CS 10:
Problem solving via Object Oriented Programming

Hashing

Java provides us faster Sets and Maps using hashing instead of Trees

- Sets hold unique objects, Maps hold Key/Value pairs
- Map Keys are unique, but Values may be duplicated
- As we saw last class, using a Tree is a natural fit for implementing Sets and Maps
- Performance with a Tree is <u>*generally*</u> better than a List
- We can do better than Tree performance by using today's topic of discussion – hashing
- Java provides the HashSet and HashMap out-of-the-box that do a lot of the hard work for us

Agenda

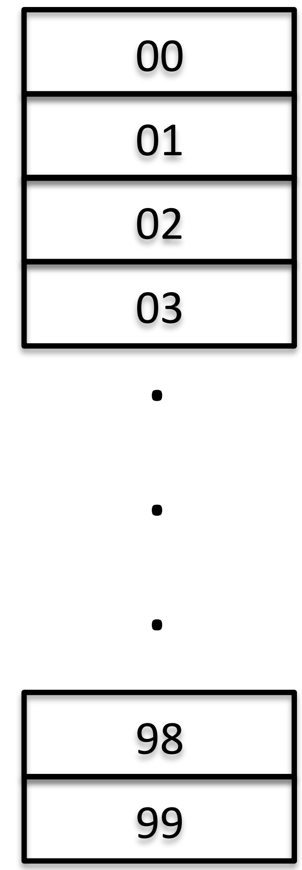
1. Hashing
2. Computing Hash functions
3. Handling collisions
    1. Chaining
    2. Open Addressing

The old Sears catalog orders illustrate how hashing works

Sears store implementation of hash table

- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)

Slots behind desk



Fixed size table

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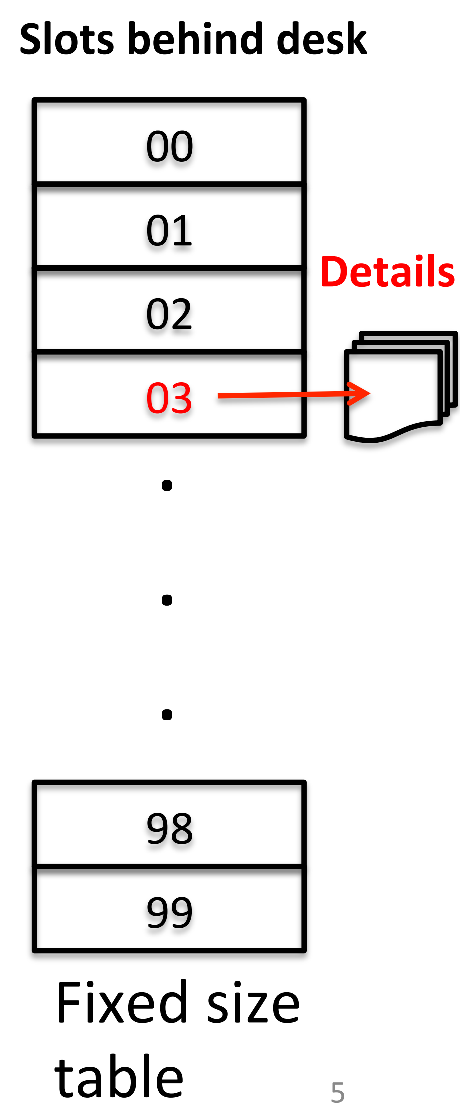

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Sears store implementation of hash table

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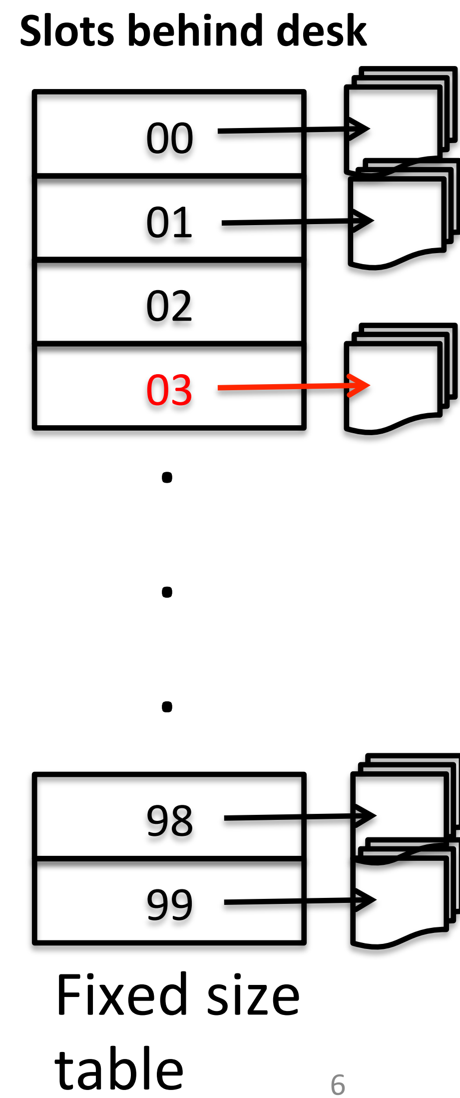
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone



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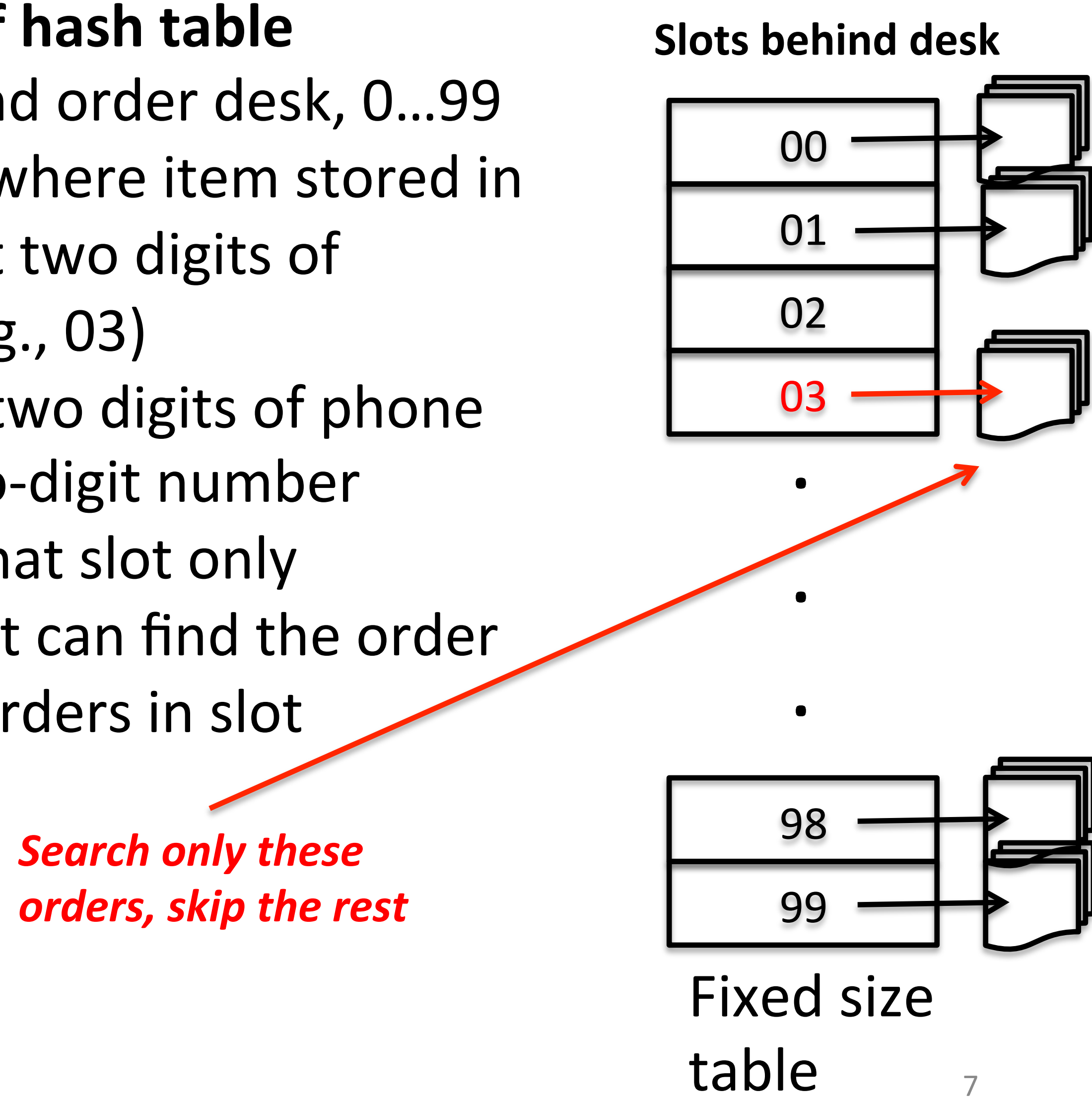
Sears store implementation of hash table

