

```

1  (*
2                                     CS51 Lab 3
3                                     Polymorphism and record types
4  *)
5
6  (*
7                                     SOLUTION
8  *)
9
10 (*=====
11 Readings:
12
13     This lab builds on material from Chapters 7.4 and 9-9.5 of the
14     textbook <http://book.cs51.io>, which should be read before the lab
15     session.
16
17 Objective:
18
19     In this lab, you'll exercise your understanding of polymorphism and
20     record types. Some of the problems extend those from Lab 2, but we'll
21     provide the necessary background code from that lab.
22     =====*)
23
24 (*=====
25 Part 1: Records and tuples
26
27 Records and tuples provide two different ways to package together
28 data. They differ in whether their components are selected by *name*
29 or by *position*, respectively.
30
31 Consider a point in Cartesian (x-y) coordinates. A point is specified
32 by its x and y values, which we'll take to be ints. We can package
33 these together as a pair (a 2-tuple), as in the following data type
34 definition: *)
35
36 type point_pair = int * int ;;
37
38 (* Then, we can add two points (summing their x and y coordinates
39 separately) with the following function:
40
41     let add_point_pair (p1 : point_pair) (p2 : point_pair) : point_pair =
42         let x1, y1 = p1 in
43         let x2, y2 = p2 in
44         (x1 + x2, y1 + y2) ;;

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45
46 .....
47 Exercise 1:
48
49 It might be nicer to deconstruct the points in a single match, rather
50 than the two separate matches in the two `let` expressions. Reimplement
51 `add_point_pair` to use a single pattern match in a single `let`
52 expression.
53 .....*)
54
55 let add_point_pair (p1 : point_pair) (p2 : point_pair) : point_pair =
56   let (x1, y1), (x2, y2) = p1, p2 in
57     x1 + x2, y1 + y2 ;;
58
59 (* Analogously, we can define a point by using a record to package up
60 the x and y coordinates. *)
61
62 type point_recd = {x : int; y : int} ;;
63
64 let example_point_recd = {x = 1; y = 3} ;;
65
66 (*.....
67 Exercise 2A:
68
69 Replace the two lines below with a single top-level `let` expression
70 that extracts the x and y coordinate values from `example_point_recd`
71 above into global variables `x1` and `y1`, respectively.
72 .....*)
73
74 let {x = x1; y = y1} = example_point_recd ;;
75
76 (*.....
77 Exercise 2B:
78
79 Implement a function `add_point_recd` to add two points of type
80 `point_recd` and returning a `point_recd` as well.
81 .....*)
82
83 (* A direct reimplementaion of `add_point_pair` would be: *)
84
85 let add_point_recd (p1 : point_recd) (p2 : point_recd) : point_recd =
86   let {x = x1; y = y1}, {x = x2; y = y2} = p1, p2 in
87     {x = x1 + x2; y = y1 + y2} ;;
88
89 (* By making use of dot notation for selecting pair elements, this
90 version may be a bit cleaner

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91
92     let add_point_recd (p1 : point_recd) (p2 : point_recd) : point_recd =
93         {x = p1.x + p2.x; y = p1.y + p2.y} ;;
94     *)
95
96     (* Recall the dot product from Lab 2. The dot product of two points
97     'x1, y1' and 'x2, y2' is the sum of the products of their x and y
98     coordinates.
99
100     .....
101     Exercise 3: Write a function 'dot_product_pair' to compute the dot
102     product for points encoded as the 'point_pair' type.
103     .....*)
104
105     let dot_product_pair (x1, y1 : point_pair) (x2, y2 : point_pair) : int =
106         x1 * x2 + y1 * y2 ;;
107
108     (* In this example, we've gone even further, performing the match
109     directly in the 'let' definition of the function itself. This is
110     actually the stylistically preferred way of implementing this in
111     OCaml, as discussed in the Style Guide. Can you adjust the solution
112     to Exercises 1 and 2B to use this syntactic sugar? *)
113
114     (*.....
