On the Prospects for a Science of Visualization

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Abstract This paper explores the extent to which a scientific framework for visualization might be possible. It presents several potential parts of a framework, illustrated by application to the visualization of correlation in scatterplots. The first is an *extended-vision thesis*, which posits that a viewer and visualization system can be usefully considered as a single system that perceives structure in a dataset, much like "basic" vision perceives structure in the world. This characterization is then used to suggest approaches to evaluation that take advantage of techniques used in vision science. Next, an *optimal-reduction thesis* is presented, which posits that an optimal visualization enables the given task to be reduced to the most suitable operations in the extended system. A systematic comparison of alternative designs is then proposed, guided by what is known about perceptual mechanisms. It is shown that these elements can be extended in various ways—some even overlapping with parts of vision science. As such, a science of some kind appears possible for at least some parts of visualization. It would remain distinct from design practice, but could nevertheless assist with the design of visualizations that better engage human perception and cognition.

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W. Huang (ed.), *Handbook of Human Centric Visualization*,
DOI 10.1007/978-1-4614-7485-2_6,

1 Introduction

Is there a best way to visually display a given dataset for a given task, and if so, can we find it? Considerable effort has been expended on this issue over the years.[1] The result has been a set of specialized disciplines concerned with the design of displays in various domains, such as cartography, diagram design, statistical graphics, visual interface design, and information visualization (e.g., [1–4]). These disciplines have achieved considerable success, often resulting in designs that are highly *effective*—i.e., that enable performance that is rapid, accurate, and relatively effortless [5].

But many important questions remain unanswered. What is the best way to measure how a given visualization works? How could we find the perceptual and cognitive factors that limit its performance? Could we determine if its design is optimal? Answering such questions will require something more than intuition and *post hoc* measures. And more than design guidelines. It will require a framework that is systematic and rigorous—ideally, one that is *scientific* in the best sense of the word, sensitive to the nature of the visualization task, the computational issues involved, and the nature of the human viewer. The issue considered here is the extent to which this might be possible for visualization. Could it—or at least part of it—even be developed into a science?

To answer this, it may first be worth considering what a science is. Few disciplines are as systematic and rigorous as physics. What then allows a domain to be considered a science? One commonly-accepted criterion is for the domain to have a framework—a *paradigm*—that explicitly describes (1) a related set of entities in the world that are of interest, (2) the kinds of questions that can be asked about them, and (3) possible ways of answering these questions [6, 7]. In other words, a science is not so much a particular body of knowledge, but an *organized way of thinking* about a coherent set of issues. A framework of this kind would ideally consist of a set of characterizations, theories and practices that are consistent with each other and connected by their reference to a common set of issues.

In what follows, several potential elements of such a framework are proposed for visualization; as a test of their suitability and consistency with each other, each is applied to the case of correlation in scatterplots.[2] The first concerns the issue of how visualization can best be characterized. An *extended-vision thesis* is proposed,

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[1]Historically, much of this work focused on *graphic displays, which* convey information using the geometric and radiometric properties of an image (as opposed to simple text alone). Meanwhile, more modern work focuses on *visual displays*, which rely on the extensive use of visual intelligence for their interpretation. For purposes here, graphic displays and visual displays are considered much the same, with the former term emphasizing the means, and the latter the ends.

[2]A single framework for all aspects of visualization (e.g., usability) is problematic, owing to the heterogeneous nature of the components (e.g., perceptual vs. motor mechanisms) and the possible lack of specificity in the tasks for which it might be used [32]. Discussion here focuses on a more restricted set of issues, viz., the extent to which visualization can enable a human to perceive some well-defined structure in a dataset. This abstracts away from details of particular tasks, and so increases the chances of a systematic framework for at least some parts of visualization. Vision

which posits that the viewer and visualization system can be considered a single system that enables structure in a dataset to be perceived in much way as “basic” vision enables perception of structure in the world. This characterization is then used as the basis of the next element: a more thoroughgoing approach to evaluation, informed by methodology drawn from vision science. Turning to issues of design, an *optimal-reduction thesis* is introduced, which states that an optimal visualization can be considered as one that reduces the given task to the most appropriate set of operations in the extended system. The next element shows how this view can motivate ways of assessing the effectiveness of various design parameters, with the results providing insight into the underlying perceptual mechanisms. Finally, it is shown that these elements can be extended to several other kinds of visualization, and that some can even begin to overlap parts of vision science. As such, it appears that at least some aspects of visualization can be handled in a more integrated and systematic way, one that can help make better use of the perceptual and cognitive abilities of the human viewer.

1.1 The Need for a Systematic Framework

Before considering particular elements, it may be helpful to say a few things about the need for a systematic framework in the first place. It might be thought, for example, that such a framework is unnecessary: designers have long explored the space of possible designs, and by now have reasonably good intuitions about what is optimal, or at least highly effective. Or it might be thought that we as observers have extensive (and perhaps privileged) experience with the operation of our visual systems, and so could authoritatively decide on the issues relevant for any particular design.

But although our intuitions about design—especially those derived from long experience—are important, they are incomplete. To begin with, many devices commonly used in static displays (e.g., box plots, small multiples) are relatively recent, so that intuition has had relatively little time to develop. And ways of visualizing complex structures such as networks are not only recent, but involve a considerable degree of complexity, making the possibility of effective intuitions even less likely. The same concerns apply equally well to dynamic displays and interactive systems (e.g., [8, 9]). All these suggest a design space of such high dimensionality that it has not been—and may never be—completely explored. Guidelines are emerging to help with this (e.g., [10]). But a systematic framework could help create such guidelines, and perhaps even go beyond them.

It might be thought that a framework of some kind might be based directly on our beliefs about how we see. But there are likewise limitations here. For example,

science uses a similar approach, focusing on well-defined functions rather than on ways that vision might help carry out some poorly-defined task [21].

although we have a strong impression that we build up a complete "picture" of our surroundings, the visual system does not operate this way: instead of a dense accumulation of data, we instead likely use a dynamic, just-in-time representation, where only a few coherent structures exist at any moment [11]. Moreover, evidence is increasing that conscious perception is only one aspect of how we see, with considerable visual intelligence in processes that operate without any conscious involvement [12]. As such, intuitions about vision—and more generally, about perception and cognition—are not enough to create a viable framework for this purpose. Something more systematic is needed, something that can enable the effective use of knowledge about the main psychological factors involved,

1.2 The Applicability of a Systematic Framework

It might be argued that even if a systematic framework of some kind were possible, it would not be of much use: how could it be general enough to apply to domains as diverse as cartography and statistical graphics, yet specific enough to guide particular designs? And what about new developments, such as a speedup in rendering, or a discovery about visual attention? Wouldn't the whole framework need to be rebuilt each time they occur?

In other design disciplines, the existence of a systematic framework for visualization is problematic neither in principle nor in practice. Architecture, for example, has long had such a framework. An architect can incorporate physical constraints into the design of a building, say, to guarantee that it will not fall over due to imbalances in weight distribution. Doing so does not interfere with design in any real way—it does not prohibit anything that is physically viable. Rather, constraints such as those based on physical forces or material properties can be applied to any design, determining whether it is viable, and sometimes even whether it is optimal. There is no a priori reason why a similar approach would not also work for visualization.