- Used to have 100 slots behind order desk, 0…99
- Shipments arrive, details of where item stored in warehouse put in slot by last two digits of customer phone number (e.g., 03)
- Customer arrives, gives last two digits of phone
- Clerk finds slot with that two-digit number
- Clerk searches contents of that slot only
- Could be multiple orders, but can find the order quickly because only a few orders in slot



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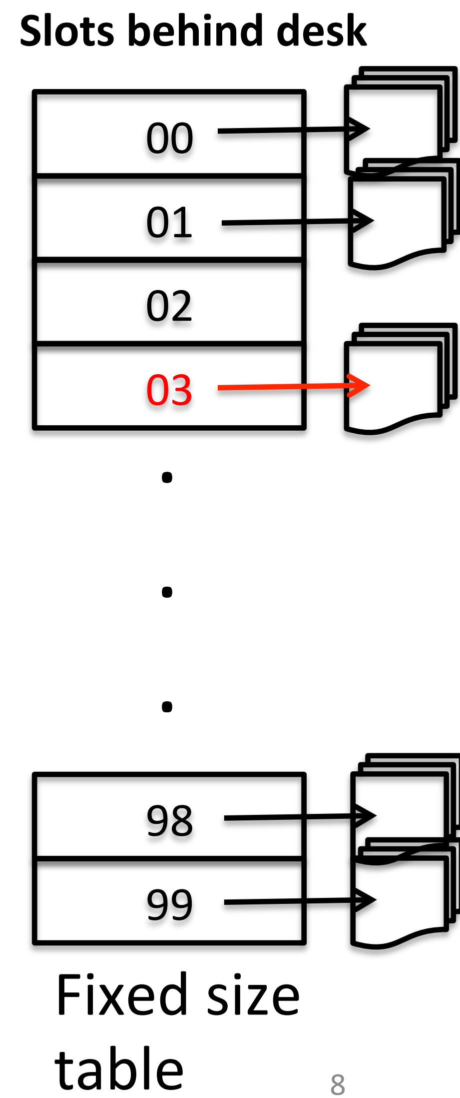
- Could be multiple orders, but can find the order quickly because only a few orders in slot
- Splits set of (possibly) hundreds or thousands of orders into 100 slots of a few items each



The old Sears catalog orders illustrate how hashing works

Sears store implementation of hash table

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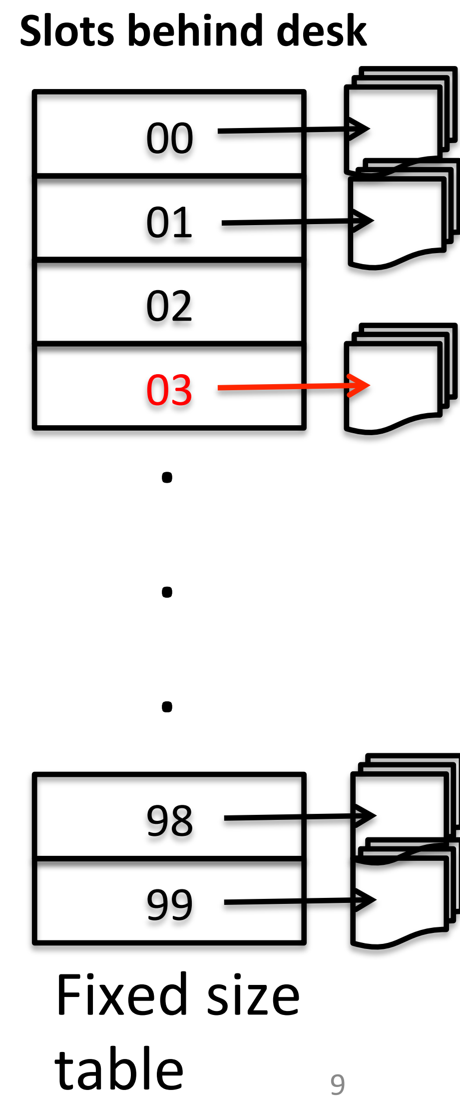
- Customer arrives, gives last two digits of phone
- Clerk finds slot with that two-digit number
- Clerk searches contents of that slot only
- Could be multiple orders, but can find the order quickly because only a few orders in slot
- Splits set of (possibly) hundreds or thousands of orders into 100 slots of a few items each
- Trick: find a hash function that spreads customers evenly
- Last two digits work, why not first two?

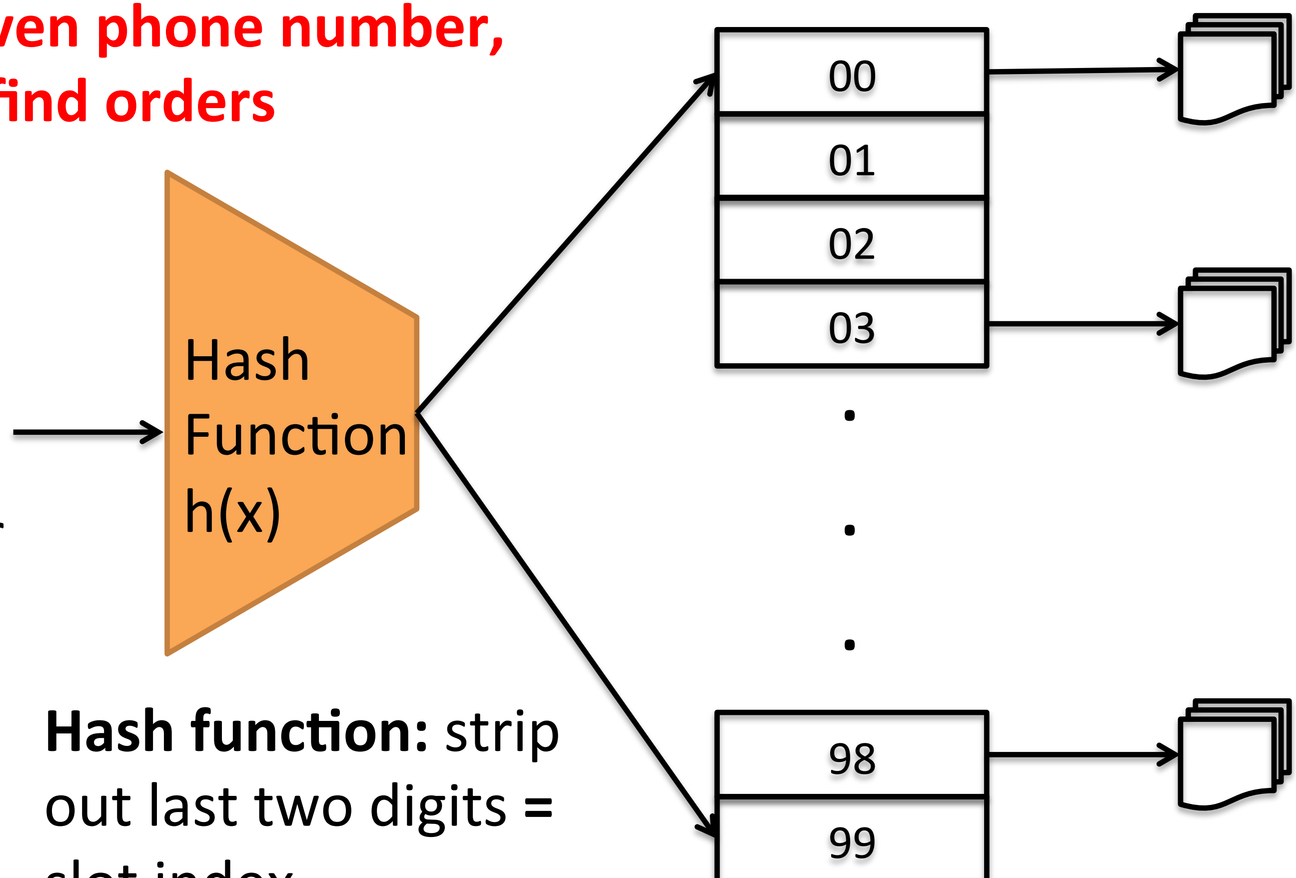


The store is using a form of hashing based on customer's phone number

Hashing phone numbers to find orders

Goal: given phone number, quickly find orders

Input: Phone number (Key)