115     Exercise 4: Write a function 'dot_product_recd' to compute the dot
116     product for points encoded as the 'point_recd' type.
117     .....*)
118
119     let dot_product_recd (p1 : point_recd) (p2 : point_recd) : int =
120         p1.x * p2.x + p1.y * p2.y ;;
121
122     (* Converting between the pair and record representations of points
123
124     You might imagine that the two representations have
125     different advantages, such that two libraries, say, might use
126     differing types for points. In that case, we may want to have
127     functions to convert between the two representations.
128
129     .....
130     Exercise 5: Write a function 'point_pair_to_recd' that converts a
131     'point_pair' to a 'point_recd'.
132     .....*)
133
134     let point_pair_to_recd (x, y : point_pair) : point_recd =
135         {x; y} ;;
136

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137 (* Note the use of pattern-matching for deconstruction directly in the
138    argument and of "field punning". Without those techniques, we'd
139    have the more cumbersome:
140
141    let point_pair_to_rec'd (p : point_pair) : point_rec'd =
142      let x, y = p in
143      {x = x; y = y} ;;
144      *)
145
146 (*.....
147 Exercise 6: Write a function 'point_rec'd_to_pair' that converts a
148 'point_rec'd' to a 'point_pair'.
149 .....*)
150
151 let point_rec'd_to_pair ({x; y} : point_rec'd) : point_pair =
152   x, y ;;
153
154 (*=====
155 Part 2: A simple database of records
156
157 A college wants to store student records in a simple database,
158 implemented as a list of individual course enrollments. The enrollments
159 themselves are implemented as a record type, called 'enrollment', with
160 'string' fields labeled 'name' and 'course' and an integer student ID
161 number labeled 'id'. The appropriate type definition is: *)
162
163 type enrollment = { name : string;
164                    id : int;
165                    course : string } ;;
166
167 (* Here's an example of a list of enrollments. *)
168
169 let college =
170   [ { name = "Pat";    id = 603858772; course = "cs51" };
171     { name = "Pat";    id = 603858772; course = "expos20" };
172     { name = "Kim";    id = 482958285; course = "expos20" };
173     { name = "Kim";    id = 482958285; course = "cs20" };
174     { name = "Sandy";  id = 993855891; course = "ls1b" };
175     { name = "Pat";    id = 603858772; course = "ec10b" };
176     { name = "Sandy";  id = 993855891; course = "cs51" };
177     { name = "Sandy";  id = 482958285; course = "ec10b" }
178   ] ;;
179
180 (* In the following exercises, you'll want to avail yourself of the
181 'List' module functions, writing the requested functions in
182 higher-order style rather than handling the recursion yourself.

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183
184 .....
185 Exercise 7: Define a function called `transcript` that takes an
186 `enrollment list` and returns a list of all the enrollments for a given
187 student as specified by the student's ID.
188
189 For example:
190
191     # transcript college 993855891 ;;
192     - : enrollment list =
193       [{name = "Sandy"; id = 993855891; course = "ls1b"};
194        {name = "Sandy"; id = 993855891; course = "cs51"}]
195     .....*)
196
197 let transcript (enrollments : enrollment list)
198   (student : int)
199   : enrollment list =
200   List.filter (fun { id; _ } -> id = student) enrollments ;;
201 (*      ^^--- field punning!
202
203 Note the use of field punning, using the `id` variable to refer to
204 the value of the `id` field.
205
206 An alternative approach is to use the dot notation to pick out the
207 record field.
208
209     let transcript (enrollments : enrollment list)
210       (student : int)
211       : enrollment list =
212       List.filter (fun studentrec -> studentrec.id = student)
213         enrollments ;;
214     *)
215
216 (*.....
217 Exercise 8: Define a function called `ids` that takes an `enrollment
218 list` and returns a list of all the ID numbers in that list,
219 eliminating any duplicates. Hint: The `map` and `sort_uniq` functions
220 from the `List` module and the `compare` function from the `Stdlib`
221 module may be useful here.
