Assume for same point in the picture, E_i keeps the same in different pictures, then we can assume that for the same point in the different picture, there is a linear relationship between exposure time and the pixel value:

$$t_j = Z_{ij} a_i + b_i$$

where t_j is the exposure time for j^{th} picture, and Z_{ij} is the pixel value for pixel at position i in the j^{th} picture.
Notation: Let M[i] be the i^{th} row of M and M_j be the j^{th} column of M
Consider construct the matrix M $\in R_{i,j+1}$ by the following rule:

$$M_{pq} = \begin{cases} t_q & 1 \leq q \leq j \\ -b_p & q = j+1 \end{cases}$$

Construct the matrix X $\in R_{j+1,j}$ by the rule:

$$X_{pq} = \begin{cases} 1 & p = j+1 \; or \; p = q \\ 0 & otherwise \end{cases}$$

Construct the diagonal matrix A $\in R_{i,i}$, where diag = $(a_1, a_2, ..., a_i)$
e.g. for i = 3, j = 2

$$M = \begin{bmatrix} t_1 & t_2 & -b_1 \\ t_1 & t_2 & -b_2 \\ t_1 & t_2 & -b3 \end{bmatrix}; X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}; A = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix}; Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \\ Z_{31} & Z_{32} \end{bmatrix}$$

And we can get that MX = AZ and A^{-1} is also a diagonal matrix with diag=$(\frac{1}{a_1}, \frac{1}{a_2}, ..., \frac{1}{a_i})$
Let K = $A^{-1}M$ and then KX = Z, we can use least square to solve for K and K $\in R_{i,j+1}$ with

$$K_{pq} = \begin{cases} \frac{t_q}{a_p} & 1 \leq q \leq j \\ -\frac{b_p}{a_p} & q = j+1 \end{cases}$$

e.g. for the same example

$$K = \begin{bmatrix} \frac{t_1}{a_1} & \frac{t_2}{a_1} & -\frac{b_1}{a_1} \\ \frac{t_1}{a_2} & \frac{t_2}{a_2} & -\frac{b_2}{a_2} \\ \frac{t_1}{a_3} & \frac{t_2}{a_3} & -\frac{b_3}{a_3} \end{bmatrix}$$

Let $s_i \in R_j$ and $s_i[c] = K_{ic} = \frac{t_c}{a_i}$, then let $r_{i+1} = \frac{a_{i+1}}{a_1} \Rightarrow a_{i+1} = a_1 r_{i+1}$, by observe we can find that $s_1 = \frac{a_{i+1}}{a_1} s_{i+1} = r_{i+1} s_{i+1}$, use least square to solve for $r_1, r_2, .., r_i$, also by the K matrix we can get:

$$a_i = r_i a_1$$

$$t_j = a_1 K_{1j}$$

$$b_i = -a_i K_{i(j+1)} = -r_i a_1 K_{i(j+1)}$$

Then we can estimate exposure time by giving a a_1