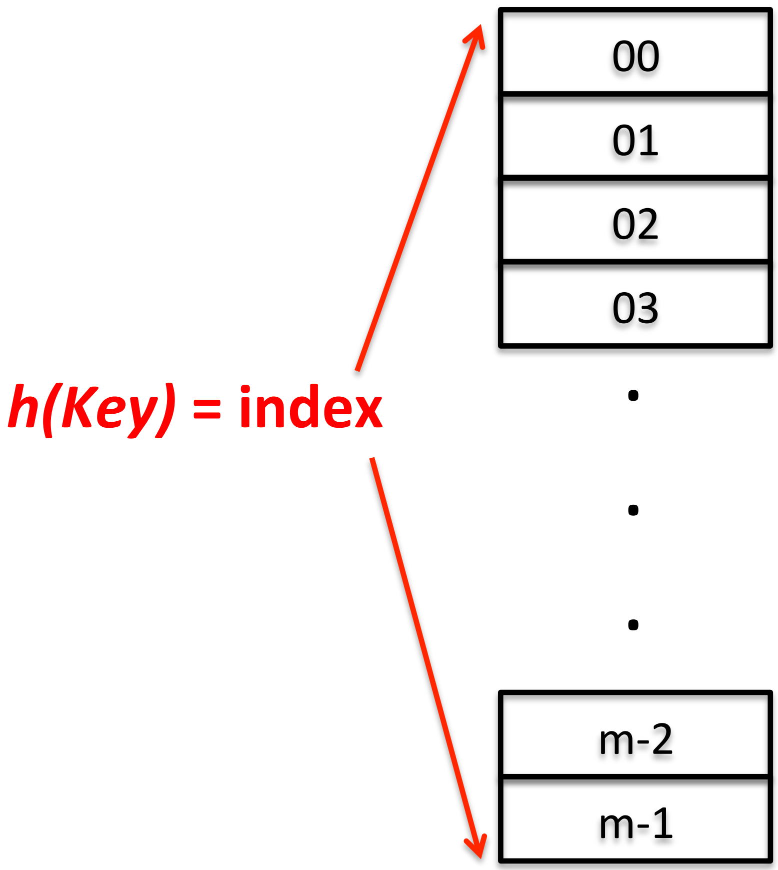
Search only small number of orders

Hash function: strip out last two digits = slot index

Fixed size table

Customer orders

Hashing's big idea: map a Key to an array index, then access is fast



Fixed size *m*
hash table

Map hash table implementation

- Begin with array of fixed size *m* (called a hash table)
- Each array index holds item we want to find (e.g., warehouse location of customer's order)
- Use hash function *h* on *Key* to give index into hash table
- ***h(Key)*** **= table index**
- Get item from hash table at index given by hash function
- *Fast* to *get/set/add/remove* items
- What about a HashSet?
- Use object itself as Key
- How to hash Key or object?

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Good hash functions map keys to indexes in table with three desirable properties

Desirable properties of a hash function

1. Hash can be computed quickly and consistently
2. Hash spreads the universe of keys evenly over the table
3. Small changes in the key (e.g., changing a character in a string or order of letters) should result in different hash value

Cryptographic hash function also:

- Difficult to determine key given the result of hash
- Unlikely that different keys will result in same hash
- We will not focus on crypto requirements

Suppose we used the first letter of people's names to hash, how would that work?

First letter of name as hash

1. It can be computed quickly

   Yes

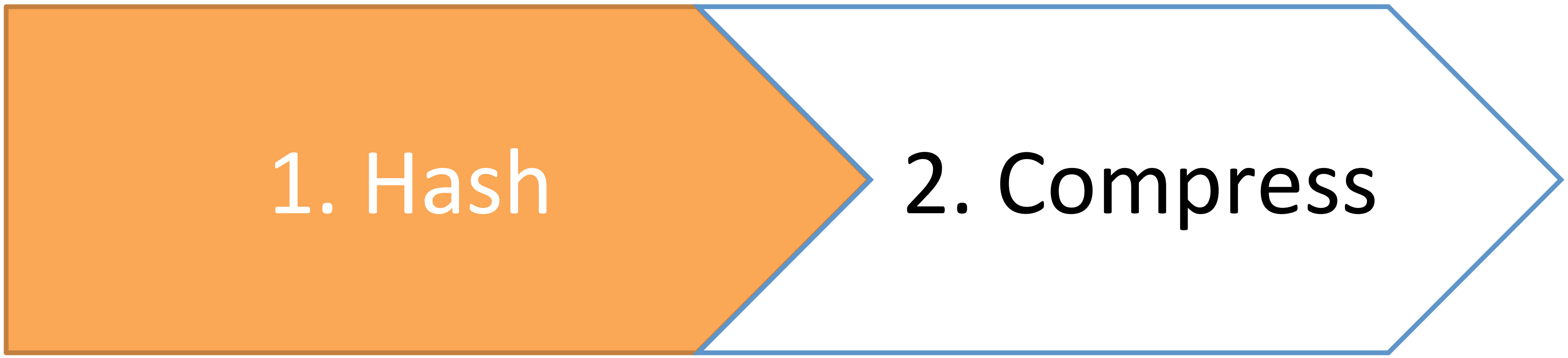
2. It spreads the universe of keys evenly over the table

   No

3. Small changes in the key (e.g., changing a character in a string or order of letters) should result in different hash value

   Not really. Different, if change first letter, otherwise not.

Hashing is often done in two steps: hash then compress



- Get an integer representation of Key
- Integer could be in range –infinity to +infinity

Constrain integer to table index [0..m)

First step in hashing is to get an integer representation of the key

Goal: given key compute an index into hash table array

Some Java objects can be directly cast to integers

- byte
- short
- int
- char

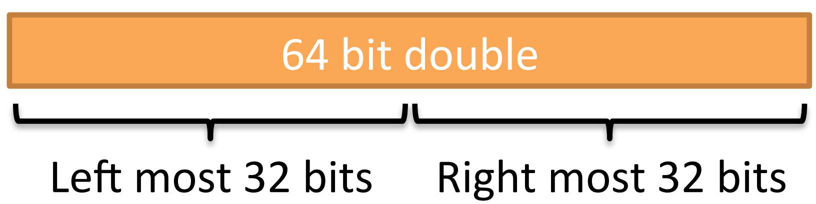
```
char a = 'a';
int b = (int)a;
```

b = 97

Some items too long cast to integers

- double (64 bits)
- long (64 bits)

- Too long to make 32 bit integers



XOR each half

Complex objects as as Strings can also be hashed to a single integer

Hashing complex objects

- Consider String `x` of length `n`
- Pick prime number `a` (book recommends 31, 37, 39 or 41)
- Cast each character in `x` to an integer
- Calculate polynomial hashcode as $x_0a^{n-1} + x_1a^{n-2} + \ldots x_{n-2}a + x_{n-1}$
- Use Horner's rule to efficiently compute hash code

```
public int hashCode() {
    final int a=37;
    int sum = x[0]; //first item in array
    for (int j=1;j<n;j++) {
        sum = a*sum + x[j]; //array element j
    }
    return sum;
}
```

- Experiments show that when using `a` as above, 50,000 English words had fewer than 7 collisions

Good news: Java provides a *hashCode()* method to compute hashes for us!

hashCode()

Java does the hashing for us for Strings and autoboxed types with *hashCode()* method

```
Character a = ‘a’;
a.hashCode() returns 97

String b = “Hello”;
b.hashCode() returns 69609650
```

Bad news: We need to override *hashCode()* and *equals()* for our own Objects

- By default Java uses memory address of objects as a *hashCode*
- But we typically want to hash based on properties of object, not whatever memory location an object happened to be assigned
- This way two objects with same instance variables will hash to the same table location (those objects are considered equal)
- Java says that two *equal* objects must return same *hashCode()*

```
public class BlobHash extends Blob{

    @Override
    public boolean equals(Object otherBlob) {
        Blob b = (Blob)otherBlob; //cast as Blob
        if (x == b.x && y == b.y && r == b.r)
            return true;
        return false;
    }

    @Override
    public int hashCode() {
        final int a=37;
        int sum = a*a + (int)x;
        sum += a * (int)y;
        sum += (int)r;
        return sum;
    }
}
```

Here we consider two Blobs *equal* if they have the same *x, y* and *r* values
<u>*equals()* IS THE RIGHT WAY TO COMPARE OBJECT EQUALITY</u> (not ==)

Override *hashCode()* to provide the same hash if two Blobs are *equal*

If don't override *hashCode()* then even though two objects are considered equal, Java will look in the wrong slot

Java *hashCode()* example

hashCode()

Some types can be directly cast to an integer

```java
    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy

```
<terminated> HashTest [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Jan 28, 2018, 5:52:17 PM)
Casting 'a' to int is: 97
```

Java *hashCode()* example

hashCode()

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Some types can be directly cast to an integer

Java does this for us for autoboxed types with *hashCode()*

Java *hashCode()* example

hashCode()

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        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
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hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650
```

Some types can be directly cast to an integer

Java does this for us for autoboxed types with *hashCode()*

***hashCode()* also works for more complex built-in types**

Java *hashCode()* example

hashCode()

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        char a = 'a';
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        //create new Blob with overridden equals and hashCode functions
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hashCode for 'a' is: 97
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```

For our own objects, we can provide our own *hashCode()* otherwise we get the memory location by default

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b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 1564 b2:1374
```

For our own objects, we can provide our own *hashCode()* otherwise we get the memory location by default