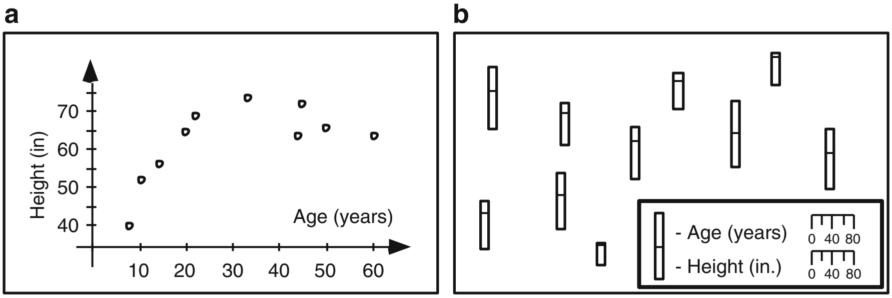
Indeed, some systematicity already exists in the design process for visualization. Guidelines exist for particular applications, such as designing a map or a graph (e.g., [1, 13, 14]); some even include explicit discussion of perceptual mechanisms, so as to enable adaptation to particular circumstances (e.g., [2, 10, 15]). A complementary approach starts with a particular perceptual mechanism and connects it to various tasks (e.g., [16, 17]). A third approach is based on general relations that exist between different kinds of tasks and different perceptual mechanisms (e.g., [5, 18]). However, none of these approaches is entirely quantitative, nor does it address all issues. What is needed is something that incorporates the best of all these, and enables visualization to be treated in a comprehensive, integrated way.

2 Extended Vision

A first step towards a more systematic framework would be to characterize—as far as possible—exactly what visualization is. Loosely speaking, visualization (in the sense considered here) can be described as *the transforming of a problem into graphical form, so as to engage the visual intelligence of a human viewer*. Said another way, the goal of visualization is to translate a given problem into the language of human vision and cognition. The effectiveness of a given design—a given *graphical representation*—is then determined by the extent to which it can be created (typically, in near real-time) while still enabling the most appropriate perceptual and cognitive mechanisms to be engaged on the task at hand (cf. [5, 19]).

Consider the ages and heights of a set of people. When these are represented via position (Fig. 1a), several trends—such as height increasing with age up to about age 20—are immediately apparent. In contrast, when these data are represented via length (Fig. 1b), these relations are virtually impossible to see. The effective design somehow engages perceptual mechanisms that are more suitable for the task. But what exactly does this mean?

Addressing such questions in a meaningful way requires a characterization of visualization that can enable sufficient articulation of the underlying issues. One such possibility is the *extended-vision thesis*: the viewer and the visualization system can be considered a single information-processing system that enables the viewer to perceive structure in the given dataset much as they would perceive structure in the world using "basic" vision. For such an "extended" system, the input is the dataset under consideration (the ages and heights of a group of people, say), the output some function of this input (correlation between age and height), and



Representation via position Representation via length

Fig. 1 Different graphical representations. (**a**) Age is represented by *horizontal position*, height by *vertical position*; size of the graphic items is irrelevant. Using this representation, the relation between age and height is immediately apparent. (**b**) Age is represented by length *above* the interior line, height by length *below*; position of the items is irrelevant. Here, the relation between the two quantities is far less obvious, even though the data represented are the same

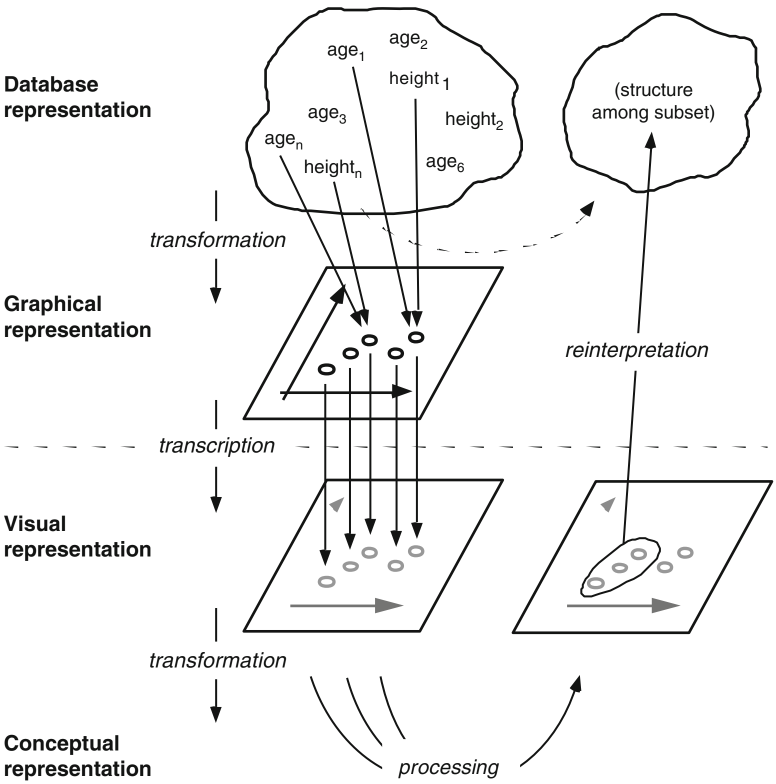


Fig. 2 Extended vision (Based on the example in Fig. 1). Translation of a given task into perceptual and cognitive operations involves three stages: (1) *transformation* of the relevant aspects of the data into a *graphical representation* on a graphical device (e.g., a sheet of paper, or a computer display), (2) *transcription* of the graphical representation to a *visual representation* in the human viewer, and (3) further *transformation* and *processing* to determine the structure present. This latter step may be done entirely via visual mechanisms, although higher-level, conceptual operations may also be used for some tasks. The result can then be reinterpreted in terms of the original data. (If processors are available in the machine component, they may also carry out some initial processing of the data; this is essentially what occurs in visual analytics)

the contents of the graphical device an intermediate step that connects the human viewer to the machine component (Fig. 2).[3]

[3]In some cases (e.g., cartography), the "machine" component may be distributed over a several different devices—and perhaps even the occasional human—with only the result contained in the display (e.g., a map). From a functional point of view, such a configuration is still considered a single visualization system. The performance of such "off-line" systems differs from "on-line" systems only in one respect: because the time scales of the two components are quite different, the effectiveness of an off-line display can usually be assessed in terms of the speed, accuracy, and

An obvious difference between a basic and an extended system is that the projection of information onto the eye of the viewer moves from the initial stage to an intermediate one. But the goal remains the same: extract useful structure in the underlying data. Interestingly, selection of which data to examine (and perhaps process), along with its transformation into graphical form typically occurs via a sequence of stages—the visualization pipeline [5]—in largely the same way that human vision transforms light into a particular perceptual structure (cf. [20, 21]). As such, the architecture of an extended system is an "extended pipeline" created by the concatenation of all these processing stages, with each focusing on a different aspect of the data (cf. [22]).

Note that the information flow between processing elements need not be unidirectional: feedback generally exists between elements in the machine component, as well as in human vision (cf. [5, 23]). Feedback likewise exists between the human and machine elements in any system that is interactive, resulting in a rough architectural consistency throughout the entire system. Indeed, the continual interactions between the components of an extended system that occur during sensemaking [5] are highly analogous to the interactions between the components of a basic system that occur during visual perception [24, 25]. As such, the interactive aspects of visualization (central to e.g., information visualization or visual analytics) are captured by this characterization in a reasonably natural way.

Because these interactive aspects involve a wide range of issues touching on much of cognitive psychology, they are not discussed in great detail here. To keep discussion focused on the main question here—viz., the possibility of a science of visualization—emphasis is placed on "static" noninteractive aspects. This recapitulates the development of vision science, where color and form perception were investigated before more interactive—and generally, more complex—processes such as scene perception [11].

3 Systematic Evaluation

Another important element of any framework for visualization involves the issues related to evaluation. Evaluation of visualization has taken on a variety of forms, ranging from careful quantitative measurement (e.g., [26]) to simple verification of basic functionality (see e.g., [27]). Such variety is understandable: tasks differ in the degree to which they can be described quantitatively, and thus, the degree to which quantitative measures can be applied [28, 29]. And sometimes the goal is merely to verify that a given visualization can be used to some extent. But if the goal is to carefully compare different designs to determine which is the best, or to understand why one works better than another, preference should be given to highly informative measurements. But if so, what exactly should be measured, and how?

effort exerted by the human component alone. (Depending on the situation, however, it may be necessary to take into account such things as the cost of producing the display.)

