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1  (*
2                                     CS51 Lab 2
3                                     More Functional Programming:
4                                     Simple Data Structures and Higher-Order Functions
5  *)
6  (*
7                                     SOLUTION
8  *)
9  (*=====
10 Readings:
11
12     This lab builds on material from Chapters 7-8 of the textbook
13     <http://book.cs51.io>, which should be read before the lab session.
14
15 Objective:
16
17     This lab is intended to introduce you to staples of functional
18     programming in OCaml, including:
19
20         * simple data structures like lists and tuples
21         * higher-order functional programming (functions as first-class
22           values)
23     =====*)
24
25  (*=====
26 Part 1: Types and type inference beyond atomic types
27
28 Exercise 1: What are appropriate types to replace the ??? in the
29 expressions below? For each expression, replace the ??? with the
30 type. Test your solution by uncommenting the examples (removing the
31 `(*` and `*)` lines at start and end) and verifying that no typing
32 error is generated.
33 .....*)
34
35 let exercise1a : float * string =
36   (0.1, "hi") ;;
37
38 let exercise1b : string list =
39   let add_to_hello_list x = ["Hello"; x]
40   in add_to_hello_list "World!";;
41
42 let exercise1c : int * float -> int =
43   fun (x, y) -> x + int_of_float y ;;
44

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45 let exercise1d : int list -> bool =
46   fun lst ->
47     match lst with
48     | [] -> false
49     | hd :: _ -> hd < hd + 1 ;;
50
51 (* Here we deconstruct our argument as a list, so the argument 'lst'
52    must be of type '___ list' (consistent with its name). In the
53    second match case, we compare the head of the list with itself plus
54    one. We use integer addition, so the head must be of type
55    'int'. All elements of a list must be of the same type, so the
56    argument is of type 'int list'. To determine the type of the
57    result, we can look to the first match case. The literal 'false' is
58    of type 'bool'. All match cases must return the same type, so the
59    result must be of type 'bool'. *)
60
61 let exercise1e : bool -> bool list =
62   fun x -> if x then [x] else [] ;;
63
64 (* The reasoning goes like this: The argument of the function is
65    'x'. Since 'x' is used as the condition part of an 'if' expression,
66    it must be of type bool. Thus the expression in the 'then' clause
67    '[x]' must be a 'bool list', and since the type of an 'if' expression
68    is the type of its 'then' and 'else' expressions, the whole 'if'
69    expression, the result of the function, must be a 'bool list'. The
70    function itself is then of function type 'bool -> bool list', as is
71    the value named 'exercise1e'. *)
72
73 (*.....
74 Exercise 2: Update each expression below by changing the 0 in the last
75 line to an integer literal so that the expression as a whole evaluates
76 to 'true'.
77 .....*)
78
79 let exercise2a =
80   let lst = [1; 2; 3; 4] in
81   let value =
82     match lst with
83     | [] -> 0
84     | [h] -> h
85     | h1 :: h2 :: t -> h2 in
86   value = 2 ;;
87
88 let exercise2b =
89   let x, y, z = 4, [1; 3], true in
90   let value =

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91     match y with
92     | [] -> 0
93     | h :: t -> h in
94     value = 1 ;;
95
96 let exercise2c =
97     let tuple_lst = [(1, 4); (5, 2)] in
98     let value =
99         match tuple_lst with
100        | [] -> 0
101        | (a, b) :: t -> a
102        | h1 :: (a, b) :: t -> a in
103        value = 1 ;;
104
105 let exercise2d =
106     let tuple_lst = [(1, 4); (5, 2)] in
107     let value =
108         match tuple_lst with
109        | [] -> 0
110        | h1 :: (a, b) :: t -> a
111        | (a, b) :: t -> a in
112        value = 5 ;;
113
114 (*.....
115 Exercise 3: Complete the following definition for a function
116 `third_element` that returns a `bool * int` pair, whose first element
117 represents whether or not its list argument has a third element, and
118 whose second element represents that element if it exists (or 0 if it
119 does not). Here are some examples of the intended behavior of this
120 function:
121
122     # third_element [1; 2; 3; 4; 5] ;;
123     - : bool * int = (true, 3)
124     # third_element [] ;;
125     - : bool * int = (false, 0)
126 .....*)
127
128 let third_element (lst : int list) : bool * int =
129     match lst with
130     | _ :: _ :: elt3 :: _ -> true, elt3
131     | _ -> false, 0 ;;
132
133 (* Because we never use the first two elements or the tail of the
134    list, naming the variables is unnecessary, so we can use anonymous
135    variables (_) as shown above.
136

