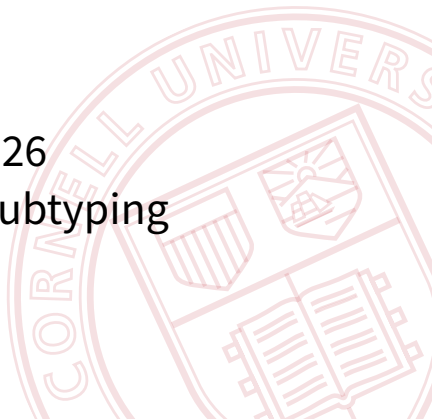


CS 4110

Programming Languages & Logics

Lecture 26 Records and Subtyping



Records

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is a record value with an integer field `foo` and a boolean field `bar`.

Its type is:

`{foo: int, bar: bool}`

Syntax

$$l \in \mathcal{L}$$

$$e ::= \dots \mid \{l_1 = e_1, \dots, l_n = e_n\} \mid e.l$$

$$v ::= \dots \mid \{l_1 = v_1, \dots, l_n = v_n\}$$

$$\tau ::= \dots \mid \{l_1 : \tau_1, \dots, l_n : \tau_n\}$$

Dynamic Semantics

$E ::= \dots$

| $\{l_1 = v_1, \dots, l_{i-1} = v_{i-1}, l_i = E, l_{i+1} = e_{i+1}, \dots, l_n = e_n\}$

| $E.l$

$$\overline{\{l_1 = v_1, \dots, l_n = v_n\}.l_i \rightarrow v_i}$$

Static Semantics

$$\frac{\forall i \in 1..n. \Gamma \vdash e_i : \tau_i}{\Gamma \vdash \{l_1 = e_1, \dots, l_n = e_n\} : \{l_1 : \tau_1, \dots, l_n : \tau_n\}}$$

$$\frac{\Gamma \vdash e : \{l_1 : \tau_1, \dots, l_n : \tau_n\}}{\Gamma \vdash e.l_i : \tau_i}$$

Example

$$\text{GETX} \triangleq \lambda p : \{x : \mathbf{int}, y : \mathbf{int}\}. p.x$$

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$\text{GETX } \{x = 4, y = 2, z = 42\}$

$\text{GETX } \{y = 2, x = 4\}$

Subtyping

Definition (Subtype)

τ_1 is a *subtype* of τ_2 , written $\tau_1 \leq \tau_2$, if a program can use a value of type τ_1 whenever it would use a value of type τ_2 .

If $\tau_1 \leq \tau_2$, we also say τ_2 is the *supertype* of τ_1 .

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$$\frac{\Gamma \vdash e:\tau \quad \tau \leq \tau'}{\Gamma \vdash e:\tau'} \text{ SUBSUMPTION}$$

This typing rule says that if e has type τ and τ is a subtype of τ' , then e also has type τ' .

Record Subtyping

We'll define a new **subtyping relation** that works together with the subsumption rule.

$$\tau_1 \leq \tau_2$$

Record Subtyping

This program isn't well-typed (yet):

$$(\lambda p: \{x : \mathbf{int}\}. p.x) \{x = 4, y = 2\}$$

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So let's add **width subtyping**:

$$\frac{k \geq 0}{\{l_1: \tau_1, \dots, l_{n+k}: \tau_{n+k}\} \leq \{l_1: \tau_1, \dots, l_n: \tau_n\}}$$

Record Subtyping

This program also doesn't get stuck:

$$(\lambda p: \{x : \mathbf{int}, y : \mathbf{int}\}. p.x + p.y) \{y = 37, x = 5\}$$

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So we can make it well-typed by adding **permutation subtyping**:

$$\frac{\pi \text{ is a permutation on } 1..n}{\{l_1 : \tau_1, \dots, l_n : \tau_n\} \leq \{l_{\pi(1)} : \tau_{\pi(1)}, \dots, l_{\pi(n)} : \tau_{\pi(n)}\}}$$

Record Subtyping

Does this program get stuck? Is it well-typed?