```java
@Override
public int hashCode() {
    final int a=37;
    int sum = a*a + (int)x;
    sum += a * (int)y;
    sum += (int)r;
    return sum;
}
```

***hashCode()* should compute hash:**

1. **Quickly and consistently**
2. **Spread keys evenly**
3. **Small changes = different hash**

Java *equals()* example

equals()

Override *equals()* to test if objects are equivalent
Otherwise *equals()* checks if same memory location

```java
    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
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        System.out.println("hashCode for 'a' is: " + z.hashCode());
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}
```

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

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b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 1564 b2:1374
```

Java *equals()* example

equals()

Override *equals()* to test if objects are equivalent
Otherwise *equals()* checks if same memory location

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        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
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        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
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        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
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    }
}
```

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

This is the right way to compare if two objects are equivalent (not b1 == b2)

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```
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b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 1564 b2:1374
b1 is equal to b2: false
```

Java *equals()* example

equals()

Override *equals()* to test if objects are equivalent
Otherwise *equals()* checks if same memory location

```java
    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy

<terminated> HashTest [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Jan 28, 2018, 5:52:17 PM)

```
Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650

b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 1564 b2:1374
b1 is equal to b2: false
after update b1 now equals b2: true
```

This is the right way to compare if two objects are equivalent (not b1 == b2)

After updating *x,y,* and *r* two Blobs are now equal

Java *equals()* example

equals()

Override *equals()* to test if objects are equivalent
Otherwise *equals()* checks if same memory location

```java
    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
        System.out.println("hashCode for 'hello' is: " + y.hashCode());
        System.out.println();

        //create new Blob with overridden equals and hashCode functions
        BlobHash b1 = new BlobHash();
        b1.x = 5; b1.y = 5; b1.r = 5;  //update b1's location
        BlobHash b2 = new BlobHash(); //create new HashBlob
        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy

<terminated> HashTest [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Jan 28, 2018, 5:52:17 PM)

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Casting 'a' to int is: 97
hashCode for 'a' is: 97
hashCode for 'hello' is: 69609650

b1 is at (x,y,r): 5.0, 5.0, 5.0
b2 is at (x,y,r): 0.0, 0.0, 5.0
hashCode b1: 1564 b2:1374
b1 is equal to b2: false
after update b1 now equals b2: true
hashCode b1: 1564 b2:1564
```

This is the right way to compare if two objects are equivalent (not b1 == b2)

After updating *x,y,* and *r* two Blobs are now equal

***hashCode()* also returns the same value for equivalent objects**

Java *equals()* example

equals()

```java
    public static void main(String[] args) {
        char a = 'a';
        int b = (int)a;
        System.out.println("Casting 'a' to int is: "+ b);
        Character z = 'a';
        System.out.println("hashCode for 'a' is: " + z.hashCode());
        String y = "Hello";
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        //create new Blob with overridden equals and hashCode functions
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        System.out.println("b1 is at (x,y,r): " + b1.x + ", " + b1.y + ", " + b1.r);
        System.out.println("b2 is at (x,y,r): " + b2.x + ", " + b2.y + ", " + b2.r);
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
        System.out.println("b1 is equal to b2: " + b1.equals(b2));
        b2.x = 5; b2.y = 5; b2.r = 5; //set b2 to same location as b1
        System.out.println("after update b1 now equals b2: " + b1.equals(b2));
        System.out.println("hashCode b1: " + b1.hashCode() + " b2:" + b2.hashCode());
    }
}
```

Problems Javadoc Declaration Console Debug Expressions Error Log Call Hierarchy

<terminated> HashTest [Java Application] /Library/Java/JavaVirtualMachines/jdk1.8.0_112.jdk/Contents/Home/bin/java (Jan 28, 2018, 5:52:17 PM)

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hashCode b1: 1564 b2:1374
b1 is equal to b2: false
after update b1 now equals b2: true
hashCode b1: 1564 b2:1564
```

Override *equals()* to test if objects are equivalent
Otherwise *equals()* checks if same memory location

```java
@Override
public boolean equals(Object otherBlob) {
    Blob b = (Blob)otherBlob; //cast as Blob
    if (x == b.x && y == b.y && r == b.r)
        return true;
    return false;
}
```

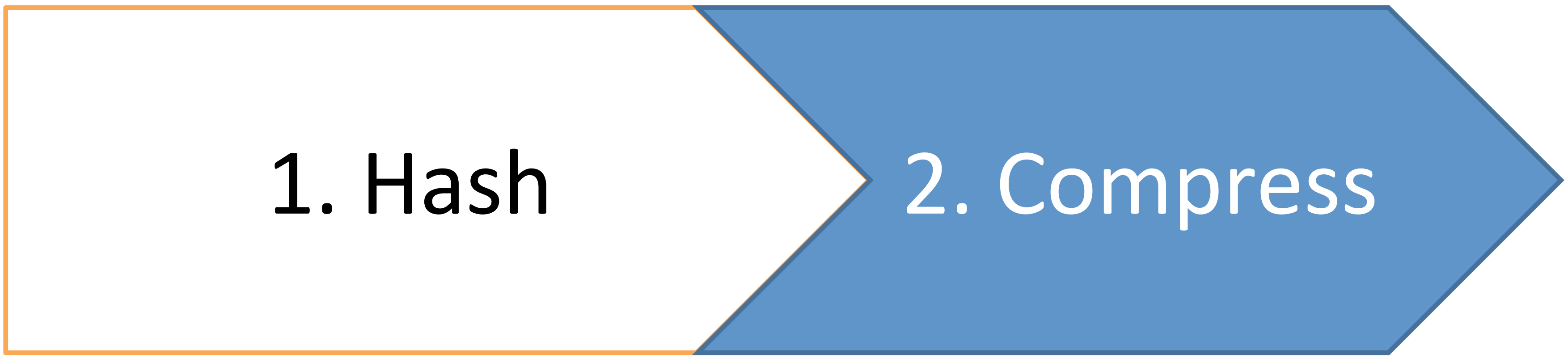
This is the right way to compare if two objects are equivalent (not b1 == b2)

After updating *x,y,* and *r* two Blobs are now equal

HashMap and HashSet will now put equivalent objects in the same slot in the table (after compression)

hashCode() also returns the same value for equivalent objects

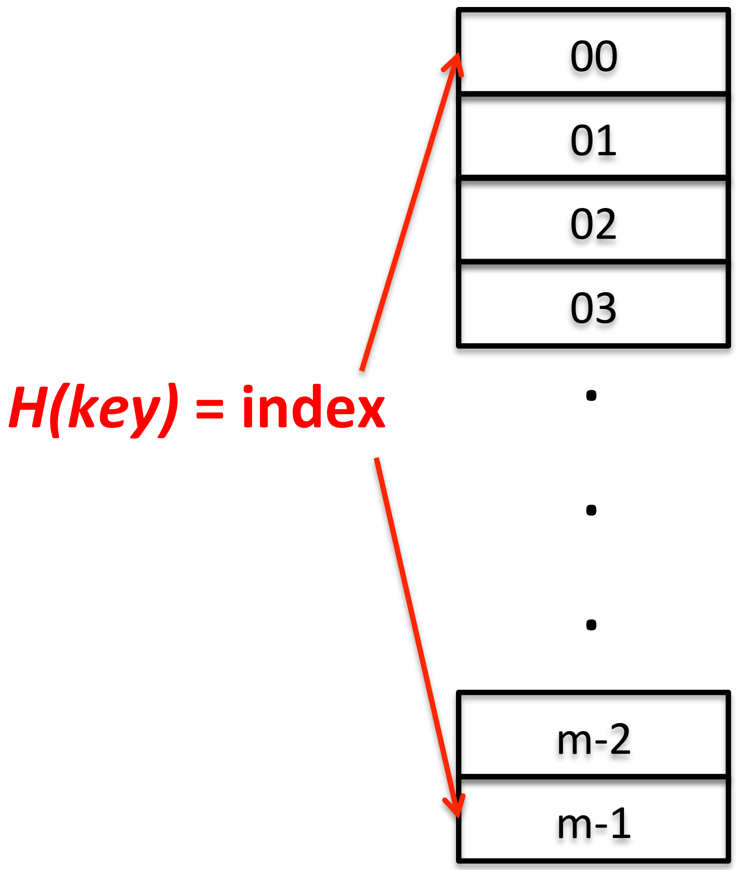
Hashing is often done in two steps: hash then compress



- Get an integer representation of Key
- Integer could be in range –infinity to +infinity

Constrain integer to table index [0..m)

May have to compress hash value to table index [0..m)



Fixed size *m* hash table

Compressing

- *hashCode()* value may be larger than the table (or negative!)
- Need to constrain value to one of the table slots [0..*m*)
- "Division method" is simple:
  h(key) = key % m
- Works well if *m* is prime
- Book gives a more advanced version called Multiply-Add-And-Divide (MAD)
- Java takes care of this for us ☺
- Eventually will encounter collisions where multiple keys map to the same slot ☹

