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1  (*)
2                                     CS51 Lab 4
3                               Error Handling, Options, and Exceptions
4  *)
5  (*)
6                               SOLUTION
7  *)
8
9
10 (*=====
11 Readings:
12
13     This lab builds on material from Chapter 10 of the textbook
14     <http://book.cs51.io>, which should be read before the lab session.
15
16     =====*)
17
18 (*=====
19 Part 1: Option types and exceptions
20
21 In Lab 2, you implemented a function `max_list` that returns the maximum
22 element in a non-empty integer list. Here's a possible implementation
23 for `max_list`:
24
25     let rec max_list (lst : int list) : int =
26         match lst with
27         | [elt] -> elt
28         | head :: tail -> max head (max_list tail) ;;
29
30 (This implementation makes use of the polymorphic `max` function from
31 the `Stdlib` module.)
32
33 As written, this function generates a warning that the match is not
34 exhaustive. Why? What's an example of the missing case? Try entering
35 the function in `ocaml` or `utop` and see what information you can
36 glean from the warning message. Go ahead; we'll wait.
37
38     .
39     .
40     .
41
42 The problem is that *there is no reasonable value for the maximum
43 element in an empty list*. This is an ideal application for option
44 types.

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45
46 .....
47 Exercise 1:
48
49 Reimplement 'max_list', but this time, it should return an 'int option'
50 instead of an 'int'. Call it 'max_list_opt'. The 'None' return value
51 should be used when called on an empty list.
52
53 (Using the suffix '_opt' is a standard convention in OCaml for
54 functions that return an option type for this purpose. See, for
55 instance, the functions 'nth' and 'nth_opt' in the 'List' module.)
56 .....*)
57
58 let rec max_list_opt (lst : int list) : int option =
59   match lst with
60   | [] -> None
61   | head :: tail ->
62     match (max_list_opt tail) with
63     | None -> Some head
64     | Some max_tail -> Some (max head max_tail) ;;
65
66 (*.....
67 Exercise 2: Alternatively, we could have 'max_list' raise an exception
68 upon discovering the error condition. Reimplement 'max_list' so that it
69 does so. What exception should it raise? (See Section 10.3 in the
70 textbook for some advice.)
71 .....*)
72
73 let rec max_list (lst : int list) : int =
74   match lst with
75   | [] -> raise (Invalid_argument "max_list: empty list")
76   | [elt] -> elt
77   | head :: tail -> max head (max_list tail) ;;
78
79 (*.....
80 Exercise 3: Write a function 'min_option' to return the smaller of its
81 two 'int option' arguments, or 'None' if both are 'None'. If exactly one
82 argument is 'None', return the other. The built-in function 'min' from
83 the Stdlib module may be useful. You'll want to make sure that all
84 possible cases are handled; no nonexhaustive match warnings!
85 .....*)
86
87 let min_option (x : int option) (y : int option) : int option =
88   match x, y with
89   | None, None -> None
90   | None, Some _right -> y

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91 | Some _left, None          -> x
92 | Some left, Some right -> Some (min left right) ;;
93
94 (*.....*)
95 Exercise 4: Write a function `plus_option` to return the sum of its two
96 `int option` arguments, or `None` if both are `None`. If exactly one
97 argument is `None`, return the other.
98 .....*)
99
100 let plus_option (x : int option) (y : int option) : int option =
101   match x, y with
102   | None,      None          -> None
103   | None,      Some _right -> y
104   | Some _left, None          -> x
105   | Some left, Some right -> Some (left + right) ;;
106
107 (=====
108 Part 2: Polymorphism practice
109
110 Do you see a pattern in your implementations of
111 `min_option` and `plus_option`? How can we factor out similar code?
112
113 .....*)
114 Exercise 5: Write a polymorphic higher-order function `lift_option` to
115 "lift" binary operations to operate on option type values, taking
116 three arguments in order: the binary operation (a curried function)
117 and its first and second arguments as option types. If both arguments
118 are `None`, return `None`. If one argument is `None`, the function
119 should return the other argument. If neither argument is `None`, the
120 binary operation should be applied to the argument values and the
121 result appropriately returned.
122
123 What is the type signature for `lift_option`? (If you're having
124 trouble figuring that out, call over a staff member, or check our
125 intended type at <https://url.cs51.io/lab4-1>.)
126
127 Now implement `lift_option`.
128 .....*)
129
130 (* SOLUTION: The type signature for `lift_option` is naturally
131    polymorphic:
132
133       ('a -> 'a -> 'a) -> 'a option -> 'a option -> 'a option
134
135    Notice the nice symmetry, which is perhaps made clearer when
136    parenthesized as