222
223 For example:
224
225     # ids college ;;
226     - : int list = [482958285; 603858772; 993855891]
227     .....*)
228

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229 (* Making good use of the recommended library functions, we have the
230    following succinct implementation: *)
231
232 let ids (enrollments: enrollment list) : int list =
233     List.sort_uniq (compare)
234         (List.map (fun student -> student.id) enrollments) ;;
235
236 (* This time we used the alternative strategy of picking out the 'id'
237    using dot notation.
238
239    By the way, the aggregation to eliminate duplicates can also be
240    done using a fold. We leave that strategy as an additional
241    exercise. *)
242
243 (* There's a big problem with this database design: nothing guarantees
244    that a given student ID is associated with a single name. The right
245    thing to do is to use a different database design where this kind of
246    thing can't happen; that would be an application of the *edict of
247    prevention*. But for the purpose of this lab, you'll just write a
248    function to verify that this problem doesn't occur.
249
250    .....
251    Exercise 9: Define a function 'verify' that determines whether all the
252    entries in an enrollment list for each of the ids appearing in the
253    list have the same name associated. Hint: You may want to use
254    functions from the 'List' module such as 'map', 'for_all',
255    'sort_uniq'.
256
257    For example:
258
259        # verify college ;;
260        - : bool = false
261
262    (Do you see why it's false?)
263    .....*)
264
265 (* We start with a function to extract all the names from the database. *)
266 let names (enrollments : enrollment list) : string list =
267     List.sort_uniq (compare)
268         (List.map (fun { name; _ } -> name) enrollments) ;;
269
270 (* Then we verify that for each id, the list of names associated with
271    the courses in that id's transcript has length 1. *)
272 let verify (enrollments : enrollment list) : bool =
273     List.for_all (fun l -> List.length l = 1)
274         (List.map

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275         (fun student -> names (transcript enrollments student))
276         (ids enrollments)) ;;
277
278 (* By the way, the computed value in the example is false because
279    Sandy appears with Kim's ID number in one of the entries. *)
280
281 (*****
282 Part 3: Polymorphism
283
284 .....
285 Exercise 10: In Lab 2, you implemented a function `zip` that takes two
286 lists and "zips" them together into a list of pairs. Here's a possible
287 implementation of `zip`:
288
289     let rec zip (x : int list) (y : int list) : (int * int) list =
290         match x, y with
291         | [], [] -> []
292         | xhd :: xtl, yhd :: ytl -> (xhd, yhd) :: (zip xtl ytl) ;;
293
294 As implemented, this function works only on integer lists. Revise your
295 solution to operate polymorphically on lists of any type. What is the
296 type of the result? Did you provide full typing information in the
297 first line of the definition? (As usual, for the time being, don't
298 worry about explicitly handling the anomalous case when the two lists
299 are of different lengths.)
300 .....*)
301
302 [@@@warning "-8"]
303 let rec zip (x : 'a list) (y : 'b list) : ('a * 'b) list =
304     match x, y with
305     | [], [] -> []
306     | xhd :: xtl, yhd :: ytl -> (xhd, yhd) :: (zip xtl ytl) ;;
307
308 (* Notice how a polymorphic typing was provided in the first line, to
309    capture the intention of the polymorphic function.
310
311    You can ignore the non-exhaustive match warning, which occurs
312    because we have no match cases for when only one of the two
313    argument lists is empty. We'll have better tools to address that
314    issue later. *)
315
316 (*.....
317 Exercise 11: Partitioning a list -- Given a boolean function, say
318
319     fun x -> x mod 3 = 0
320

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321 and a list of elements, say,
322
323     [3; 4; 5; 10; 11; 12; 1]
324
325 we can partition the list into two lists, the list of elements
326 satisfying the boolean function ([3; 12]) and the list of elements
327 that don't ([4; 5; 10; 11; 1]).
328
329 The library function List.partition partitions its list argument in
330 just this way, returning a pair of lists. Here's an example:
331
332     # List.partition (fun x -> x mod 3 = 0) [3; 4; 5; 10; 11; 12; 1] ;;
333     - : int list * int list = ([3; 12], [4; 5; 10; 11; 1])
334
335 What is the type of the partition function, keeping in mind that it
336 should be as polymorphic as possible?