Agenda

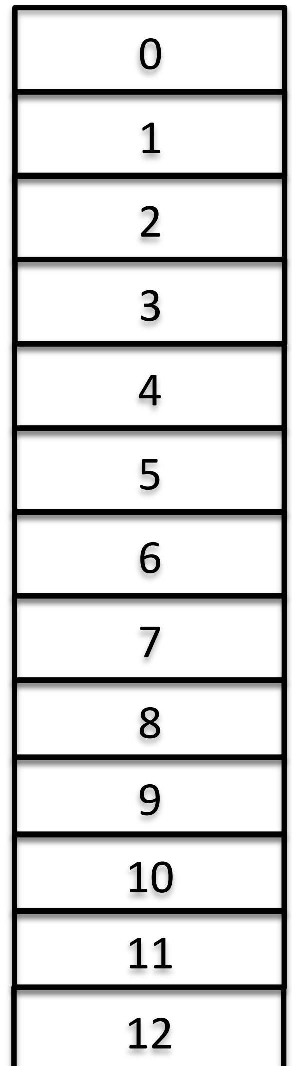
1. Hashing
2. Computing Hash functions
3. Handling collisions
   1. Chaining
   2. Open Addressing

Collisions happen when multiple keys map to the same table index

Integer keys

Given table size m = 13

Compute **h(key) = (key % m)**



m = 13

Collisions happen when multiple keys map to the same table index

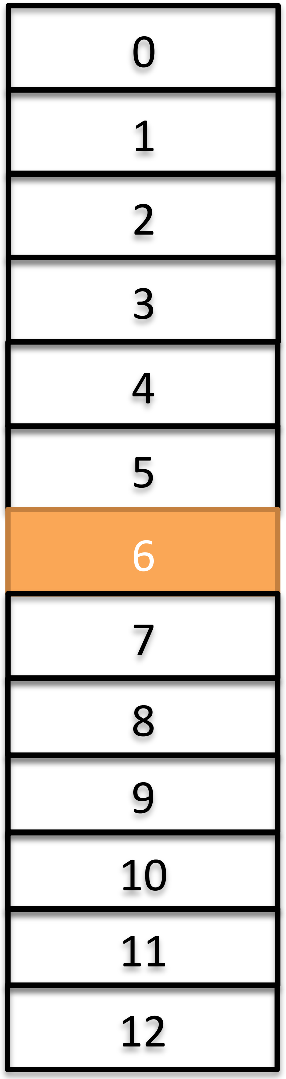
Integer keys

Given table size m = 13

Compute h(key) = (key % m)

Example

- **h(6) = 6**



m = 13

Collisions happen when multiple keys map to the same table index

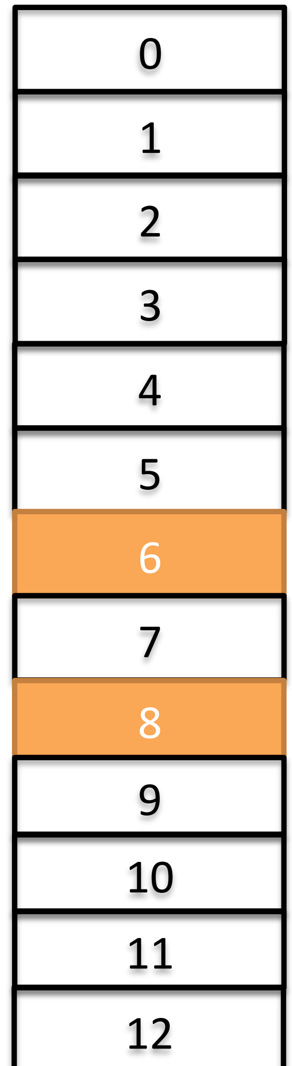
Integer keys

Given table size m = 13

Compute h(key) = (key % m)

Example

- h(6) = 6
- **h(8) = 8**



m = 13

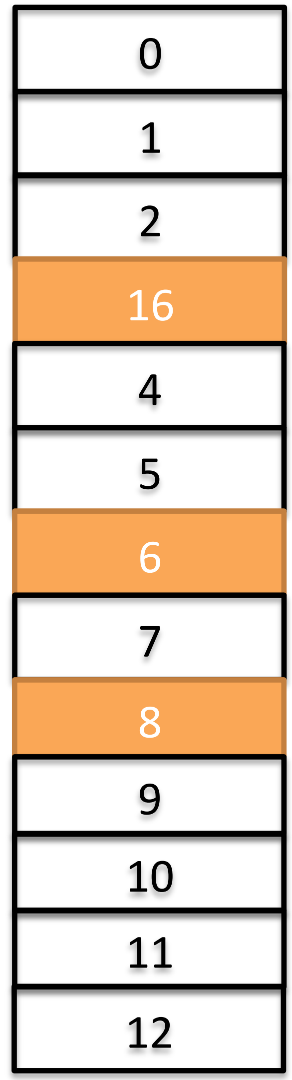
Collisions happen when multiple keys map to the same table index

Integer keys

Given table size m = 13

Compute h(key) = (key % m)

Example
- h(6) = 6
- h(8) = 8
- **h(16) = 3**



m = 13

Collisions happen when multiple keys map to the same table index

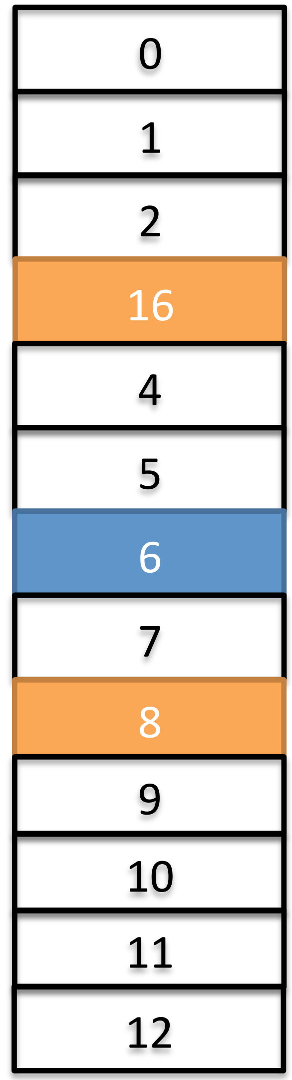
Integer keys

Given table size m = 13

Compute h(key) = (key % m)

Example

- h(6) = 6
- h(8) = 8
- h(16) = 3
- **h(19) = 6**



m = 13

Collision!
6 and 19 mapped to the same index

h(6)=h(19)