This is in general a complex issue, making it difficult to find definitive answers. Fortunately, however, decades of work in vision science have been spent on developing high-precision and robust techniques to measure how well graphical structures can be perceived (e.g., [8], Appendix C; [30], Appendix A). The deep similarity between vision and visualization posited by the extended-vision thesis suggests that many of these approaches (along with their foundations in measurement theory) could be applied almost directly to the evaluation of visualizations, resulting in the development of evaluation techniques with a high degree of utility. Said another way: vision science not only offers possible mechanisms to help with visualization design [8], but also possible methodologies.

In what follows, this point will be illustrated via its application to scatterplots. Although scatterplots appear to be simple (at least, once learned), and are widely used to enable correlation to be easily seen (e.g., [1, 31]), relatively little is known about how well they work, or why. As such, the evaluation of these provides an ideal example of what might be done.

3.1 Task Specification

The first step in a rigorous evaluation of a perceptual system—extended or not—is a clear specification of its *function*. In the case of scatterplots, something like "enable discovery of interesting structure in the data" might initially be thought sufficient. But what does "interesting" mean? Finding outliers? Finding trends? If trends, is it correlation or something else? And if correlation, what kind? Spearman? Pearson? Some tasks are inherently vague; requests to "just find something interesting" are not uncommon [29, 32]. But a precise specification should be attempted whenever possible. It can often be achieved.

The next step is to determine an appropriate set of *inputs* for testing. (In the jargon of vision science, these would be the *stimuli*.) Ideally, these are representative of the data encountered in everyday applications; failing that, they should at least have sufficient range to enable determination of how performance depends on various properties of interest. In the case of scatterplots, inputs might be specified as a set of ordered (scalar) pairs drawn from a bivariate gaussian distribution with particular means, variances, and correlation, along with a particular number of pairs in each set. The number of instances and viewers tested should be large enough to ensure sufficient statistical power in the results of the evaluation. In addition, the representation used to display these values would require specification of all properties pertaining to its graphical nature, such as dot size and color.

The final step would then be to specify an appropriate set of *measures* by which performance could be evaluated. In the case of scatterplots, evaluation would determine how well the given representation supports the perception of correlation in the test dataset, based on measures such as accuracy, variability, and timecourse.

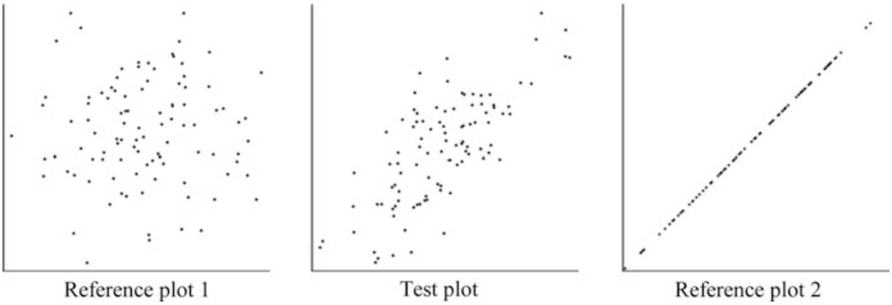


Fig. 3 **Example of bisection task**. Observers adjusted the correlation of the central test plot until its correlation was halfway between those of the reference plots. Plots were $5° \times 5°$ in extent. Here, the value of the test plot is $r = 0.74$, corresponding to the subjective midpoint $g = 0.5$

3.2 Accuracy

One of the most basic performance measures (one often used in evaluating visualizations) is *accuracy*: how well on average the extended system extracts structure from a given dataset.[4] For the case of Pearson correlation r in scatterplots, various studies of accuracy have been carried out (e.g., [26, 33, 34]). Almost all tested observers by asking for direct numerical estimates, viz., a number between 0 and 1 corresponding to the correlation seen in the scatterplot (For a review, see [35]). Several important results have been discovered this way, including the finding that perceived correlation *g* tends to underestimate physical correlation r (especially for intermediate levels of correlation), and that essentially no correlation is perceived when $|r| < 0.2$.

However, the use of direct numerical estimates is usually not optimal. To begin with, its central assumption—that numbers can consistently be assigned to perceived magnitudes—may be incorrect [36]. Indeed, assigning a number to a perceived quantity is a somewhat unnatural task; human perception typically focuses on relations rather than absolutes [37]. Consequently, a better approach—one often used in vision science—might be *bisection*, a technique that takes advantage of the ability of humans to easily and accurately determine the midpoint of a structure. In [38], for example, observers were shown a display containing two *reference plots* (one with a high level of correlation, one with a low) along with a *test plot* between these. Observers adjusted the correlation of the test plot until it appeared to be halfway between the correlations of the reference plots (Fig. 3).

[4]Although inaccuracies are usually caused by mechanisms in the human viewer, they can also be due to the machine component—e.g., insufficient sampling, or a bias in an algorithm. According to the extended-vision thesis, the source is irrelevant: the measure of interest is based on the performance of the *entire* system. For the most part, discussion here focuses on the human viewer, since this is typically the largest source of inaccuracy. But if need be, the accuracy of the machine component could also be evaluated. Similar considerations apply to other measures of performance.

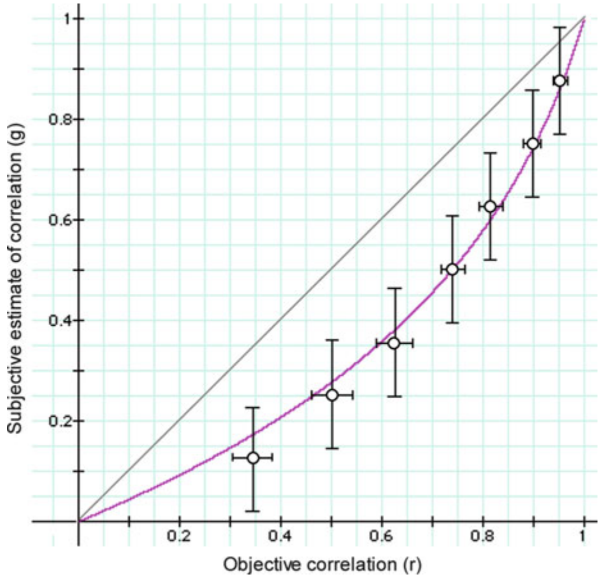


Fig. 4 Perceived correlation via bisection. The curve describing the perceived correlation is g(r) = ln(1−br)/ln(1−b); best fit to data is for b = 0.88. The *straight line* g(r) = r is for comparison; accuracy is based on the difference between this function and g(r). Note the severe underestimation of correlation, especially around r = 0.6

Experiments were run with 20 observers, on scatterplots containing 100 dots. In a first round of testing, observers judged the halfway point between the extremes r = 0 and r = 1. (These corresponded to g = 0 and g = 1, respectively; the halfway point was g = 1/2). A second round applied this test recursively, with each observer now asked to find the value that appeared to be halfway between g = 0 and 1/2, and the value between g = 1/2 and 1, resulting in estimates for g = 1/4 and g = 3/4 respectively. Finally, this method was applied again to determine the values for the subjective estimates g = 1/8, 3/8, 5/8, and 7/8. All conditions were appropriately counterbalanced (For details, see [38]).

Results are shown in Fig. 4. Consistent with the results of other studies, an underestimation of correlation appears, especially around r = 0.6. The results are also broadly consistent with two previous proposals for perceived correlation: the square function $g(r) = r^2$ [33, 34], and a more complex double-power function $g(r) = 1-(1-r)^a(1 + r)^b$, where a and b are free parameters [39]. But the data were precise enough to show that a better fit was with the logarithmic function

$$g(r) = \ln(1 - br)/\ln(1 - b) \tag{1}$$

where b is a real number such that $0 < b < 1$; $g(r) = r$ when $b = 0$ [38].