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137         ('a -> 'a -> 'a) -> ('a option -> 'a option -> 'a option)      .
138
139
140     To think about: Both the first and second argument of `f` must be
141     of the same type as the result type of `f` (and hence of each
142     other). Do you see why?
143
144     *)
145     let lift_option (f : 'a -> 'a -> 'a) (x : 'a option) (y : 'a option)
146         : 'a option =
147         match x, y with
148         | None,      None      -> None
149         | None,      Some _right -> y
150         | Some _left, None      -> x
151         | Some left,  Some right -> Some (f left right) ;;
152
153     (*.....*)
154     Exercise 6: Now rewrite `min_option` and `plus_option` using the
155     higher-order function `lift_option`. Call them `min_option_2` and
156     `plus_option_2`.
157
158     Note: You might encounter inexplicable "weak type variable"
159     warnings. If you do, you should make sure to type the arguments of
160     the function. For more information read the detailed explanation
161     in the lab's solution comments or Section 9.6 of the textbook.
162     .....*)
163
164     let min_option_2 : int option -> int option -> int option =
165         lift_option min ;;
166
167     (* You may have not added in the specific type information in your
168     definition of `min_option_2`, and received an inscrutable warning
169     involving "weak type variables", and type problems when submitting
170     your code. Here's an example of that behavior:
171
172     # let min_option_2 =
173         lift_option min ;;
174     val min_option_2 : '_weak1 option -> '_weak1 option -> '_weak1 option = <fun>
175     # min_option_2 (Some 3) (Some 4) ;;
176     - : int option = Some 3
177     # min_option_2 (Some 4.2) (Some 4.1) ;;
178     Error: This expression [namely, the 4.2] has type float but an expression
179         was expected of type int
180
181     The type variables like `'_weak1` (with the underscore) are "weak
182     type variables", not true type variables. Weak type variables are
183     discussed briefly in Section 9.7 of the textbook. They arise

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183 because in certain situations OCaml's type inference can't figure
184 out how to express the most general types and must resort to this
185 weak type variable approach.
186
187 When a function with these weak type variables is applied to
188 arguments with a specific type, the polymorphism of the function
189 disappears. Notice that the first time we apply min_option_2 above
190 to int options, things work fine. But the second time, applied to
191 float options, there's a type clash because the first use of
192 min_option_2 fixed the weak type variables to be ints. Since our
193 unit tests try using min_option_2 in certain ways inconsistent with
194 weak type variables, you'll get an error message saying that "The
195 type of this expression, '_weak1 option -> '_weak1 option ->
196 '_weak1 option, contains type variables that cannot be
197 generalized."
198
199 To correct the problem, you can add in specific typing information
200 (as we've done in the solution above) or make explicit the full
201 application of 'lift_option':
202
203     let min_option_2 x y =
204         lift_option min x y ;;
205
206 rather than the partial application we used. Either of these
207 approaches gives OCaml sufficient hints to infer types more
208 accurately.
209
210 For the curious, if you want to see what's going on in detail, you
211 can check out the discussion in the section "A function obtained
212 through partial application is not polymorphic enough" at
213 <https://v2.ocaml.org/learn/faq.html#Typing>. *)
214
215 let plus_option_2 : int option -> int option -> int option =
216     lift_option (+) ;;
217
218 (*.....
219 Exercise 7: Now that we have 'lift_option', we can use it in other
220 ways. Because 'lift_option' is polymorphic, it can work on things other
221 than 'int option's. Define a function 'and_option' to return the boolean
222 AND of two 'bool option's, or 'None' if both are 'None'. If exactly one
223 is 'None', return the other.
224 .....*)
225
226 let and_option : bool option -> bool option -> bool option =
227     lift_option (&&) ;;
228

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229 (*.....
230 Exercise 8: In Lab 3, you implemented a polymorphic function `zip` that
231 takes two lists and "zips" them together into a list of pairs. Here's
232 a possible implementation of `zip`:
233
234     let rec zip (x : 'a list) (y : 'b list) : ('a * 'b) list =
235         match x, y with
236         | [], [] -> []
237         | xhd :: xtl, yhd :: ytl -> (xhd, yhd) :: (zip xtl ytl) ;;
238
239 A problem with this implementation of `zip` is that, once again, its
240 match is not exhaustive and it raises an exception when given lists of
241 unequal length. How can you use option types to generate an alternate
242 solution without this property?
243
244 Do so below in a new definition of `zip` -- called `zip_opt` to make
245 clear that its signature has changed -- which returns an appropriate
246 option type in case it is called with lists of unequal length. Here
247 are some examples:
248
249     # zip_opt [1; 2] [true; false] ;;
250     - : (int * bool) list option = Some [(1, true); (2, false)]
251     # zip_opt [1; 2] [true; false; true] ;;
252     - : (int * bool) list option = None
253     .....*)
254
255 let rec zip_opt (x : 'a list) (y : 'b list) : (('a * 'b) list) option =
256     match x, y with
257     | [], [] -> Some []
258     | xhd :: xtl, yhd :: ytl ->
259         (match zip_opt xtl ytl with
260          | None -> None
261          | Some ztl -> Some ((xhd, yhd) :: ztl))
262     | _, _ -> None ;;
263
264 (*=====
265 Part 3: Factoring out None-handling
266
267 Recall the definition of `dotprod` from Lab 2. Here it is, adjusted to
268 an option type:
269
270     let dotprod_opt (a : int list) (b : int list) : int option =
271         let pairs_opt = zip_opt a b in
272         match pairs_opt with
273         | None -> None
274         | Some pairs -> Some (sum (prods pairs)) ;;