Agenda

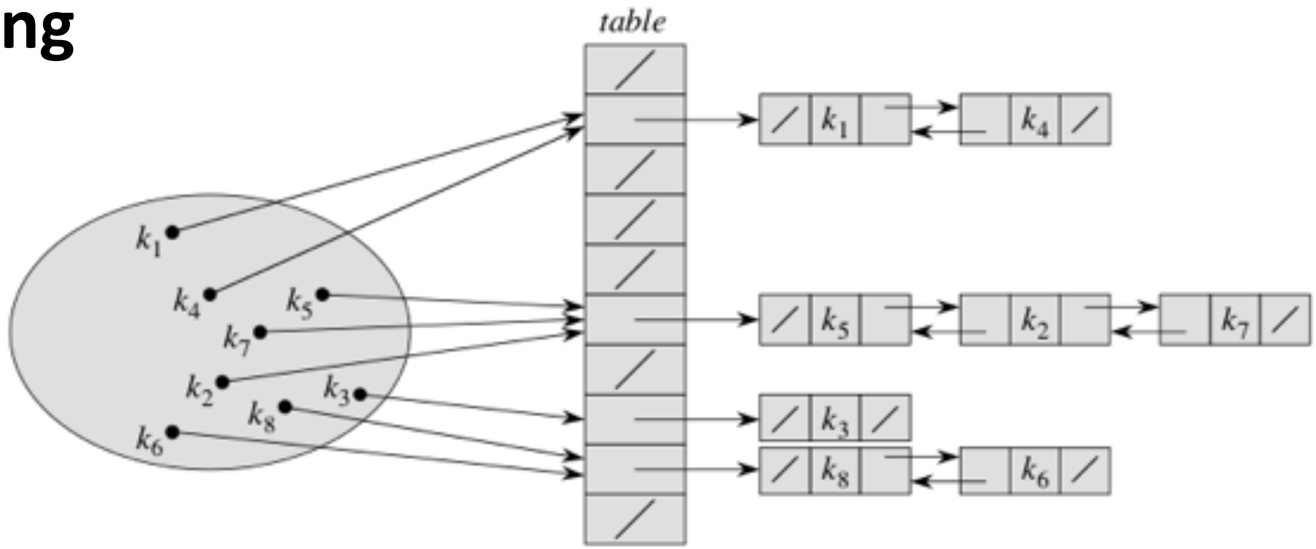
1. Hashing

2. Computing Hash functions

3. Handling collisions
   1. Chaining
   2. Open Addressing

Chaining handles collisions by creating a linked list for each table entry

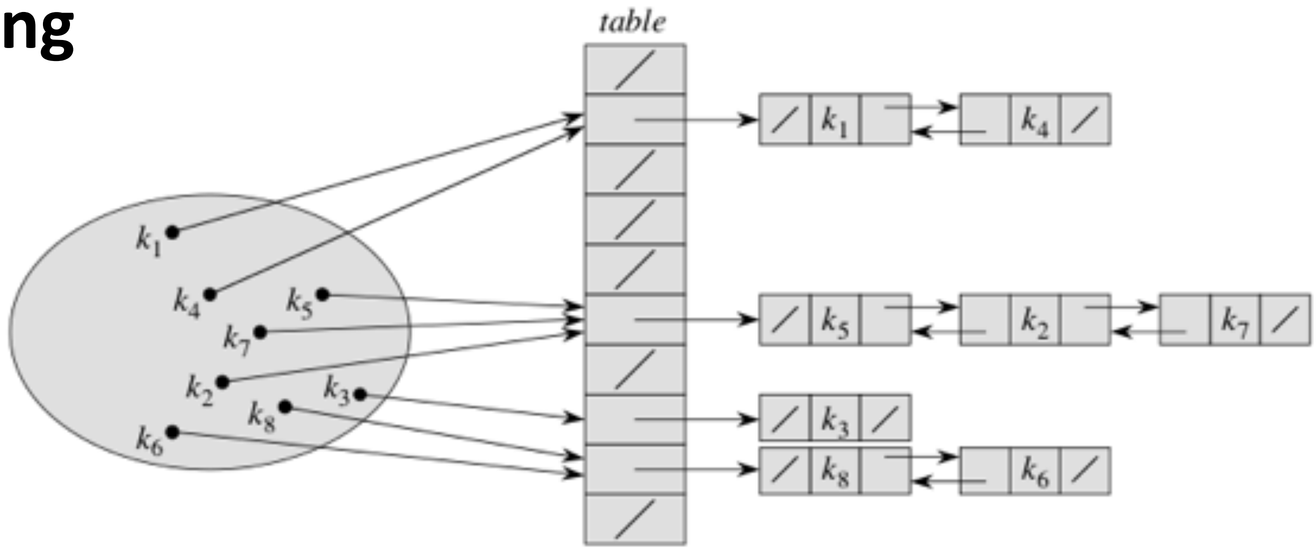
Chaining



- Create a table pointing to linked list of items that hash to the same index (similar to last class word positions)
- Slot *i* holds all keys *k* for which *h(k) = i*
- Splice in new elements at head for O(1) performance
- NOTE: Values associated with Keys are not shown, here just showing Keys

Load factor measures number of items in the list that must be searched on average

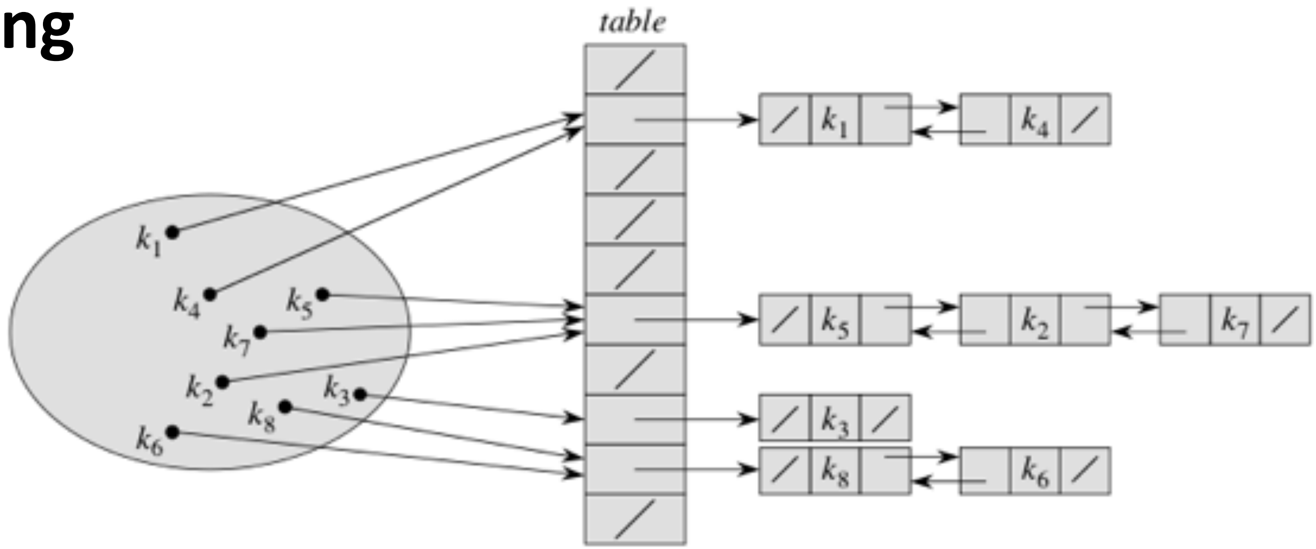
Chaining



- Assume table with *m* slots and *n* keys are stored in it
- On average, we expect *n/m* elements per collision list
- This is called the ***load factor*** *(λ=n/m)*
- <u>*Expected*</u> search time is *Θ(1+λ)*, assuming ***simple uniform hashing*** (each possible key equally likely to hash into a particular slot), worst case *Θ(n)* if bad hash function

If the load factor gets too high, then we should increase the table size

Chaining



- If *n* (# elements) becomes larger than *m* (table size), then collisions are inevitable and search time goes up
- Java increases <u>*table size*</u> by 2X and *rehashes* into new table when λ > 0.75 to combat this problem
- Problem: memory fragmentation with link lists spread out all over, might not be good for embedded systems

Agenda

1. Hashing

2. Computing Hash functions

3. Handling collisions
   1. Chaining
   2. Open Addressing

Open addressing is different solution, everything is stored in the table itself

Open addressing using linear probing

- Insert item at hashed index (no linked list)
- For key *k* compute *h(k)=i,* insert at index *i*
- If collision, a simple solution is called ***linear probing***
  - Try inserting at *i+1*
  - If slot *i+1* full, try *i+2*… until find empty slot
  - Wrap around to slot 0 if hit end of table at *m-1*
  - If λ <1 will find empty slot
  - If $\lambda \approx 1$, increase table size (*m*2*) and rehash
- Search analogous to insertion, compute key and probe until find item or empty slot (key not in table)

Linear probing is one way of handling collisions under open addressing

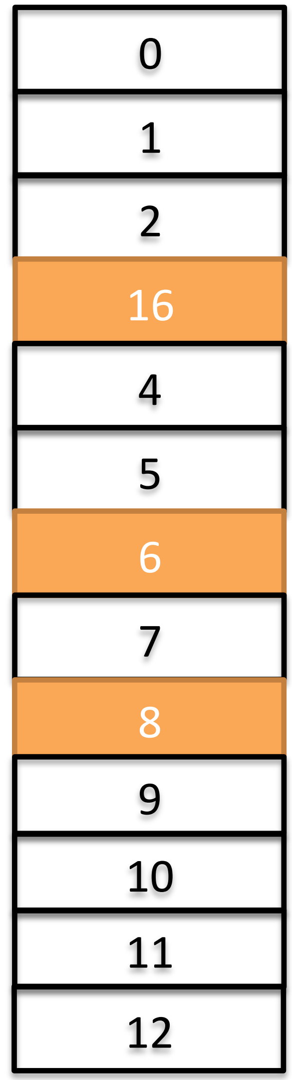
Integer keys

Given table size m = 13

Compute h(key) = (key %m)

Example

- h(6) = 6
- h(8) = 8
- h(16) = 3



m = 13

Linear probing is one method of open addressing

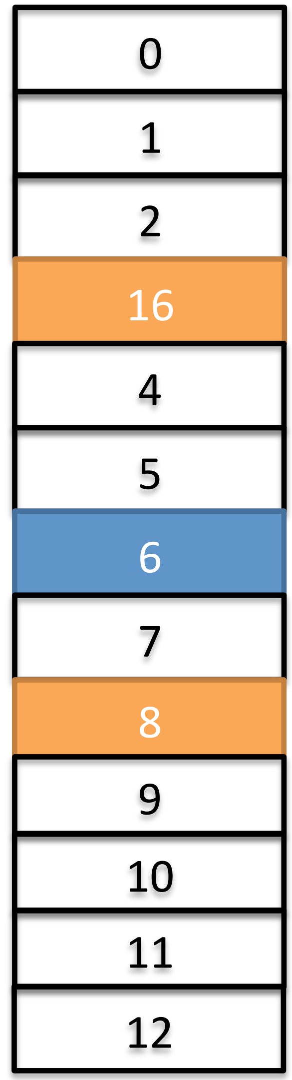
Integer keys

Given table size m = 13

Compute h(key) = (key %m)

Example

- h(6) = 6
- h(8) = 8
- h(16) = 3
- **h(19) = 6**



m = 13

Collision!