The accuracy for a given design can be determined by the difference between this function and the physical correlation r. Note that it can be described by a single value (b), covering the entire range of correlations possible. A measure can even be determined based on the relative frequency at which various correlations might be encountered in a task [38].

Interestingly, Eq. 1 is a form of *Fechner's law*, which states that perceived magnitude is proportional to the logarithm of physical magnitude; it applies to the perception of several simple properties, such as brightness [40]. To make this more explicit, Eq. 1 can be rewritten as

$$\mathrm{g}(u) = \ln(u)/\ln(1-b) \tag{2}$$

where $0 < \mathrm{b} < 1$, and $u = 1-br$, the distance *away* from complete correlation. The dependence on u suggests that the relevant factor may be related to the *dispersion* of the dots in the scatterplot, rather than correlation per se. As such, the greater sensitivity of the bisection technique not only provides a better estimate of accuracy (and possibly, a better way to describe it), but also begins to cast some light on the underlying mechanisms.

3.3 Variability

Although accuracy is important for evaluation, other measures are also useful. One of these is *variability*, the extent to which the extended system gives the same answers when given the same data. (Recall that the mean values of these determine accuracy.) Equivalently, it describes the *discriminative power* of the system—i.e., its ability to distinguish between data values that are somewhat similar. Variability can also provide insight into the maximum and minimum values the system might provide, and how often these might occur.

For scatterplots, variability can be assessed by an approach commonly used in vision science: determining how much two properties must differ in order to be *discriminated*, i.e., to see that they are not the same. More precisely, for any objective correlation r, the goal is to find the *just noticeable difference* (*jnd*), the value of Δ for which correlations r and $\mathrm{r} \pm \Delta$ can be discriminated 75 % of the time ([30], Appendix A). The greater the jnd, the greater the separation needed to see that two scatterplots have different correlations, and thus, the greater the variability.

Variability was evaluated this way using 20 observers, on scatterplots containing 100 dots each [38]. A set of base correlations was examined, ranging from $\mathrm{r} = 0$ to $\mathrm{r} = 1$ in increments of 0.1. For each base correlation, two side-by-side scatterplots were shown—one with the base, the other with a variant correlation. Observers were asked to select the scatterplot appearing to be more highly correlated (Fig. 5). All conditions were appropriately counterbalanced (For details, see [38]).

The results of this test are shown in Fig. 6. Here, the absolute value of the jnd is plotted against the adjusted correlation $\mathrm{r_A}$, the average correlation of the

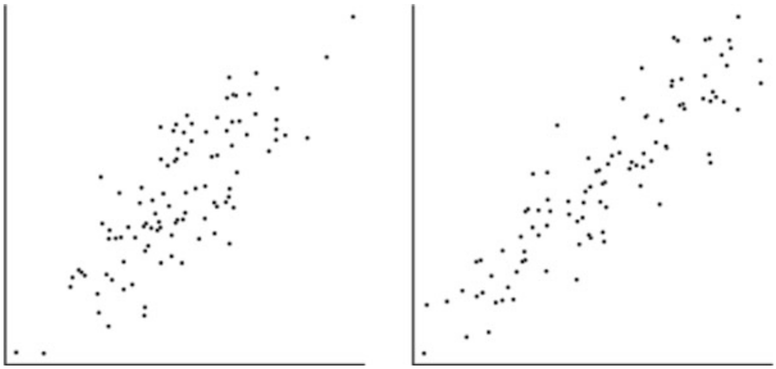


Fig. 5 Example of discrimination task. Observers are asked to choose which scatterplot is more highly correlated. Plots were 5° × 5° in extent. In this example, base correlation is 0.8; jnd is from above (i.e., the variant against which the base is tested has a higher correlation)

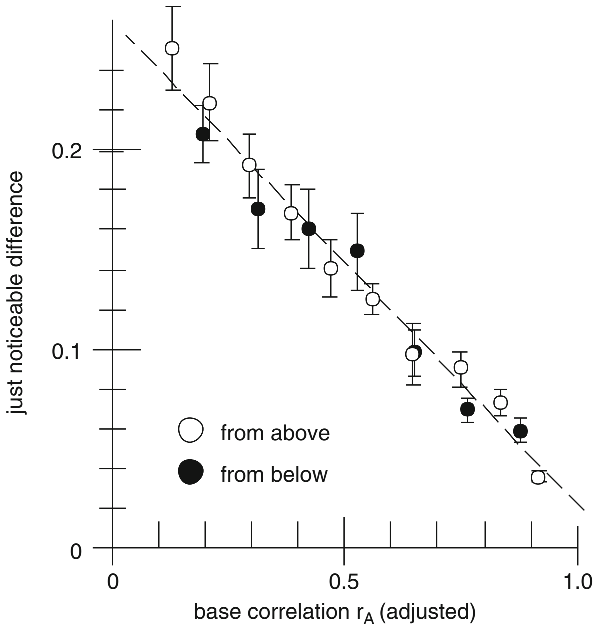


Fig. 6 Discriminability of correlation in scatterplots. *White dots* indicate that estimates are made by comparing a scatterplot with the base correlation against a higher correlation; *black dots* indicate that the comparison is made against a lower correlation. As is evident, these give the same estimates. Best fit of slope k is k = −0.22. Error bars denote standard error of the mean. Importantly, the value of b for the best-fitting line (b = 0.91) is similar to the value of b for accuracy (b = 0.88)

scatterplots being compared ($r_A = r + jnd(r)/2$). Portrayed this way, two patterns become evident. First, there is no dependence of the absolute value of the jnd on the direction of the variant—the same value is found regardless of whether the variant has a correlation higher or lower than the base. Second, a striking linearity exists in the way that jnd depends on adjusted correlation. This behavior can be described to a high degree of precision by

$$\mathrm{jnd}\,(r) = k\,(1/b - r_A) \tag{3}$$

where k and b are real numbers such that $0 < k, b < 1$. Note that for both k and b, smaller values denote lesser variability, in that jnd is lower.

Interestingly, as in the case of accuracy, the relevant variable appears to be the distance *away* from complete correlation. Indeed, Eq. 3 can be rewritten

$$\mathrm{jnd}\,(u) = k u \tag{4}$$

where $0 < k < 1$, and, as before, $u = 1 - br$, with $0 < b < 1$. This is a form of *Weber's Law*, which states that jnd is proportional to perceived magnitude; this appears to hold for the perception of several simple physical properties, such as vibration frequency and brightness [40]. In any event, the simplicity of this behavior allows variability to be described by just two scalar values: k and b.

Under some conditions, Weber's law for discrimination leads to Fechner's law for perceived quantity [41]. This appears to be the case for correlation, in that not only do both laws hold, but they are systematically related, with the value of b for perceived quantity having much the same value as its counterpart for discrimination [38]. Thus, only two values (k and b) are needed to describe accuracy and variability.

3.4 Timecourse

Although accuracy and variability are sometimes all that needs to be evaluated, other aspects of performance can also be important. One of these is *timecourse*, the minimum time needed to extract a structure from a dataset, such that more time will not lead to further improvement in performance. Although there may be delays in the machine component, speed of performance is usually limited by the human viewer. As such, measuring the minimum time needed to determine a structure from its graphical representation will often be an important part of evaluation. Not only can the result inform decisions about timing in dynamic displays, but it may also provide some information about the underlying mechanisms. Indeed, studies of timecourse are a common way to investigate various aspects of visual perception [30].