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Let's add **depth subtyping**:

$$\frac{\forall i \in 1..n. \tau_i \leq \tau'_i}{\{l_1 : \tau_1, \dots, l_n : \tau_n\} \leq \{l_1 : \tau'_1, \dots, l_n : \tau'_n\}}$$

Record Subtyping

Putting all three forms of record subtyping together:

$$\frac{\forall i \in 1..n. \exists j \in 1..m. \quad l'_i = l_j \quad \wedge \quad \tau_j \leq \tau'_i}{\{l_1:\tau_1, \dots, l_m:\tau_m\} \leq \{l'_1:\tau'_1, \dots, l'_n:\tau'_n\}} \text{S-RECORD}$$

Standard Subtyping Rules

We always make the subtyping relation both reflexive and transitive.

$$\frac{}{\tau \leq \tau} \text{ S-REFL} \qquad \frac{\tau_1 \leq \tau_2 \quad \tau_2 \leq \tau_3}{\tau_1 \leq \tau_3} \text{ S-TRANS}$$

Think of every type describing a set of values. Then $\tau_1 \leq \tau_2$ when τ_1 's values are a subset of τ_2 's.

Top Type

It's sometimes useful to define a *maximal* type with respect to subtyping:

$$\tau ::= \dots \mid \top$$

$$\frac{}{\tau \leq \top} \text{S-Top}$$

Everything is a subtype of \top , as in Java's `Object` or Go's `interface{}`.

Subtype All the Things!

We can also write subtyping rules for sums and products:

$$\frac{\tau_1 \leq \tau'_1 \quad \tau_2 \leq \tau'_2}{\tau_1 + \tau_2 \leq \tau'_1 + \tau'_2} \text{ S-SUM}$$

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$$\frac{\tau_1 \leq \tau'_1 \quad \tau_2 \leq \tau'_2}{\tau_1 \times \tau_2 \leq \tau'_1 \times \tau'_2} \text{ S-PRODUCT}$$

Function Types

How should we decide whether one function type is a subtype of another?

$$\frac{???}{\tau_1 \rightarrow \tau_2 \leq \tau'_1 \rightarrow \tau'_2} \text{ S-FUNCTION}$$

Desiderata

We'd like to have:

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In general, to prove:

$$\tau_1 \rightarrow \tau_2 \leq \tau'_1 \rightarrow \tau'_2$$

we'll require:

- Argument types are **contravariant**: $\tau'_1 \leq \tau_1$
- Return types are **covariant**: $\tau_2 \leq \tau'_2$

Function Subtyping

Putting these two pieces together, we get the subtyping rule for function types:

$$\frac{\tau'_1 \leq \tau_1 \quad \tau_2 \leq \tau'_2}{\tau_1 \rightarrow \tau_2 \leq \tau'_1 \rightarrow \tau'_2} \text{ S-FUNCTION}$$

Reference Subtyping

What should the relationship be between τ and τ' in order to have $\tau \text{ **ref** } \leq \tau' \text{ **ref** }$?

Example

If r' has type τ' **ref**, then $!r'$ has type τ' .

Imagine we replace r' with r , where r has a type τ **ref** that we've somehow decided is a subtype of τ' **ref**.

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Then $!r$ should still produce something can be treated as a τ' . In other words, it should have a type that is a *subtype* of τ' .

So the referent type should be covariant:

$$\frac{\tau \leq \tau'}{\tau \text{ **ref** } \leq \tau' \text{ **ref**}}$$

Example

If v has type τ' , then $r' := v$ should be legal.

If we replace r' with r , then it must still be legal to assign $r := v$.
So r would then produce a value of type τ' .

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$$\frac{\tau' \leq \tau}{\tau \text{ ref} \leq \tau' \text{ ref}}$$

Reference Subtyping

In fact, subtyping for reference types must be *invariant*: a reference type τ **ref** is a subtype of τ' **ref** if and only if $\tau \leq \tau'$ and $\tau' \leq \tau$.

$$\frac{\tau \leq \tau' \quad \tau' \leq \tau}{\tau \text{ **ref** } \leq \tau' \text{ **ref** }} \text{ S-REF}$$

Java Arrays

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Suppose that Cow is a subtype of Animal.

Code that only reads from arrays typechecks:

```
Animal[] arr = new Cow[] { new Cow("Alfonso") };  
Animal a = arr[0];
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Code that only reads from arrays typechecks:

```
Animal[] arr = new Cow[] { new Cow("Alfonso") };  
Animal a = arr[0];
```

but writing to the array can get into trouble:

```
arr[0] = new Animal("Brunhilda");
```

Specifically, this generates an `ArrayStoreException`.