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137     The order of the two match cases is crucial. If they are flipped,
138     the second match case will never be invoked, since the first
139     match case will always match. *)
140
141     (=====
142     Part 2: First-order functional programming with lists
143
144     We'll continue with some "finger exercises" defining simple functions
145     before moving on to more complex problems. The intention in this part
146     of the lab is for you to implement these functions by *explicit
147     recursion*. Only later, in part 3 of this lab, will we make use of the
148     'map'/'fold'/'filter' higher-order functions.
149
150     As a reminder, here's the definition for the 'length' function of type
151     'int list -> int' implemented in this explicit recursion style:
152
153         let rec length (lst : int list) : int =
154             match lst with
155             | [] -> 0
156             | _head :: tail -> 1 + length tail ;;
157
158     .....
159     Exercise 4: In lab 1, we defined a function that could square its
160     input. Now, define a function 'square_all' that squares all of the
161     elements of an integer list. We've provided a bit of template code,
162     supplying the first line of the function definition, but the body of
163     the template code just causes a failure by forcing an error using the
164     built-in 'failwith' function. Edit the code to implement 'square_all'
165     properly.
166
167     Test out your implementation of 'square_all' by modifying the template
168     code below to define 'exercise4' to be the 'square_all' function
169     applied to the list containing the elements '3', '4', and '5'. You'll
170     want to replace the '[]' with the correct function application.
171
172     Thorough testing is important in all your work, and we hope to impart
173     this view to you in CS51. Testing will help you find bugs, avoid
174     mistakes, and teach you the value of short, clear functions. In the
175     file 'lab2_tests.ml', we've put some prewritten tests for 'square_all'
176     using the testing method of Section 6.7 in the book. Spend some time
177     understanding how the testing function works and why these tests are
178     comprehensive. Then test your code by compiling and running the test
179     suite:
180
181         % ocamlbuild -use-ocamlfind lab2_tests.byte
182         % ./lab2_tests.byte

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183
184 You may want to add some tests for other functions in the lab to get
185 some practice with automated unit testing.
186 .....*)
187
188 let rec square_all (lst : int list) : int list =
189     match lst with
190     | [] -> []
191     | head :: tail -> (head * head) :: (square_all tail) ;;
192
193 let exercise4 =
194     square_all [3; 4; 5] ;;
195
196 (*.....*)
197 Exercise 5: Define a recursive function `sum` that sums the values in
198 its integer list argument. (What's a sensible return value for the sum
199 of the empty list?)
200 .....*)
201
202 let rec sum (lst : int list) : int =
203     match lst with
204     | [] -> 0
205     | head :: tail -> head + sum tail ;;
206
207 (*.....*)
208 Exercise 6: Define a recursive function `max_list` that returns the
209 maximum element in a non-empty integer list. Don't worry about what
210 happens on an empty list. You may be warned by the compiler that "this
211 pattern-matching is not exhaustive." You may ignore this warning for
212 this lab.
213 .....*)
214
215 (* Here's a first cut at a solution, using just the portion of OCaml
216 already introduced. Notice that there's no branch in the pattern
217 match that matches the empty list, because there is no maximum
218 element in the empty list! For that reason, the ocaml interpreter
219 warns us with an "inexhaustive pattern match" warning.
220
221     let rec max_list (lst : int list) : int =
222         match lst with
223         | [elt] -> elt
224         | head :: tail ->
225             let max_tail = max_list tail in
226             if head > max_tail then head else max_tail ;;
227
228 This is the solution we expected people to come up with. And it