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275
276 It uses `zip_opt` from Exercise 8, `prods` from Lab 3, and a function
277 `sum` to sum up all the integers in a list. The `sum` function is
278 simply *)
279
280 let sum : int list -> int =
281   List.fold_left (+) 0 ;;
282
283 (* and a version of `prods` is *)
284
285 let prods =
286   List.map (fun (x, y) -> x * y) ;;
287
288 (* Notice how in `dotprod_opt` and other option-manipulating functions
289 we frequently and annoyingly have to test if a value of option type is
290 `None`; this requires a separate match, and passing on the `None`
291 value in the "bad" branch and introducing a `Some` in the "good"
292 branch. This is something we're likely to be doing a lot of. Let's
293 factor that out to simplify the implementation.
294
295 .....
296 Exercise 9: Define a function called `maybe` that takes a first
297 argument, function of type `'arg -> 'result`, and a second argument,
298 of type `'arg option`, and "maybe" applies the first (the function) to
299 the second (the argument), depending on whether its argument is a
300 `None` or a `Some`. The `maybe` function either passes on the `None`
301 if its second argument is `None`, or if its second argument is `Some
302 v`, it applies its first argument to that `v` and returns the result,
303 appropriately adjusted for the result type.
304
305 What should the type of the `maybe` function be?
306
307 Now implement the `maybe` function.
308 .....*)
309
310 let maybe (f : 'arg -> 'result) (x : 'arg option) : 'result option =
311   match x with
312   | None -> None
313   | Some v -> Some (f v) ;;
314
315 (*.....
316 Exercise 10: Now reimplement `dotprod_opt` to use the `maybe`
317 function. (The previous implementation makes use of functions `sum`
318 and `prods`, which we've provided for you above.) Your new solution
319 for `dotprod` should be much simpler than the version we provided
320 above at the top of Part 3.

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321 .....*)
322
323 let dotprod_opt (a : int list) (b : int list) : int option =
324   maybe (fun pairs -> sum (prods pairs))
325         (zip_opt a b) ;;
326
327 (*.....
328 Exercise 11: Reimplement `zip_opt` using the `maybe` function, as
329 `zip_opt_2` below.
330 .....*)
331
332 (* We remove the embedded match using a maybe: *)
333
334 let rec zip_opt_2 (x : 'a list) (y : 'b list) : (('a * 'b) list) option =
335   match x, y with
336   | [], [] -> Some []
337   | xhd :: xt1, yhd :: yt1 ->
338     maybe (fun zt1 -> ((xhd, yhd) :: zt1))
339           (zip_opt_2 xt1 yt1)
340   | _, _ -> None ;;
341
342 (*.....
343 Exercise 12: [Optional] For the energetic, reimplement `max_list_opt`
344 as `max_list_opt_2` along the same lines. There's likely to be a
345 subtle issue here, since the `maybe` function always passes along the
346 `None`.
347 .....*)
348
349 let rec max_list_opt_2 (lst : int list) : int option =
350   match lst with
351   | [] -> None
352   | [single] -> Some single
353   | head :: tail ->
354     maybe (fun max_tail -> max head max_tail)
355           (max_list_opt_2 tail) ;;
356
357 (* The subtle issue is this. Recall the previous definition of
358 `max_list_opt` above:
359
360     let rec max_list_opt (lst : int list) : int option =
361       match lst with
362       | [] -> None
363       | head :: tail ->
364         match (max_list_opt tail) with
365         | None -> Some head
366         | Some max_tail -> Some (max head max_tail) ;;

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367
368 In this version, no special match case is needed for the case of a
369 singleton list. Instead, we can recur all the way to the empty list
370 case, and handle the singleton case in the first case in the
371 embedded match, where the `None` from the recursive call becomes
372 `Some head`. However, when using the `maybe` in the corresponding
373 case in `max_list_opt_2`, we can't allow the recursion to proceed
374 all the way to the empty list, where `None` would be returned,
375 because `maybe` always preserves `None`s; we can't "promote" the
376 `None` to a `Some`. Consequently, we need to handle the singleton
377 case explicitly. *)