Linear probing is one method of open addressing

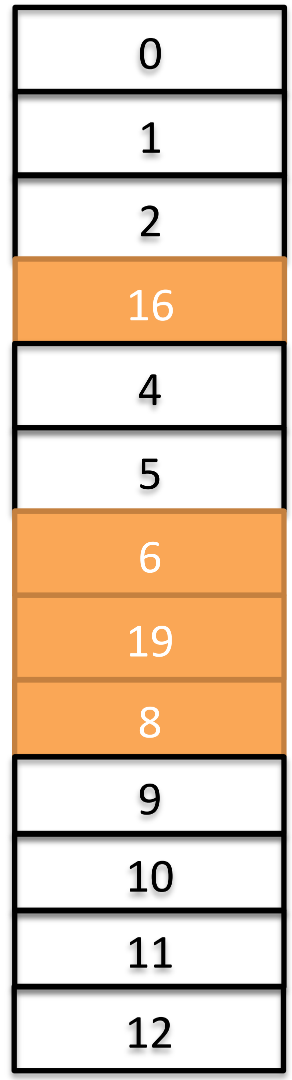
Integer keys

Given table size m = 13

Compute h(key) = (key %m)

Example

- h(6) = 6
- h(8) = 8
- h(16) = 3
- h(19) = 6



m = 13

Insert at *i+1* = 7

To find items later, hash to table index, then probe until find item or hit empty slot

Deleting items is tricky, need to mark deleted spot as available but not empty

Problems deleting items under linear probing

- Insert k_1, k_2, and k_3 where $h(k_1)=h(k_2)=h(k_3)$
- All three keys hash to the same slot in this example
- k_1 in slot i, k_2 in slot $i+1$, k_3 in slot $i+2$
- Remove k_2, creates hole at $i+1$
- Search for k_3
  - Hash k_3 to i, slot i holds $k_1 \neq k_3$, advance to slot $i+1$
  - Find hole at $i+1$, assume k_3 not in hash table
- Can mark deleted spaces as available for insertion, and search skips over marked spaces
- This can be a problem if many deletes create many marked slots, search approaches linear time

Clustering of keys can build up and reduce performance

Clustering problem

- Long runs of occupied slots (clusters) can build up increasing search and insert time
- Clusters happen because empty slot preceded by t full slots gets filled with probability $(t+1)/m$, instead of $1/m$ (e.g., t keys can now fill open slot instead of just 1 key)
- Clusters can bump into each other exacerbating the problem

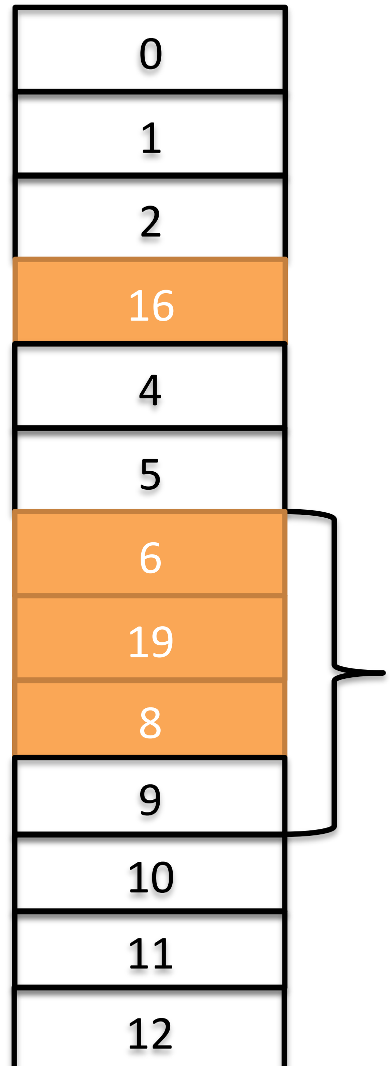
Clustering of keys can build up and reduce performance

Integer keys

Given table size m = 13

Compute h(key) = (key %m)

Example
- h(6) = 6
- h(8) = 8
- h(16) = 3
- h(19) = 6



m = 13

Hashing 6,7,8, or 9 go into index 9

Makes index 9 more likely to be filled than other slots

Double hashing can help with the clustering problem

Double hashing

- **<u>Big idea</u>: instead of stepping by 1 at each collision like linear probing, step by a different amount where the step size depends on the key**
- Use two hash functions h_1 and h_2 to make a third h'
- $h'(k,p)=(h_1(k) + ph_2(k))$ mod m, where p number of probes
- First probe $h_1(k)$, $p=0$, then p incremented by 1 on each collision until space is found
- Result is a step by $h_2(k)$ on each collision (then mod m to stay inside table size), instead of 1
- Need to design hashes so that if $h_1(k_1)=h_1(k_2)$, then ***unlikely*** $h_2(k_1)=h_2(k_2)$

Double hashing can help with the clustering problem

Integer keys

h_1 same as before
h_2 new hash function
p = probe number (initially 0)

Given table size m = 13
Compute

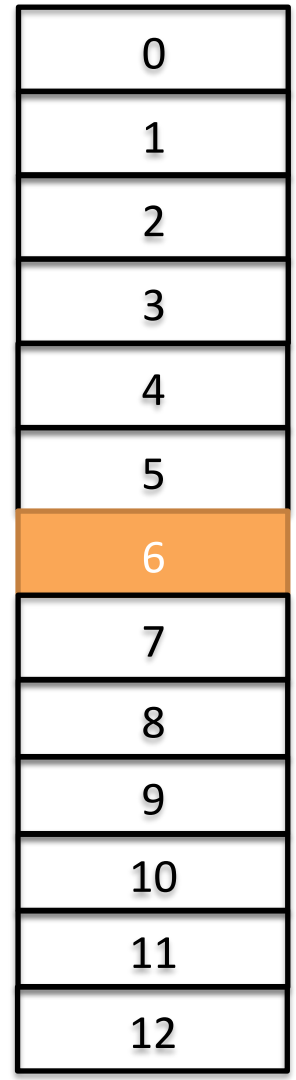
$h_1(key) = (key \% m)$

$h_2(key) = 1 + (key \% (m-1))$

$h'(k,p)=(h_1(k) + p h_2(k)) \% m$

Example

Key	p	h_1	h_2	h'
6	0	6	7	(6+0*7)%13 = 6



m = 13

Double hashing can help with the clustering problem

Integer keys

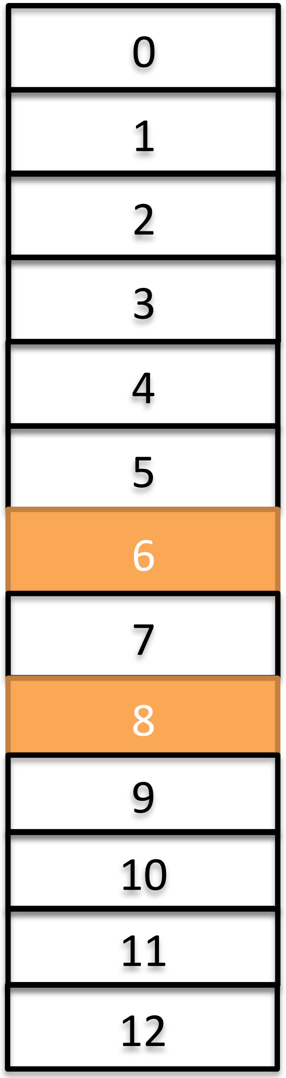
h_1 same as before
h_2 new hash function
p = probe number (initially 0)

Given table size m = 13
Compute

$$h_1(key) = (key \% m)$$
$$h_2(key) = 1 + (key \% (m-1))$$
$$h'(k,p)=(h_1(k) + ph_2(k)) \% m$$

Example

Key	**p**	**h_1**	**h_2**	**h'**
6	0	6	7	(6+0*7)%13 = 6
8	0	8	9	(8+0*9)%13 = 8



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Double hashing can help with the clustering problem

Integer keys

h_1 same as before
h_2 new hash function
p = probe number (initially 0)