Returning to the case of scatterplots, the form of the laws describing performance suggests that correlation is—or at least, is associated with—a perceptually simple property. As such, it might be determined extremely quickly. To examine this possibility, discrimination was measured for scatterplots shown for controlled

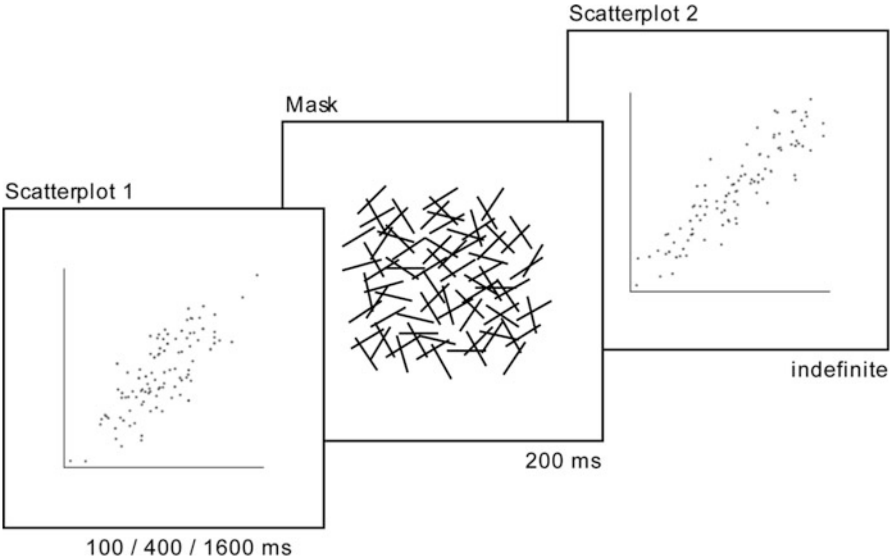


Fig. 7 **Measurement of timecourse of correlation perception**. Plots were 5° × 5° in extent. Observers were asked to choose which scatterplot is more highly correlated (b = 0.88)

amounts of time [42]. Here, scatterplots were presented sequentially instead of side-by-side; the first was presented for 100, 400, or 1,600 milliseconds (ms) before being followed by a mask that effectively stopped it from being processed further. A second scatterplot was then presented and remained on until the observer responded (Fig. 7). Jnds for these were measured as before.

Results for 20 observers are given in Fig. 8. These show jnd to be linear for all timescales examined. Performance for the 400 and 1,600 ms conditions was almost identical. Performance for the 100 ms condition showed a slight deterioration, but was otherwise much the same, indicating that the process was largely complete by that time. As such, these results indicate that correlation in scatterplots can indeed be determined by the visual system quite rapidly—likely within the 150 ms typical for processes such as object recognition and estimation of averages (e.g., [43, 44]).

4 Optimal Reduction

Although the extended-vision thesis can help understand what visualization is, and more systematic techniques can help with evaluation, they cannot address all issues dealing with visualization, such as those concerned with optimal visual representation. For example, even when a clear specification of a task exists, they still cannot help specify what an optimal representation might be, or whether a particular representation is the best one possible. Of course, it may often be

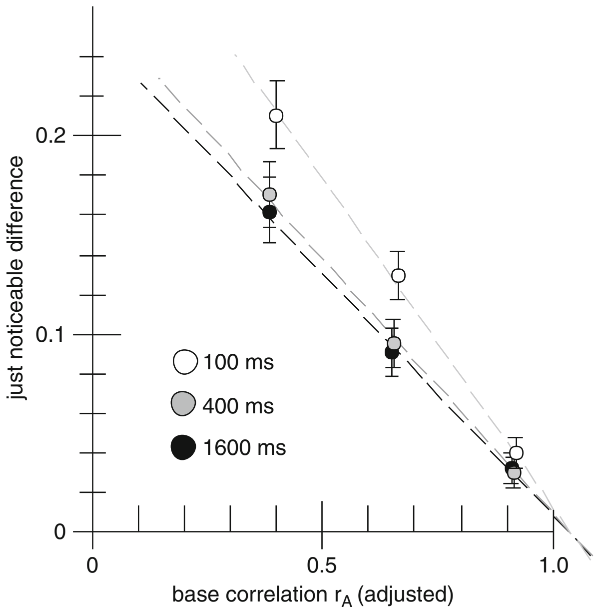


Fig. 8 Discrimination as a function of presentation time (Data from [42]). *Error bars* denote standard error of the mean. Note that performance is virtually identical for the 400 and 1,600 ms presentations. Although performance for 100 ms differs to some extent, it is still fairly similar, with a high degree of linearity

that an optimal representation simply does not exist. But it is nevertheless worth investigating the extent to which such representations might exist and might be found, or failing that, at least determine the extent to which representations would be able to avoid as many poor features as possible.

Traditionally, guidelines have been a major way to improve the likelihood of good design (e.g., [2, 8, 10]). And the relative suitability of a representation can be determined to some extent simply by evaluating a set of alternatives and selecting the one that yields the best performance. But such approaches cannot *guarantee* that a particular design is the best one, much less give a complete account of *why* it would (or would not) be optimal.

It is therefore worth returning to a more abstract perspective. As mentioned in Sect. 2, visualization can be characterized as the transformation of an original problem of finding abstract structure into one of finding geometrical structure. Once this geometrical structure has been found (by whatever means), it can be re-interpreted in terms of the original dataspace.[5] Considered in this way, visualization

[5]This assumes that the viewer can interpret the visual (or possibly other) representations used as a correlation. Developing such a "return route" to enable the inverse mapping may be an important part of training. Note that some of the performance limitations could occur at this stage, rather than the formation of the visual result proper.

is essentially a *similarity transform*—a commonly-used technique that transforms a given representation into one that enables solutions to be more easily found [45]. More precisely, it *reduces* the given problem to a simpler one.

This view then suggests that the issue of optimal representation might be approached in terms of an *optimal-reduction thesis*: the optimality of a particular design can be usefully viewed in terms of its ability to reduce a task to the most suitable set of operations in the extended system. (Note that this includes the processes used in the transformations themselves, many of which would be part of the machine component.) A given task can generally be reduced in various ways; in some sense this is similar to an exercise in programming, where different algorithms are used. However, different algorithms require different amounts of time or space (e.g., [46]); different kinds of processing elements create different levels of noise. Only a few possible designs—perhaps just one—will engage the most appropriate mechanisms [19]. The goal of the design process is to find this.

To make this a bit more concrete, return again to the example of correlation. Correlation can be determined not only via scatterplots of different designs (involving different kinds of axes or symbols, say), but also via devices such as parallel coordinates, bar charts, and line graphs [31]. The choice of representation determines the particular reduction—i.e., the particular processes used. Apart from the algorithms needed to create it, use of a scatterplot might involve operations in the human viewer such as grouping, determination of shape, and perhaps measurement of aspect ratio. Meanwhile, parallel coordinates might require isolating the lines between corresponding elements and determining the variance of their orientations. The goal for design is then to search through the space of possible processes to find those that lead to performance that is fastest, most accurate, and requires least effort from the viewer.

4.1 Operations Inventory

Perhaps the most direct way to approach design based on the optimal-reduction thesis is via an *operations inventory*. This would be a catalog of all possible operations (in both the human and the machine component) that could be applied in a given visualization. The description of each would include things such as its associated costs (in terms of e.g., time and space) and performance on various kinds of inputs. The goal would be to find the sequence of operations from this inventory that led to the fastest performance for the given task, consistent with constraints on overall performance (accuracy and variability, say). Finding this sequence would still be a problem, but it would at least be a more objective and well-defined one. In some ways, it would be similar to the problem faced by a compiler—viz., reducing a process to a given set of machine operations, such that it is carried out as quickly as possible.

Operations in the inventory could be organized into several sets. One might comprise basic elementary geometric and radiometric operations that could be directly applied to an image, e.g., grouping, determining the convex hull of a set of points, following a curve, finding the point of maximum intensity in a given region. More complex methods constructed from these could then form a second set—e.g., finding the set of similar shapes among items with various sizes and orientations. The inventory could also include constraints on interactions between operations, such as which ones cannot be applied concurrently. It might even contain a library of optimal—or at least relatively efficient—procedures that could be used for any visualization task (cf. [47]); another (not necessarily disjoint) set might contain tasks that can be carried out on the basis of visual operations alone [48]. Note that many operations could be carried out either by the machine or by the human (e.g., a figure could be rotated either graphically or mentally). In such cases, the particular choice would result from considering the costs and performance limitations with respect to the entire task.

