CS 4110

Programming Languages & Logics

Lecture 24
Parametric Polymorphism

Roadmap

We've extended a simple type system for the λ-calculus with support for a few interesting language constructs. But the "power" of the underlying type system has remained more or less the same.

Today, we'll develop a far more fundamental change to the simply-typed λ-calculus: *parametric polymorphism*, the concept at the heart of OCaml's type system and underlying generics in Java and similar languages.

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- *Subtype polymorphism* allows values of type τ to masquerade as values of type τ', provided that τ is a subtype of τ'.
- *Ad-hoc polymorphism*, also called overloading, allows the same function name to be used with functions that take different types of parameters.
- *Parametric polymorphism* refers to code that is written without knowledge of the actual type of the arguments; the code is parametric in the type of the parameters.

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$$\text{doubleInt} \triangleq \lambda f{:}\,\mathbf{int} \to \mathbf{int}.\ \lambda x{:}\,\mathbf{int}.\ f\,(f\,x)$$

Now suppose we want the same function for Booleans, or functions...

$$\begin{aligned}\text{doubleBool} &\triangleq \lambda f{:}\,\mathbf{bool} \to \mathbf{bool}.\ \lambda x{:}\,\mathbf{bool}.\ f\,(f\,x)\\ \text{doubleFn} &\triangleq \lambda f{:}\,(\mathbf{int} \to \mathbf{int}) \to (\mathbf{int} \to \mathbf{int}).\ \lambda x{:}\,\mathbf{int} \to \mathbf{int}.\ f\,(f\,x)\\ &\ \vdots\end{aligned}$$

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Definition (Abstraction Principle)

Every major piece of functionality in a program should be implemented in just one place in the code. When similar functionality is provided by distinct pieces of code, the two should be combined into one by abstracting out the varying parts.

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In the doubling functions, the varying parts are the types.

We need a way to abstract out the type of the doubling operation, and later instantiate it with different concrete types.

Polymorphic λ-Calculus

Invented independently in 1972–1974 by a computer scientist John Reynolds and a logician Jean-Yves Girard (who called it System F).

Key feature: Function abstraction and application, just like in λ-calculus terms, but *at the type level!*

Notation:

- $\Lambda\alpha.\, e$: type abstraction
- $e[\tau]$: type application

Example:
$\Lambda\alpha.\, \lambda x\!:\!\alpha.\, x$

Polymorphic λ-Calculus

Syntax

$$
\begin{aligned}
e &::= n \mid x \mid \lambda x\!:\!\tau.\, e \mid e_1\, e_2 \\
v &::= n \mid \lambda x\!:\!\tau.\, e
\end{aligned}
$$

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Dynamic Semantics

$$E ::= [\cdot] \mid E\, e \mid v\, E \mid E\, [\tau]$$

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$$\frac{e \to e'}{E[e] \to E[e']} \qquad \frac{}{(\lambda x\!:\!\tau.\, e)\, v \to e\{v/x\}}$$

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Typing Judgment: $\Delta, \Gamma \vdash e : \tau$

- Γ a mapping from variables to types
- Δ a set of types in scope
- e an expression
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In the polymorphic λ-calculus, however, we can type this expression using a polymorphic type:

$$\vdash \quad \lambda x{:}\forall\alpha.\,\alpha \to \alpha.\, x\,[\forall\alpha.\,\alpha \to \alpha]\, x : (\forall\alpha.\,\alpha \to \alpha) \to (\forall\alpha.\,\alpha \to \alpha)$$

(However, all expressions in polymorphic λ-calculus still halt. There is no way to give a type to the *self-application* of this term.)

Example: Products

We can encode products in polymorphic λ-calculus without adding any additional types!

The encodings are based on the (untyped) Church encodings:

$$\tau_1 \times \tau_2 \triangleq \forall R.\, (\tau_1 \to \tau_2 \to R) \to R$$

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\pi_1 &\triangleq \Lambda T_1.\, \Lambda T_2.\, \lambda v : T_1 \times T_2.\, v\, [T_1]\, (\lambda x : T_1.\, \lambda y : T_2.\, x)
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\mathit{erase}(e\ [\tau]) & = & \mathit{erase}(e)\ (\lambda x.\, x) &
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Type Erasure

The following theorem states this translation is adequate:

Theorem (Erasure Adequacy)

For all expressions e and e', we have $e \to e'$ iff $erase(e) \to erase(e')$.

Type Inference

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See Chapter 23 of Pierce for further discussion, as well as restrictions for which type reconstruction is decidable.