337
338 Now implement the function yourself (without using List.partition of
339 course, though other List module functions may be useful).
340 .....*)
341
342 (* Let's start by working out the type. The partition function takes
343 two arguments, a boolean condition and a list of elements. The
344 boolean condition might apply to elements of any type, so it should
345 be a function of type 'a -> bool'. The list must contain elements
346 appropriate to apply the condition to, that is, elements of type
347 'a', so the list itself is of type 'a list'. The result is a pair
348 of lists, each of which contains elements of type 'a', that is,
349 'a list * 'a list'. The type of partition itself is then
350
351     ('a -> bool) -> 'a list -> 'a list * 'a list
352
353 The implementation is really straightforward if we just reuse the
354 filtering functionality of the List.filter function. *)
355
356 let partition (condition : 'a -> bool) (lst : 'a list)
357     : 'a list * 'a list =
358     let open List in
359     filter condition lst, filter (fun x -> not (condition x)) lst ;;
360
361 (* If, instead, we want to perform the walking of the list directly,
362 we might have
363
364     let rec partition (condition : 'a -> bool) (lst : 'a list)
365         : 'a list * 'a list =
366         match lst with

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367 | [] -> [], []
368 | hd :: tl ->
369     let yeses, noes = partition condition tl in
370     if condition hd then (hd :: yeses), noes
371     else yeses, (hd :: noes) ;;
372
373 An implementation with a single fold is also possible.
374
375 let partition (condition : 'a -> bool) (lst : 'a list)
376     : 'a list * 'a list =
377     List.fold_right (fun elt (yeses, noes) ->
378         if condition elt then (elt :: yeses), noes
379         else yeses, (elt :: noes))
380         lst
381         ([], []) ;;
382
383 To think about: Which of these do you like best? What are the
384 advantages and disadvantages of each?
385 *)
386
387 (*=====
388 Part 4: Implementing polymorphic application, currying, and uncurrying
389
390 .....
391 Exercise 12: We can think of function application itself as a
392 polymorphic higher-order function (:exploding_head:). It takes two
393 arguments -- a function and its argument -- and returns the value
394 obtained by applying the function to its argument. In this exercise,
395 you'll write this function, called 'apply'. You might use it as in the
396 following examples:
397
398 # apply pred 42 ;;
399 - : int = 41
400 # apply (fun x -> x ** 2.) 3.14159 ;;
401 - : float = 9.86958772809999907
402
403 An aside: You may think such a function isn't useful, since we
404 already have an even more elegant notation for function
405 application, as in
406
407 # pred 42 ;;
408 - : int = 41
409 # (fun x -> x ** 2.) 3.14159 ;;
410 - : float = 9.86958772809999907
411
412 But we'll see a quite useful operator that works similarly --

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413         the backwards application operator -- in Chapter 11 of the
414         textbook.
415
416     Start by thinking about the type of the function. We'll assume it
417     takes its two arguments curried, that is, one at a time.
418
419     1. What is the most general (polymorphic) type for its first argument
420        (the function to be applied)?
421
422     2. What is the most general type for its second argument (the argument
423        to apply it to)?
424
425     3. What is the type of its result?
426
427     4. Given the above, what should the type of the function `apply` be?
428
429     Now write the function.
430
431     Can you think of a reason that the `apply` function might in fact be
432     useful?
433     .....*)
434
435     (* Thinking through the types of the `apply` function:
436
437     1. Its first argument, the function to be applied, itself takes an
438        argument of some generic type, call it `arg`. (We're not
439        restricted to type variables like `a`, `b`, `c`. We might as
440        well use a good mnemonic type variable name like `arg`.) The
441        result type for the function to be applied we'll call
442        `result`. So the type of the first argument is `arg ->
443        result`.
444
445     2. Its second argument is the argument to apply that function to,
446        and must thus be of type `arg`.