Of course, the search for an optimal reduction will succeed only to the extent that such a reduction actually exists. It might be, for example, that instead of a single candidate, a *family* of candidates exists, corresponding to different trade-offs [49]. Or even if a unique candidate exists, it may not be possible to find it in reasonable time if the space of possible designs is too large. But as with intractable problems in general, however, there could exist a systematic approach to at least some aspects of the problem (e.g., [50]), or that would find candidates that are at least adequate [51].

Another limitation is that although there are many models of perceptual and cognitive processes in humans [52], much is still unknown [30]. As such, the evaluation of candidate designs will not be able to take everything into account. Nevertheless, existing knowledge might still enable candidates to be found that, even if not optimal, are still likely to be good.

4.2 General Representational Principles

A complementary approach to design—one that may overcome some of the limitations of an operations-based approach—is to focus not on operations, but on *form*, viz., the form of the graphical representation. Indeed, design has often relied on guidelines that essentially constrain the set of candidate representations considered (e.g., [10, 15, 53]). Many of these guidelines were developed for particular applications, such as tables and graphs. And although they are very important in these areas, they are often of limited generality. As such, they need to be complemented by a set of more general guidelines that would apply to any visualization.

A more general approach of this kind could be based entirely on (1) general information-processing principles, and (2) well-established knowledge about the perceptual or cognitive mechanisms of the human viewer [18]. The result would be a set of *general representational principles* (GRPs) that focuses not on a particular representation for a particular task, but on properties that *any* graphical representation should have for *any* task. Examples of these include:

- *Invertibility*. The graphical representation must support a 1:1 mapping between data values and the visual representations in the viewer. (For example, greater value along some data dimension would map to greater contrast, or to greater height.) If different values mapped to the same representation, information would be lost. Conversely, if different visual structures corresponded to the same data value in the context of a single task, different visual processes would be involved, causing interference.
- *Distinctness*. Values that need to be distinguished for the task must map onto visual representations that are distinct. (For example, if it is important to notice that two values are different, they must map to contrasts or positions that can easily be seen as different.) Otherwise, important information would be lost, or at the very least, performance would be slowed.
- *Uniformity*. Values along a single data dimension must map onto a single visual dimension, such as height, colour, or orientation. If different visual dimensions were used, different processes would become involved; both information and time would be lost trying to combine the results of these.
- *Ordering*. Data values that are ordered in some way must map to a visual property that is similarly ordered over the relevant range. (For example, greater value could map to greater height, if mapped to orientation or color, only a subset could be used, since these properties are cyclic.) If this is not done, the ability of the visual system to use perceptual order cannot be harnessed; indeed, if it operates against its natural ordering, performance could degrade substantially.
- *Separability*. Data involving separate dimensions must map to visual properties that are separable—i.e., can be attended separately (e.g., size and orientation). Otherwise, these values may become parts of a perceptually integrated structure, with the separate components then being difficult to access.

Such principles are not entirely new—several comprise the basis (often unstated) for much of good design (cf. [8, 18]). For example, the principle of separability is obeyed in the effective scatterplot of Fig. 1a, which uses the separable properties of horizontal and vertical for each data dimension. In contrast, the ineffective scatterplot of Fig. 1b violates this principle, using properties that become parts of a perceptually integrated structure—viz., the upper and lower parts of the structure corresponding to each glyph. Likewise, the principle of ordering has often been recommended for displays (e.g., [8, 18]), and the principle of distinctness is related to that of the smallest effective difference [54]. What is proposed here is that such principles be explicitly distinguished from task-specific constraints and identified as a distinct group. Moreover, given that they are based on universal considerations, it may also be possible to develop and organize them in a more systematic way (cf. [55]), or give them a more quantitative character.

GRPs constrain the kinds of transformations permitted between data values and graphical representations, eliminating many possible reductions right from the start. And because they are general, they will tend to be consistent with each other; the simultaneous use of several such principles will not lead to a clash, but to a more tightly-bounded space of possibilities, increasing the chances for an effective design. Such principles could also be easily combined with more task-specific ones, guiding design for particular applications.

5 Assessment of Alternative Designs

Given the difficulties faced in searching through all the alternatives possible for a design, and the fact that much is still unknown about the perceptual and cognitive mechanisms involved, the search for optimal—or even good—designs must be supplemented by empirical assessment. As in the case of evaluation, however, it may be possible to carry out such assessments in a relatively systematic way, informed in part by elements of vision science. These not only can suggest particular techniques, but also some of the design parameters to consider. Indeed, if alternatives are examined in an appropriate way, the results can also provide considerable insight into the mechanisms involved.

5.1 Different Parameter Values

To see how performance can be assessed for different values along a single design parameter, consider the issue of how many dots—or more precisely, symbols—a scatterplot should display. Too many would cause *overplotting*, where important information is crowded out. Displaying only a randomly-chosen subset of data points would solve this. But how many should be selected? Too few would cause correlation to be conveyed poorly. Consequently, it would be useful to know how performance depends on the number of dots displayed.

To investigate this, accuracy and variability was measured for a set of scatterplots containing various numbers of dots (12, 24, 48, 100, and 200), with the other parameters remaining fixed (Fig. 9) [56]. Evaluations were similar to those described in Sect. 3. The form of the laws governing accuracy and variability meant that only a few correlations needed to be tested: accuracy could be measured via the physical correlation corresponding to a perceived correlation of 1/2 (the first phase of the approach outlined in Sect. 3), while variability was measured using just three base correlations. This meant that all conditions could be tested on a single observer within a single experimental session. Such a *within-observer* design allows a far more sensitive measure of the effects of a parameter (in this case, number), in that it minimizes noise caused by the use of different observers [57].

Results for 12 observers are shown in Fig. 10. No differences were found in subjective perception (and thus, accuracy) for the various numbers of dots. (A slight difference appeared for $n = 200$, but was not statistically significant.) As might be expected on the basis of sampling, variability decreased as more dots were shown, but stabilized at 48 dots. Given that a within-observer design was used, this result

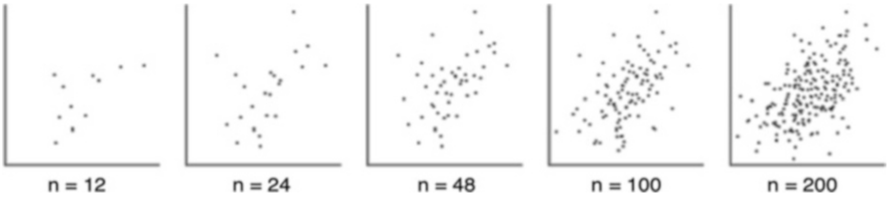


Fig. 9 Examples of tests of number of dots. For each observer, accuracy and variability was measured for each condition, and then compared

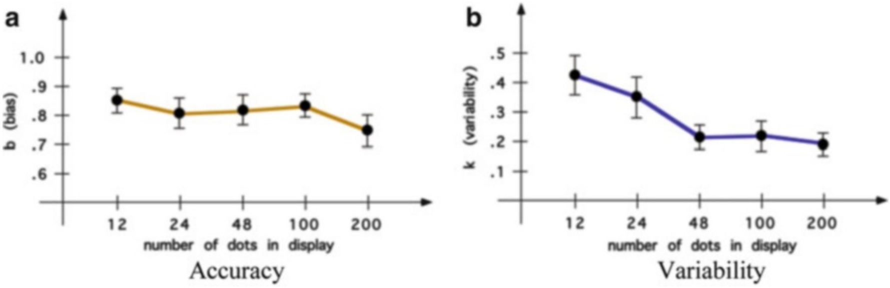


Fig. 10 Performance as a function of number of dots (Data from [56]). (**a**) Accuracy, as described by the measure b, derived from perceived correlation. (**b**) Variability, as described by the measure k. Each condition used the same set of observers

provides strong evidence that performance is not affected as long as the number of dots is 48 or more (up to 200), at least for the kind of dots tested here. A similar approach could also be used to investigate the upper limit, viz., how many dots can be displayed until overplotting begins to be a problem.

