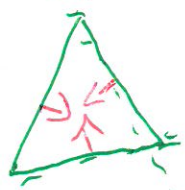
Is a point in a triangle

Approach #1 Inside Test



if the point is on the "inside" of all 3 legs, it is inside the triangle

Pro: Straight Forward

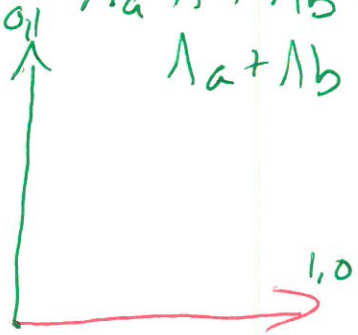
Con: Only get true/false

Approach #2 - Barycentric Coordinate

Barycentric Coordinate (x,y)

$\Lambda_a \cdot A + \Lambda_b B + \Lambda_c C$

$\Lambda_a + \Lambda_b + \Lambda_c = 1$



Inside a Triangle Test

#1 Decompose triangle into vectors

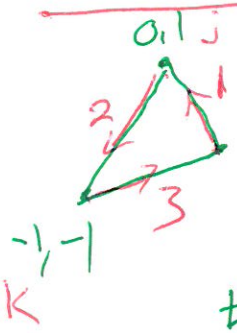
#2 Find the $Ax+By+C=0$ definitions of those vectors

#3 Plug in point to see if it is inside

→ Point independent

$A^2 = B^2 + C^2$

$A = \sqrt{B^2 + C^2}$



#1

$$\frac{(j-i)}{|j-i|} \Rightarrow \frac{(-1,1)}{\sqrt{-1^2+1^2}} \Rightarrow \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \quad \frac{1}{2}+\frac{1}{2}$$

$$\frac{(k-j)}{|k-j|} \Rightarrow \frac{-1,-2}{\sqrt{-1^2+-2^2}} \Rightarrow \left(\frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}\right) \quad \frac{1}{5}+\frac{4}{5}$$

$$\frac{(i-k)}{|i-k|} \quad \frac{(2,1)}{\sqrt{2^2+1^2}} \Rightarrow \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \quad \frac{4}{5}+\frac{1}{5}$$

$C = -Ax - By$

$C_1 = +\left(\frac{+1}{\sqrt{2}}\right)x + \frac{+1}{\sqrt{2}}y$

$C_2 = -\frac{2}{\sqrt{5}}x + \left(\frac{+1}{\sqrt{5}}\right)y$

$C_3 = +\frac{+1}{\sqrt{5}}x - \frac{2}{\sqrt{5}}y$

$(1,0) = (x,y)$

$(0,1) = (x,y)$

$(-1,-1) = (x,y)$

$C_1 = \frac{1}{\sqrt{2}}$

$C_2 = \frac{1}{\sqrt{5}}$

$C_3 = \frac{-1}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{1}{\sqrt{5}}$

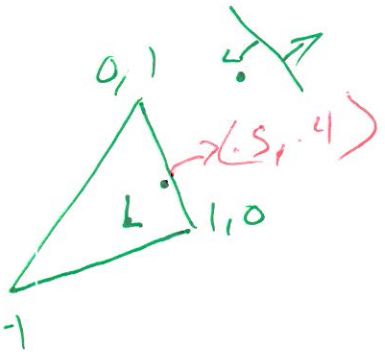
Inside a Triangle Test #2

Sanity Test

$\text{Line}1 = \frac{-1}{\sqrt{2}}x + \frac{-1}{\sqrt{2}}y + \frac{1}{\sqrt{2}} = 0$ | $(1,0)$ | $\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$

$\text{Line}2 = \frac{+2}{\sqrt{5}}x + \frac{-1}{\sqrt{5}}y + \frac{1}{\sqrt{5}} = 0$ | $(0,1)$ | $\frac{-1}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 0$

$\text{Line}3 = \frac{-1}{\sqrt{5}}x + \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 0$ | $(-1,-1)$ | $\frac{1}{\sqrt{5}} - \frac{2}{\sqrt{5}} + \frac{1}{\sqrt{5}} = 0$



IF I put an arbitrary point in line $Ax + By + C$,
the point is on the line if the answer is 0
the point is "in front/inside" if the answer is positive
the point is "behind/outside" if the answer is negative

Test (0,0) $\text{Line}_1 = \frac{1}{\sqrt{2}}$ ✓
$\text{Line}2 = \frac{1}{\sqrt{5}}$ ✓
$\text{Line}3 = \frac{1}{\sqrt{5}}$ ✓
→ inside

Test (2,0) $\text{Line}1 = \frac{-2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$ ☹

$(.5, .4)$ ① $\frac{-.5}{\sqrt{2}} + \frac{-.4}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{.1}{\sqrt{2}}$ ✓

② $\frac{+1}{\sqrt{5}} - \frac{.4}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{1.6}{\sqrt{5}}$ ✓

$\frac{-.5}{\sqrt{5}} + \frac{.8}{\sqrt{5}} + \frac{1}{\sqrt{5}} = \frac{1.3}{\sqrt{5}}$ ✓