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229     seems to work.
230
231     # max_list [1; 3; 2] ;;
232     - : int = 3
233
234     What happens when we apply this function to the empty list?
235
236     # max_list [] ;;
237     Exception: Match_failure ("//toplevel//", 2, 2).
238
239     It generates a `Match_failure` exception. (We'll talk more about
240     error handling and exceptions later in the course, and use them
241     starting in Lab 4.) This `Match_failure` exception is a symptom of
242     a deeper underlying problem, namely, that the function `max_list`
243     was called with an invalid argument. A better solution, then, and
244     one that not coincidentally eliminates the "inexhaustive pattern
245     match" warning, is to explicitly raise a more appropriate exception
246     like `Invalid_argument`, as we've done in the solution below. *)
247
248     let rec max_list (lst : int list) : int =
249       match lst with
250       | [] -> raise (Invalid_argument "max_list: empty list")
251       | [elt] -> elt
252       | head :: tail ->
253         let max_tail = max_list tail in
254         if head > max_tail then head else max_tail ;;
255
256     (* It turns out that the `Stdlib` module has a `max` function that
257     returns the larger of its two arguments. Using that function, we
258     can simplify a bit.
259
260     let rec max_list (lst : int list) : int =
261       match lst with
262       | [elt] -> elt
263       | head :: tail -> max head (max_list tail) ;;
264     *)
265
266     (*.....
267     Exercise 7: Define a function `zip`, that takes two `int list`
268     arguments and returns a list of pairs of ints, one from each of the
269     two argument lists. Your function can assume the input lists will be
270     the same length. You can ignore what happens in the case the input
271     list lengths do not match. You may be warned by the compiler that
272     "this pattern-matching is not exhaustive." You may ignore this warning
273     for this lab.
274

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275 For example,
276
277 fun lst -> zip [1; 2; 3] [4; 5; 6] ;;
278 - : (int * int) list = [(1, 4); (2, 5); (3, 6)]
279
280 To think about: Why wouldn't it be possible, in cases of mismatched
281 length lists, to just pad the shorter list with, say, `false` values, so
282 that, `zip [1] [2; 3; 4] = [(1, 2); (false, 3); (false, 4)]`?
283 .....*)
284
285 let rec zip (x : int list) (y : int list) : (int * int) list =
286   match x, y with
287   | [], [] -> []
288   | xhd :: xtl, yhd :: ytl -> (xhd, yhd) :: (zip xtl ytl) ;;
289
290 (* This was the solution we expected people to come up with. It
291    generates a warning about the pattern match not being
292    exhaustive. As in `max_list` above, the ramifications of this issue
293    and how best to address it are discussed at length in Chapter 10,
294    Section 10.2. That discussion is beyond the scope of lab 2, but feel
295    free to read ahead if you're interested. *)
296
297 (*.....
298 Exercise 8: Recall from Chapter 7 the definition of the function `prods`.
299 *)
300
301 let rec prods (lst : (int * int) list) : int list =
302   match lst with
303   | [] -> []
304   | (x, y) :: tail -> (x * y) :: (prods tail) ;;
305
306 (* Using `sum`, `prods`, and `zip`, define a function `dotprod` that
307    takes the dot product of two integer lists (that is, the sum of the
308    products of corresponding elements of the lists; see
309    https://en.wikipedia.org/wiki/Dot\_product if you want more
310    information, though it shouldn't be necessary). For example, you
311    should have:
312
313    # dotprod [1; 2; 3] [0; 1; 2] ;;
314    - : int = 8
315    # dotprod [1; 2] [5; 10] ;;
316    - : int = 25
317
318 Even without looking at the code for the functions, carefully looking
319 at the type signatures for `zip`, `prods`, and `sum` should give a
320 good idea of how you might combine these functions to implement

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321 `dotproduct`.
322
323 If you've got the right idea, your implementation should be literally
324 a single short line of code. If it isn't, try it again, getting into
325 the functional programming zen mindset.
326 .....*)
327
328 let dotprod (a : int list) (b : int list) : int =
329     sum (prods (zip a b)) ;;
330
331 (*=====
332 Part 3: Higher-order functional programming with map, filter, and fold
333
334 In these exercises, you should use the built-in functions `map`,
335 `filter`, and `fold_left` and `fold_right` provided in the OCaml List
336 module to implement these simple functions.
337
338 * IMPORTANT NOTE 1: When you make use of these functions, you'll
339 either need to prefix them with the module name, for example,
340 `List.map` or `List.fold_left`, or you'll need to open the `List`
341 module with the line
342
343     open List ;;
344
345 You can place that line at the top of this file if you'd like.
346
347 * IMPORTANT NOTE 2: In these labs, and in the problem sets as well,
348 we'll often supply some skeleton code that looks like this:
349
350     let somefunction (arg1 : type) (arg2 : type) : returntype =
351         failwith "somefunction not implemented"
352
353 We provide this to give you an idea of the function's intended
354 name, its arguments and their types, and the return type. But
355 there's no need to slavishly follow that particular way of
356 implementing code to those specifications. In particular, you may
357 want to modify the first line to introduce, say, a `rec` keyword
358 (if your function is to be recursive):
359
360     let rec somefunction (arg1 : type) (arg2 : type) : returntype =
361         ...your further code here...
362
363 Or you might want to define the function using anonymous function
364 syntax. (If you haven't seen this yet, come back to this comment
365 later when you have.)
366