More generally, a systematic approach of this kind can be used for the assessment of any design parameter. For example, in a study examining the effect of dot (symbol) luminance, color, and size [56], observers were tested over a wide range of values for each parameter, along with a condition in which the values were mixed (Fig. 11).

Each parameter was tested separately, with 12 observers per parameter. Results showed an interesting amount of invariance: for all parameters tested, value had no measurable effect on either accuracy or variability [56]. Thus—at least for purposes of conveying correlation—designs varying along these dimensions appear to be equivalent.

Such invariance is also informative in terms of the possible mechanisms that underlie correlation perception in scatterplots. The indifference to size, for example, rules out the involvement of simple operations such as blurring, since the overall shape of the dot cloud does not seem to matter greatly. Instead, it suggests that the operations involved may rely primarily on the locations of the centers of the symbols.

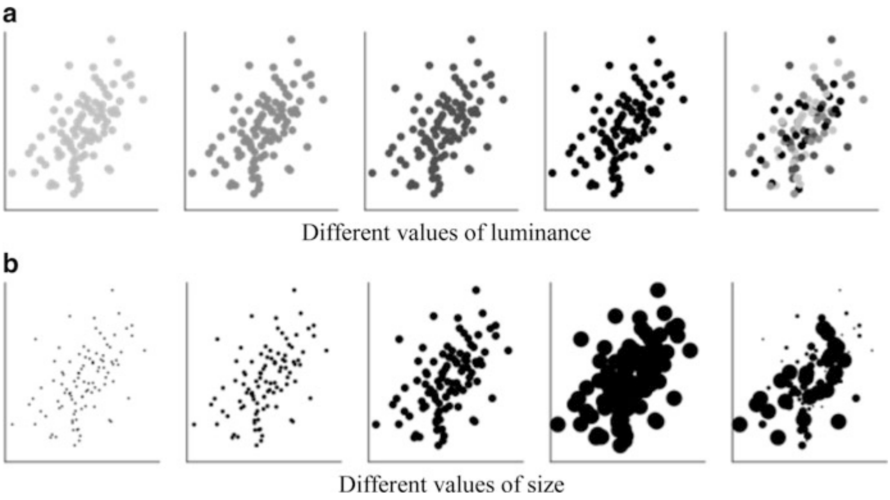


Fig. 11 **Examples of tests of parameter values**. (**a**) Luminance. Here, four different values were tested, along with a condition in which they were equally mixed. (**b**) Size. Again, four different values were tested, along with a condition in which they were equally mixed. For each of these, accuracy and variability was measured and compared across observers

5.2 Other Dimensions

This way of assessing alternative designs can in principle be applied more generally—for example, assessing designs that use different *dimensions*. To clarify what is meant here, note that a scatterplot represents the first data dimension by horizontal position and the second by vertical; for both data dimensions, space "carries" information. But it may be that carriers need not be spatial—several perceptual dimensions (or "visual variables") exist that are similar to position in many ways [8, 18, 30]. It might therefore be possible to use these dimensions to convey correlation, provided they obey the GRPs of Sect. 4.

As an example of this, consider "augmented stripplots" where the first data dimension is represented via horizontal position (as for scatterplots) and the second data dimension via size (diameter); correlation is then conveyed via the relation between these two properties (Fig. 12). Note the interaction here between visualization and vision science: the use of dimensions other than space is suggested by findings from vision science about perceptual dimensions; in return, the extent to which these dimensions can be used in a visualization design lets us learn more about their nature.

Owing to their isomorphism with scatterplots, augmented stripplots can be evaluated via the same kinds of techniques. Accuracy and variability were tested for a group of observers [56] based on correlations over the range 0–1, in increments of 0.1. Preliminary results from 18 observers are shown in Fig. 13.

Interestingly, both accuracy and variability appear to obey laws that are quite similar to those for scatterplots. Among other things, this means that the assessment

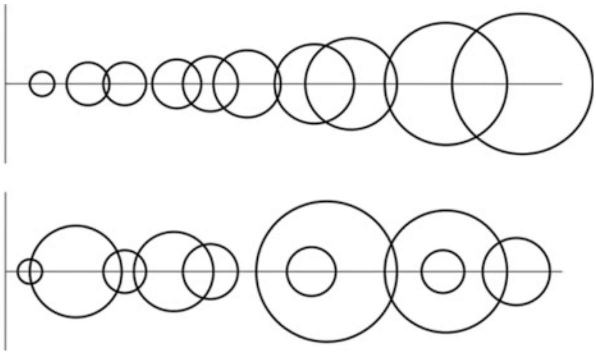


Fig. 12 Examples of augmented stripplots. *Upper figure* has correlation r = 1; *lower figure* has correlation r = 0

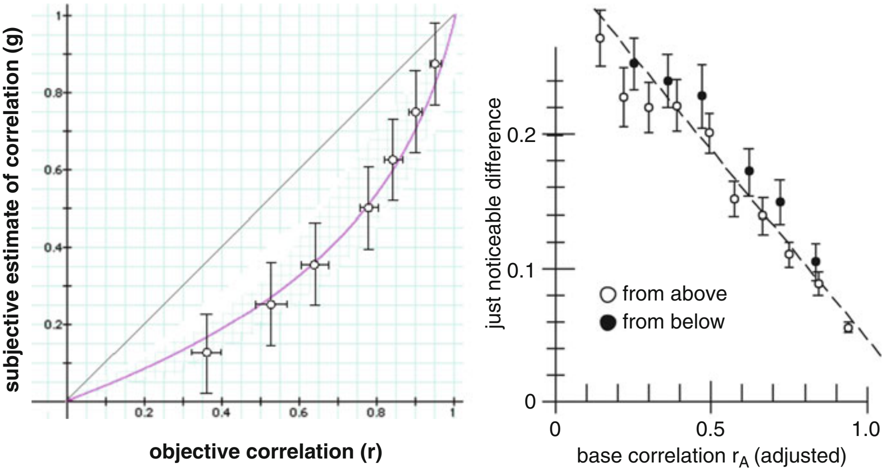


Fig. 13 Performance for augmented stripplots (Data from [56]). (**a**) Accuracy, as given by perceived correlation. Best-fit line to results is for b = 0.91 (cf b = 0.88 for scatterplots). (**b**) Variability, as given by jnd. Best-fine line is for k = 0.26 and b = 0.86 (cf. k = 0.22 and b = 0.91 for scatterplots). Both measures were obtained using the same set of observers

of different values of design parameters in these kinds of visualizations can be simplified: since only a few scalars (k and b) are sufficient to describe performance, only a few base correlations need to be tested. It also suggests that the perception of correlation in augmented stripplots is much the same as is it in scatterplots, perhaps reflecting an ability more general than previously believed. It remains an open question as to the mechanisms involved in these kinds of representations, and which other properties might also result in such behavior. But the approach sketched here is clearly capable of suggesting interesting new directions of research, and then examining them.

6 Limits of the Framework

The preceding sections have shown how several sets of issues in visualization—pertaining to both theory and practice—can be addressed by systematic approaches that can be considered parts of a common framework. These approaches can help us understand how visualizations work, how they might be evaluated, and even to some extent how they might be designed. But this has been shown in detail only for those aspects pertaining to correlation in scatterplots. Returning now to the question that motivated this paper: to what extent is a systematic framework—perhaps even a science—possible for visualization in general?

