

# 3D Triangle Collision

3 points

↳ Three vectors between points  $\rightarrow |p_2 - p_1|$ ,  $|p_3 - p_2|$ ,  $|p_1 - p_3|$

↳ orthogonal  $\rightarrow \textcircled{1} \times \textcircled{2}$

↳ ABCD

↳  $ABC = [\text{orthogonal}.x, \text{orthogonal}.y, \text{orthogonal}.z]$

↳  $D = -(Ax + By + Cz)$

↳  $D = -\text{orthogonal} \cdot \textcircled{1}$

Given Point  $\textcircled{4}$  in inside triangle  $\textcircled{1} \textcircled{2} \textcircled{3}$

find 3 bounding planes

↳ ABCD of the three planes

↳  $ABC \Rightarrow$  orthogonal to the bounding plane  $\rightarrow$

↳  $D = -(\text{point} \cdot \text{orthogonal})$

plug point into bounding planes

2 points ~~from~~ the triangle. How do I find the 3rd point on the bounding plane?  
 $p_3 = p_1 + \text{orthogonal}$

## Computational Cost

3 subtractions, 3 ||  
1 cross product  $\times$   
1 dot product  $\cdot$   
3 additions +  
3 dot products  
3 dot products  $\cdot$

Start  
1. multiply  
1. ~~add~~ comparison  
1. Addition

1. Guess  
2. Guess<sup>2</sup>  
3. Compare Guess<sup>2</sup> to  $x$   
if  $\sqrt{x}$   
4. Adjust Guess

3 + cheap - 3 operations  
3 - cheap - 3 operations

Expensive 3 ||  $\rightarrow$  3 divisions,  $\sqrt{x^2 + y^2 + z^2} \rightarrow$  3 multiplies, 2 additions,  $\sqrt{\quad}$   
1  $\times$  middle - 6 multiplies, 3 additions  
7  $\cdot$  middle - 3 multiplies + 2 additions

Practical Ray Tracing advice.

- Trace only 1 triangle at first  
- Optimize at the pain points.  $\rightarrow$  Square Roots/length

is  $x < y$  length  
is  $x < \sqrt{abc}$   
is  $x^2 < abc^2$

# Linear Algebra For Computer Graphics

Translation  $\rightarrow$  Position

Scale

Rotation

1. All matrices in computer graphics are  $4 \times 4$
2. All points are of length 4  $[x, y, z, w]^T$
3.  $w$  is always 1  $\rightarrow [x, y, z, 1]^T$

Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} \text{ScaleX} & & & \\ & \text{ScaleY} & & \\ & & \text{ScaleZ} & \\ & & & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} = \begin{bmatrix} a \cdot S_x \\ b \cdot S_y \\ c \cdot S_z \\ 1 \end{bmatrix}$$

## Translation

$$\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ 1 \end{bmatrix} = \begin{bmatrix} a + T_x \\ b + T_y \\ c + T_z \\ 1 \end{bmatrix}$$

## Rotation

Roll  $\rightarrow Z$   
Pitch  $\rightarrow X$   
Yaw  $\rightarrow Y$