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367         let somefunction =
368             fun (arg1 : type) (arg2 : type) : returntype ->
369                 ...your further code here...
370
371     This will be especially pertinent in this section, where functions
372     can be built just by applying other higher order functions
373     directly, without specifying the arguments explicitly, for
374     example, in this implementation of the 'double_all' function,
375     which doubles each element of a list:
376
377         let double_all : int list -> int list =
378             map (( * ) 2) ;;
379
380     * END IMPORTANT NOTES
381
382     .....
383     Exercise 9: Reimplement 'sum' using 'fold_left', naming it 'sum_ho'
384     (for "higher order").
385     .....*)
386
387     let sum_ho : int list -> int =
388         List.fold_left (+) 0 ;;
389
390     (* One of the key advantages of curried functions (like 'fold_left')
391        is that they can be partially applied. (See Section 8.2 in the
392        textbook.) We've taken advantage of that in the definition above,
393        by defining 'sum_ho' as a partially applied 'fold_left', rather
394        than as
395
396         let sum_ho (lst : int list) : int =
397             List.fold_left (+) 0 lst ;;
398
399     The latter will work, but lacks the elegance of the more idiomatic
400     approach here.
401
402     The same technique is used in the exercises below. It may be
403     useful, in order to understand what's going on, to try typing in
404     longer and longer prefixes of an expression like 'List.fold_left
405     (+) 0 [1; 2; 3]' and watch the types closely.
406
407         # List.fold_left ;;
408         - : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a = <fun>
409         # List.fold_left (+) ;;
410         - : int -> int list -> int = <fun>
411         # List.fold_left (+) 0 ;;
412         - : int list -> int = <fun>

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413     # List.fold_left (+) 0 [1; 2; 3] ;;
414     - : int = 6
415
416     You may also note the use of parentheses in the expression
417     `(+)`. The `+` operator is an example of an "infix" operator, an
418     operator that goes in between its arguments rather than in front of
419     them. When using `+` as an argument to higher-order functions, we
420     generally need to remove that infix property, so that it will be
421     parsed as a prefix operator like most other functions. Wrapping the
422     operator in parentheses induces this behavior.
423
424     # 3 + 4 ;;
425     - : int = 7
426     # (+) 3 4
427     - : int = 7
428     *)
429
430     (*.....*)
431     Exercise 10: Reimplement prods : `(int * int) list -> int list` using
432     the `map` function. Call it `prods_ho`.
433     .....*)
434
435     let prods_ho : (int * int) list -> int list =
436         List.map (fun (x, y) -> x * y) ;;
437
438     (*.....*)
439     Exercise 11: The OCaml List module provides -- in addition to the `map`,
440     `fold_left`, and `fold_right` higher-order functions -- several other
441     useful higher-order list manipulation functions. For instance, `map2` is
442     like `map`, but takes two lists instead of one along with a function of
443     two arguments and applies the function to corresponding elements of the
444     two lists to form the result list. (You can read about it at
445     https://caml.inria.fr/pub/docs/manual-ocaml/libref/List.html#VALmap2.)
446     Use `map2` to reimplement `zip` and call it `zip_ho`.
447     .....*)
448
449     let zip_ho : int list -> int list -> (int * int) list =
450         List.map2 (fun first second -> first, second) ;;
451
452     (* Note the rejiggering of the first line to allow the function
453     `zip_ho` to take advantage of partial application, so that `zip_ho`
454     is the functional output of the higher-order function
455     `map2`. Without the rejiggering, you'd probably implement it as:
456
457     let zip_ho (x : int list) (y : int list) : (int * int) list =
458         List.map2 (fun first second -> first, second) x y ;;

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459 *)
460
461 (*.....
462 Exercise 12: Define a function `evens`, using these higher-order
463 functional programming techniques, that returns a list of all of the
464 even numbers in its argument list in the same order. For instance,
465
466     # evens [1; 2; 3; 6; 5; 4] ;;
467     - : int list = [2; 6; 4]
468 .....*)
469
470 (* Again, without partial application:
471
472     let evens (lst : int list) : int list =
473         List.filter (fun n -> n mod 2 = 0) lst;;
474
475     and with partial application: *)
476
477 let evens : int list -> int list =
478     List.filter (fun n -> n mod 2 = 0) ;;

```