Clearly, the elements discussed here do not cover all aspects of visualization. For instance, issues concerning the nature and role of interaction have not been discussed in any great detail. More fundamentally, perhaps, little has been said about what would constitute an adequate explanation for *why* a particular visualization works (cf. vision science, where standards of explanation are much more developed [21]). The elements proposed here also say little about how an optimal design could actually be found, or even how to tell if one exists in the first place. And little consideration has been given to the nature of the design process itself (cf. [58–60]).

But the goal of this paper is simply to get a sense of whether at least some aspects of visualization could be handled via a scientific framework, and if so, what this might look like. The discussion here has shown that systematic and coherent approaches can be developed to address several important issues concerning the study and design of visualizations. The central issue now becomes the extent to which these elements could be developed into a framework of greater range and power.

6.1 Generality

The proposals here have largely used as their touchstone the determination of correlation in scatterplots. This was done primarily to show that these proposals are consistent with each other, and are useful for at least one real-world domain. In a way, this use of scatterplots is like the use of fruit flies by geneticists to speed up their investigation of complex systems: focus on a real system complex enough to involve the issues at hand, but simple enough to be studied in a relatively straightforward way. The issue now is the extent to which the approaches outlined above can be generalized to more than just these “fruit flies”.

In regards to scatterplots themselves, these approaches could likely be used to investigate design parameters of various kinds—not only properties of individual symbols, but aspects such as the size and shape of the cloud formed by the set of datapoints. They could also be used to examine issues such as the effect of outliers on performance, or the presence of a second group of symbols representing an irrelevant population. Again, such investigations would likely yield not only important knowledge about how well scatterplots work, but also additional insights into the mechanisms involved.

Importantly, these approaches do not depend on anything particular to scatterplots or to correlation. They could therefore be applied to correlation as conveyed by other kinds of representation, such as bar charts, parallel coordinates, or line graphs [31]. Likewise, they could also be applied to other descriptive statistics, such as means and variances. They could even be applied to tasks such detection of outliers or the detection of two different populations in a given dataset. A considerable amount of work would be required to carry out such studies, but at least the general outlines of the approach are clear.

One possible limit, however, is the complexity of the structures (and related operations) to be visualized. For example, applying these approaches to the visualization of connectivity patterns in networks might lead to issues that are too complex to resolve, or at least, resolve in a reasonable amount of time. On the other hand, it appears that some kinds of complex structure can be effectively handled via metaphor, which engages the appropriate higher-level cognitive processes (e.g., [61]); if so, evaluation techniques for these might be modeled on those used in cognitive science. Settling this issue is an important goal for future work. It is likely connected the general issue of the extent to which different kinds of information can be represented visually [62].

Another possible limit is the extent to which approaches developed for "static" aspects of visualization can apply to dynamic aspects such as interaction. Interaction has its origins in the fact that there is often too much information to be displayed at any time, requiring an active process of exploration [5, 63]. A parallel situation exists in visual perception, where the inability to represent more than a fraction of the visual input in stable form means that vision must rely on a dynamic system in which objects are represented in a "just-in-time" manner [11]. This parallelism may explain why interaction can be such a natural activity if visualization is designed correctly; indeed, GRPs can be developed towards this end [64]. Going beyond this would not only require knowledge of how scenes are perceived, but also why a viewer might represent a particular object at a particular time. It is increasingly clear that these issues are complex. Whether they are insurmountable remains an open question.

6.2 Visualization, Vision, and Science

Given that at least some of what has been proposed here could likely be generalized further, to what extent could it also be developed into a truly scientific framework, or even an outright science of visualization?

As mentioned earlier, a science is essentially an organized way of thinking about a connected set of issues. Looking at visualization in terms of issues, at least three such sets can be identified.[6] Some appear able to support the development of a science; others do not.

The first set concerns visualization as *artifact*—i.e., as a system that already exists. Issues here include determining its characteristics, ascertaining how it works, and describing how it relates to others like it (i.e., taxonomy). Work has been done on some of these issues—e.g., taxonomies of data types [65], operation types [63], and algorithms [66]—and things could likely be developed further in these directions. Indeed, according to the extended-vision thesis, visualization systems can be treated much the same as basic vision systems, with a similar handling of basic entities, questions, and methodologies. In the case of vision (or biology more generally), coherence in subject matter is obtained in part by the need for systems to be reasonably effective at what they do, creating a tendency to converge on common architectures [21, 67, 68]. Similar pressures clearly exist on visualization systems. As such, the coherence needed for a science almost certainly exists. Moreover, it appears likely that systematic approaches could be developed for many tasks that are well-defined (such as correlation perception), resulting in an *artifact-level science* of visualization. Several of the approaches discussed here could be part of this. It is unclear how many such "islands" of well-defined tasks (or subtasks) could be handled this way, but if vision science is any guide, there would be quite a few.

A very different set of issues concerns the *practice* (or activity) of design—e.g., how to design a visualization that enables discovery of interesting structure in some set of high-dimensional data. Such activity is generally regarded as not a science [69]: the immense space of possibilities, the often ill-defined nature of what is requested of the system, and the need to include subjective factors such as aesthetic preferences make it difficult to carry out via a simple set of approaches. Design as applied to visualization inherits this [29, 32]. (For example, even defining a clear purpose for a visualization system can be difficult—if not impossible—in some circumstances [32].) But although this *design practice* of visualization cannot be a science, parts of it could nevertheless be made more systematic; effective design methods (cf. software programming) could make good use of what is known in related sciences—including the "islands" of the artifact-level science of visualization—as well as any other kinds of relevant knowledge. (Several of the approaches discussed in Sects. 4 and 5 could be part of this.) The result would be an open-ended activity similar in many ways to engineering, architecture, or any other organized design discipline [69].

A third set of issues concerns the *nature* of the design process (as opposed to issues that arise during a design activity). For example, how can design itself be characterized? Which aspects of the design process can be formalized, and which cannot? What kinds of design processes have been developed, and how do they relate to each other? It has been suggested that such issues could be the basis

[6] Other issues exist, such as those concerned with visualization as a technology [32]. However, these are not directly relevant to concerns about the nature of a scientific framework, and so are not discussed here.

for a science under some circumstances [70]; a few studies (e.g., [59, 60]) are possible beginnings of this in the context of visualization. The extent to which these developments will eventually result in a *design-level science* of visualization (or equivalently, a science of visualization design) is currently unclear. But there appear to be no a priori objections standing in the way.

Finally, it should be mentioned that the proposals discussed here also point towards the possibility of even closer connections between visualization and vision science. Knowledge about perceptual mechanisms can be usefully applied to visualization (e.g., [5, 9]), and Sects. 3 and 5 show this to be true for methodology as well. But there is still another connection that appears to be emerging. In Sect. 5, it was shown that systematic techniques could be used to determine how the number of dots in a scatterplot affects the accuracy and variability of correlation estimates. Although this was a study of how performance depended on different values of a design parameter (the number of dots), it could also be viewed as a controlled experiment in vision science, with the results shedding a bit of light on the mechanisms involved. Similar considerations apply to the other examples in that section, such as the study of how correlation can be conveyed by size as well as by position. More generally, then, if researchers in visualization expand their set of techniques to include those typically used by vision scientists (while keeping the same kind of stimuli), while vision scientists expand their set of stimuli to include those typically used by visualization researchers (while keeping the same kind of methodologies), the possibility arises of a class of studies that belong in both fields, with results of genuine interest to each.

Acknowledgements Many thanks to Stephen Few, Dave Kasik, Minjung Kim, Tamara Munzner, Vicki Lemieux, Michael Sedlmair, Ben Shneiderman, and Jack van Wijk for their comments on earlier versions of this paper. Thanks also to Tamara Munzner for feedback on the idea of the operations inventory, and to Jack van Wijk for interesting discussions about the limits of a possible science of visualization. Support for this work and several of the studies mentioned was provided by the Natural Sciences and Engineering Research Council of Canada (NSERC), and The Boeing Company.

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