Lines

3 flavors
- Infinite Lines
- Ray
- Line Segment

Vector is a ray where one end is at (0,0,0)

2 points -> Line definition

~~$y = mx + b$~~

Bad in this class

$ax + by + c = 0$ [2D]

$ax + by + cz + d = 0$ [3D]

2D

$P_1 = (2,3)$

$P_2 = (4,5)$

$a \cdot 2 + b \cdot 3 + c = 0$

$c = -2a - 3b$

$ax + by - 2a - 3b = 0$

$a \cdot 4 + b \cdot 5 - 2a - 3b = 0$

$a(4-2) + b(5-3) = 0$

$2a + 2b = 0$

$a + b = 0 \Rightarrow a = -b$

$a = 1$

$1x + -1y - 2 + 3 = 0$

$1x - 1y + 1 = 0$

$a = 1$

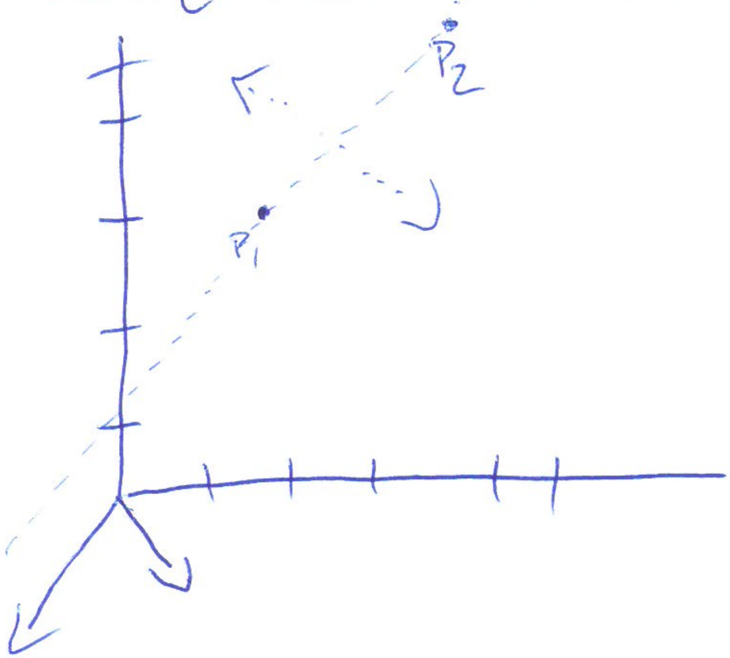
$b = -1$

$c = 1$

$2 - 3 + 1 = 0$

$4 - 5 + 1 = 0$

Equation of a Line



Tangent $\simeq$ Slope $= P_1 - P_2$

Pependicular $\perp$

2, 3

4, 5

$P_1 - P_2 = (-2, -2)$

Perpendicular

$(a, b, c) = (\boxed{1, -1}, 1)$

perpendicular

Bad, bad, bad! [So Far]

Tangent $= \dfrac{P_1 - P_2}{\|P_1 - P_2\|} \rightarrow \sqrt{-2^2 + -2^2} \Rightarrow \sqrt{4+4} \Rightarrow \sqrt{8} \Rightarrow \dfrac{-2}{\sqrt{8}}, \dfrac{-2}{\sqrt{8}}$

$\dfrac{-2}{2\sqrt{2}}, \dfrac{-2}{2\sqrt{2}}$

$-1/\sqrt{2} + \dfrac{-1}{\sqrt{2}}$

Perpendicular $= \dfrac{(a, b)}{\|a - b\|} \rightarrow \sqrt{1^2 + (-1)^2} \Rightarrow \sqrt{2}$

$a = \dfrac{1}{\sqrt{2}}$ $b = \dfrac{-1}{\sqrt{2}}$

Perpendicular in 2&3D

perpendicular of tangent $(a,b) = (-b,a)$ or $(b,-a)$

2D only

3D there isn't a simple solution

Dot Product $\cdot \Rightarrow a \cdot d + b \cdot e + c \cdot f \Rightarrow \cos\theta$ between vectors

$(a,b,c) \cdot (d,e,f) = 0$ iff $(a,b,c) \perp (d,e,f)$

Practice:

$(0,0,0) + (1,1,1) \rightarrow$ Tangent $= (1,1,1) \Rightarrow$ Normalized $= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$a \cdot \frac{1}{\sqrt{3}} + b \cdot \frac{1}{\sqrt{3}} + c \cdot \frac{1}{\sqrt{3}} = 0$

$a + b + c = 0$

$a = 0$

$b = 0$

$c = 0$

(a,b,c)

Cross Product

Right-handed Cross Product = $v_1 \times v_2$

$C_x = a_y b_z - a_z b_y$

$C_y = a_z b_x - a_x b_z$

$C_z = a_x b_y - a_y b_x$

why?

The cross product of two vectors is <u>always</u> perpendicular to these vectors.