447
448     3. The type of the result of the application is, of course,
449        `result`.
450
451     4. So the type for apply is given by the typing:
452
453         apply : (arg -> result) -> arg -> result
454
455     Types in hand, the apply function itself is truly trivial to
456     implement: *)
457
458     let apply (func : 'arg -> 'result) (arg : 'arg) : 'result =

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459     func arg ;;
460
461     (* Something to think about: One reason the `apply` function might be
462        useful is that it might be handy as *an argument to another
463        higher-order function*. *)
464
465     (*.....
466     Exercise 13: In the next two exercises, you'll define polymorphic
467     higher-order functions `curry` and `uncurry` for currying and uncurrying
468     binary functions (functions of two arguments). The functions are named
469     after mathematician Haskell Curry '1920. (By way of reminder, a
470     curried function takes its arguments one at a time. An uncurried
471     function takes them all at once in a tuple.)
472
473     We start with the polymorphic higher-order function `curry`, which
474     takes as its argument an uncurried binary function and returns the
475     curried version of its argument function.
476
477     Before starting to code, pull out a sheet of paper and a pencil and
478     with your partner work out the answers to the following seven
479     questions.
480
481         *****
482         Do not skip this pencil and paper work.
483         *****
484
485     1. What is the type of the argument to the function `curry`? Write down
486        a type expression for the argument type.
487
488     2. What is an example of a function that `curry` could apply to?
489
490     3. What is the type of the result of the function `curry`? Write down a
491        type expression for the result type.
492
493     4. What should the result of applying the function `curry` to the
494        function from (2) be?
495
496     5. Given (1) and (2), write down a type expression for the type of the
497        `curry` function itself.
498
499     6. What would a good variable name for the argument to `curry` be?
500
501     7. Write down the header line for the definition of the `curry` function.
502
503     Call over a staff member to go over your answers to these
504     questions. Once you fully understand all this, its time to implement

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505 the function `curry`.
506 .....*)
507
508 (* In order to think through this problem, it helps to start with the
509    types of the functions. The `curry` function is a *function*; it has
510    a function type, of the form  $\_ \rightarrow \_$ . It is intended to take an
511    uncurried binary function as its argument, and return the
512    corresponding curried function. An uncurried binary function is a
513    function that takes its two arguments both "at the same time", that
514    is, as a pair. Generically, the type of such a function is thus
515
516          `a * 'b -> 'c`      (that's the answer to question (1) above)
517
518    An example (2) would be the function that adds the elements of an
519    `int` pair:
520
521          `fun (x, y) -> x + y`
522
523    A curried binary function takes its two arguments "one at a time".
524    Its type is
525
526          `a -> ('b -> 'c)`
527
528    which is the appropriate result type for the curry function (3). For
529    instance, the curried version of the integer addition function is
530    just the `(+)` operator itself (4).
531
532    Putting these together, the type of curry should be (5)
533
534          ((`a * 'b) -> 'c) -> ('a -> ('b -> 'c))      .
535
536    Dropping extraneous parentheses since the ` $\rightarrow$ ` type operator is right
537    associative (and of lower precedence than ` $*$ `, we can also write this
538    as
539
540          (`a * 'b -> 'c) -> 'a -> 'b -> 'c      .
541
542    A good name for the argument of the curry function is `uncurried`
543    (6), to emphasize that it is an uncurried function.
544
545    This type information already gives us a big hint as to how to
546    write the curry function. We start with the first line giving the
547    argument structure (7):
548
549          let curry (uncurried : 'a * 'b -> 'c) : 'a -> 'b -> 'c = ...
550

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551 The return type is a function type, so we'll want to build a
552 function value to return. We use the `'fun _ -> _'` anonymous
553 function construction to do so, carefully labeling the type of the
554 function's argument as a reminder of what's going on:

```
555         let curry (uncurried : 'a * 'b -> 'c) : 'a -> 'b -> 'c =  
556             fun (x : 'a) -> ...