Given table size m = 13

Compute

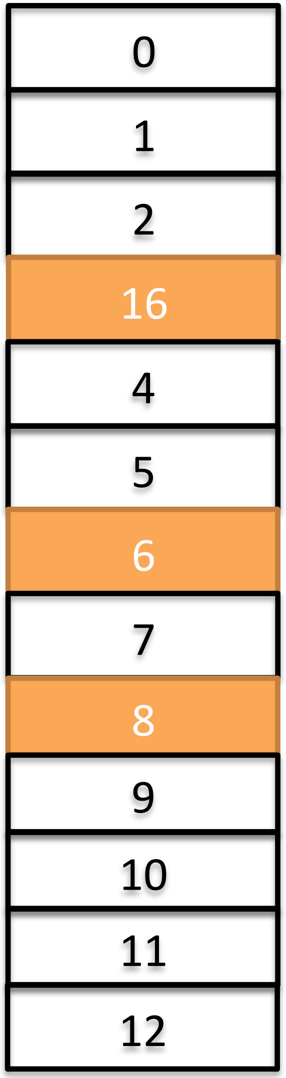
$$h_1(\text{key}) = (\text{key} \% m)$$

$$h_2(\text{key}) = 1 + (\text{key} \% (m-1))$$

$$h'(k,p) = (h_1(k) + p h_2(k)) \% m$$

Example

Key	**p**	**h_1**	**h_2**	**h'**
6	0	6	7	(6+0*7)%13 = 6
8	0	8	9	(8+0*9)%13 = 8
16	0	3	5	(3+0*5)%13 = 3



m = 13

Double hashing can help with the clustering problem

Integer keys

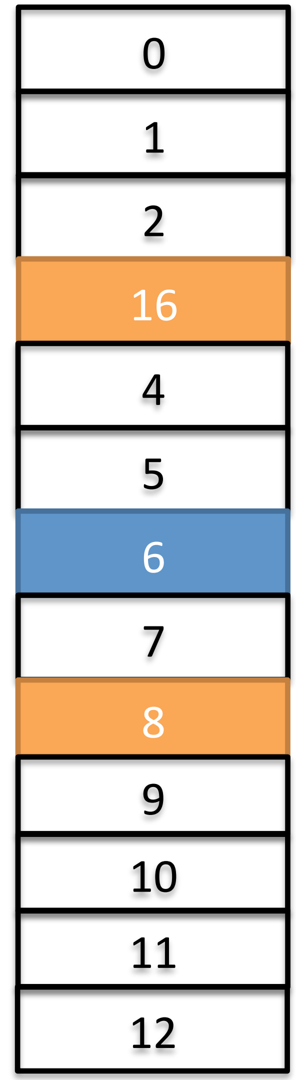
h_1 same as before
h_2 new hash function
p = probe number
(initially 0)

Given table size m = 13
Compute

$$h_1(key) = (key \% m)$$
$$h_2(key) = 1 + (key \% (m-1))$$
$$h'(k,p)=(h_1(k) + ph_2(k)) \% m$$

Example

Key	p	h_1	h_2	h'
6	0	6	7	(6+0*7)%13 = **6**
8	0	8	9	(8+0*9)%13 = 8
16	0	3	5	(3+0*5)%13 = 3
19	0	6	8	(6+0*8)%13 = **6**



Collision!

m = 13

Double hashing can help with the clustering problem

Integer keys

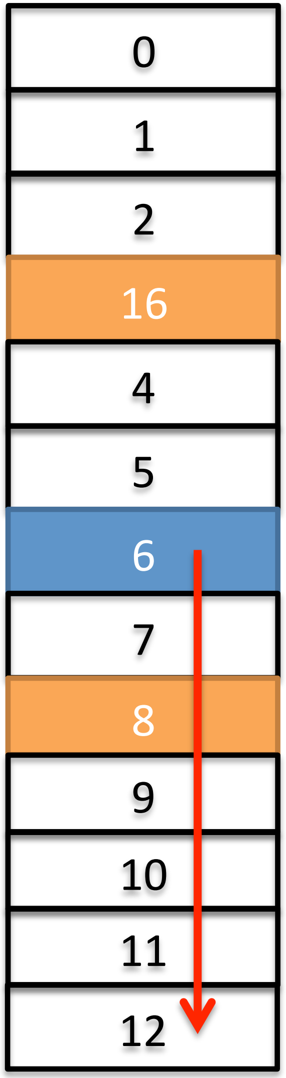
h_1 same as before
h_2 new hash function
p = probe number (initially 0)

Given table size m = 13
Compute

$$h_1(key) = (key \% m)$$
$$h_2(key) = 1 + (key \% (m-1))$$
$$h'(k,p) = (h_1(k) + ph_2(k)) \% m$$

Example

Key	p	h_1	h_2	h’
6	0	6	7	(6+0*7)%13 = 6
8	0	8	9	(8+0*9)%13 = 8
16	0	3	5	(3+0*5)%13 = 3
19	0	6	8	(6+0*8)%13 = 6
19	**1**	6	**8**	(6+**1*8**)%13 = **1**



m = 13

Collision!

Increment p

Step forward by h_2(key) = 8 spaces

Wrap around if needed

Double hashing can help with the clustering problem

Integer keys

Given table size m = 13

Compute

$h_1(\text{key}) = (\text{key} \% m)$

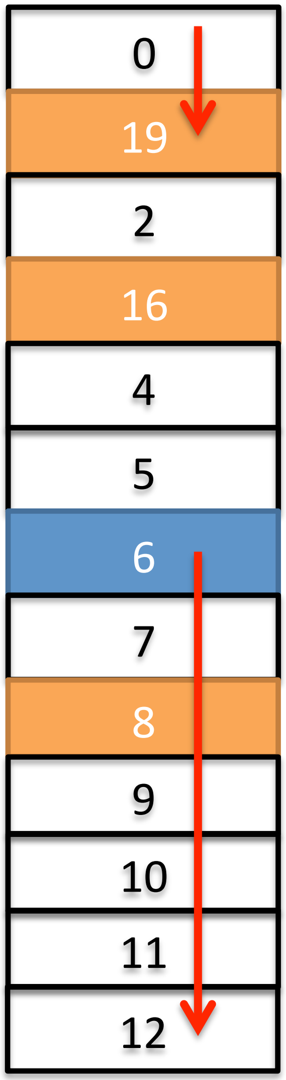
$h_2(\text{key}) = 1 + (\text{key} \% (m-1))$

$h'(k,p)=(h_1(k) + p h_2(k)) \% m$

h_1 same as before
h_2 new hash function
p = probe number (initially 0)

Example

Key	p	h_1	h_2	h'
6	0	6	7	(6+0*7)%13 = 6
8	0	8	9	(8+0*9)%13 = 8
15	0	2	4	(2+0*4)%13 = 2
19	0	6	8	(6+0*8)%13 = 6
19	**1**	6	**8**	(6+**1*8**)%13 = **1**



m = 13

Insert here

Collision!

Increment p

Step forward by $h_2(\text{key})$ = 8 spaces

Wrap around if needed

Run time degrades as λ gets large, so keep λ small by growing hash table

Expected insert and search time

- Average number of probes is approximately $1/(1-\lambda)$
- As λ ->1, expected number of probes becomes large, when λ small, number of probes approaches 1
- If table 90% full, then expect about 10 probes for insert or unsuccessful search
- Successful search generally a little faster, about 2.5 probes (math on course web page and in book)
- Must grow table and ***rehash*** when copying to new table to keep the table sparsely populated or performance suffers

Assuming load factor λ is small and hashing spreads keys, core operations are O(1)

Operation	**<u>Expected</u> run time**	**Notes**
hash(k)	O(1)	• Math operations on key *k* to hash and compress, outputs 0...m-1 • Constant time, does not depend on number of items in Set or Map

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hash(k)	O(1)	• Math operations on key *k* to hash and compress, outputs 0...m-1 • Constant time, does not depend on number of items in Set or Map

find(k)	O(1)	• Once have index of table due to hash:
		  • Chaining: traverse linked list O(λ) = O(1)
		  • Probing: probe until find O(1/(1-λ)) = O(1)

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hash(k)	O(1)	• Math operations on key *k* to hash and compress, outputs 0...m-1 • Constant time, does not depend on number of items in Set or Map

find(k)	O(1)	• Once have index of table due to hash:
		    • Chaining: traverse linked list $O(\lambda) = O(1)$
		    • Probing: probe until find $O(1/(1-\lambda)) = O(1)$

get(k)	$O(1+1) = O(1)$	• *Hash + find*: • chaining = $O(1+\lambda) = O(1)$, probing = $O(1+(1/(1-\lambda))) = O(1)$

Assuming load factor λ is small and hashing spreads keys, core operations are O(1)

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get(k)	O(1+1) = O(1)	• *Hash + find*: • chaining = O(1+λ) = O(1), probing = O(1+(1/(1-λ))) = O(1)

put(k,v)	O(1)
	<u>+O(1)</u>
	O(1)
		• *Hash + find* = O(1)
		• Plus update or add element:
		    • Chaining: update value or add at head O(1)
		    • Probing: store value in array O(1)

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hash(k)	O(1)	• Math operations on key *k* to hash and compress, outputs 0...m-1 • Constant time, does not depend on number of items in Set or Map

find(k)	O(1)	• Once have index of table due to hash:
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get(k)	O(1+1) = O(1)	• *Hash + find*: • chaining = O(1+λ) = O(1), probing = O(1+(1/(1-λ))) = O(1)

put(k,v)	O(1)
	+O(1)
	O(1)
		• *Hash + find* = O(1)
		• Plus update or add element:
		  • Chaining: update value or add at head O(1)
		  • Probing: store value in array O(1)

remove(k)	O(1)
	+O(1)
	O(1)
		• *Hash + find* = O(1)
		• Plus remove element:
		  • Chaining: update one pointer O(1)
		  • Probing: mark space empty O(1)

Assuming a small load factor and uniform hashing, the core operations of HashSets and HashMaps are constant time!