```

558 The type of the argument of this anonymous function is `'a`' because
559 its type as a whole -- the return type of `'curry'` itself -- is `'a`
560 `-> ('b -> 'c)'`. This function should return a function of type `'b`
561 `-> 'c'`. We'll construct that as an anonymous function as well:

```
563         let curry (uncurried : 'a * 'b -> 'c) : 'a -> 'b -> 'c =  
564             fun (x : 'a) ->  
565                 fun (y : 'b) -> ...
```

567 Now, how should we construct the value (of type `'c'`) that this
568 inner function should return? Remember that `curry` should return a
569 curried function whose value is the same as the uncurried function
570 would have delivered on arguments `'x'` and `'y'`. So we can simply
571 apply `'uncurried'` to `'x'` and `'y'` (in an uncurried fashion, of
572 course), to obtain the value of type `'c'`:

```
574         let curry (uncurried : 'a * 'b -> 'c) : 'a -> 'b -> 'c =  
575             fun (x : 'a) ->  
576                 fun (y : 'b) -> uncurried (x, y) ;;
```

584 You'll note that all of these anonymous functions are a bit
585 cumbersome, and we have a nicer notation for defining functions in
586 `let` expressions incorporating the arguments in the definition part
587 itself. We've already done so for the argument `uncurried`. Let's use
588 that notation for the `'x'` and `'y'` arguments as well.

```
589         let curry (uncurried : 'a * 'b -> 'c) (x : 'a) (y : 'b) : 'c =  
590             uncurried (x, y) ;;
```

591 To make clearer what's going on, we can even drop the explicit
592 types to show the structure of the computation:

```
593         let curry uncurried x y = uncurried (x, y) ;;
```

594 Here, we see what's really going on: `'curry uncurried'` when applied
595 to `'x'` and `'y'` in curried fashion gives the same value that
596 `'uncurried'` gives when applied to `'x'` and `'y'` in uncurried fashion.

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597 By a similar argument (which it might be useful to carry out
598 yourself), uncurry is implemented as
599
600     let uncurry curried (x, y) = curried x y ;;
601
602 Below, we use the version with explicit types, as we generally want
603 to do to make our typing intentions known to the
604 compiler/interpreter. *)
605
606 let curry (uncurried : 'a * 'b -> 'c) (x : 'a) (y : 'b) : 'c =
607     uncurried (x, y) ;;
608
609 (*.....
610 Exercise 14: Now implement the polymorphic higher-order function
611 `uncurry`, which takes as its argument a curried function and returns
612 the uncurried version of its argument function. You may want to go
613 through the same 7-step process to get started.
614 .....*)
615
616 let uncurry (curried : 'a -> 'b -> 'c) (x, y : 'a * 'b) : 'c =
617     curried x y ;;
618
619 (*.....
620 Exercise 15: OCaml's built in binary operators, like `+` and `*` are
621 curried. You can tell from their types:
622
623     # ( + ) ;;
624     - : int -> int -> int = <fun>
625     # ( * ) ;;
626     - : int -> int -> int = <fun>
627
628 Using your `uncurry` function, define uncurried versions of the `+` and
629 `*` functions. Call them `plus` and `times`.
630 .....*)
631
632 let plus = uncurry ( + ) ;;
633
634 let times = uncurry ( * ) ;;
635
636 (* Did you write something like this?
637
638     let plus x y =
639         ...more stuff here...
640
641 Remember, functions are first-class values in OCaml; they can be
642 returned by other functions. So you don't always need to give the

```

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643     arguments explicitly in a function definition.  *)
644
645     (*.....*)
646 Exercise 16: Recall the `prods` function from Lab 1:
647
648     let rec prods (lst : (int * int) list) : int list =
649         match lst with
650         | [] -> []
651         | (x, y) :: tail -> (x * y) :: (prods tail) ;;
652
653 Now reimplement `prods` using `map` and your uncurried `times`
654 function. Why do you need the uncurried `times` function?
655 .....*)
656
657 let prods = List.map times ;;
658
659 (* Elegant, no? *)

```