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1  (*
2                                     CS51 Lab 4
3                               Error Handling, Options, and Exceptions
4  *)
5  (*
6                               SOLUTION
7  *)
8
9
10 (*=====
11 Readings:
12
13     This lab builds on material from Chapter 10 of the textbook
14     <http://book.cs51.io>, which should be read before the lab session.
15
16     =====*)
17
18 (*=====
19 Part 1: Option types and exceptions
20
21 In Lab 2, you implemented a function `max_list` that returns the maximum
22 element in a non-empty integer list. Here's a possible implementation
23 for `max_list`:
24
25     let rec max_list (lst : int list) : int =
26         match lst with
27         | [elt] -> elt
28         | head :: tail -> max head (max_list tail) ;;
29
30 (This implementation makes use of the polymorphic `max` function from
31 the `Stdlib` module.)
32
33 As written, this function generates a warning that the match is not
34 exhaustive. Why? What's an example of the missing case? Try entering
35 the function in `ocaml` or `utop` and see what information you can
36 glean from the warning message. Go ahead; we'll wait.
37
38     .
39     .
40     .
41
42 The problem is that *there is no reasonable value for the maximum
43 element in an empty list*. This is an ideal application for option
44 types.

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45
46 .....
47 Exercise 1:
48
49 Reimplement 'max_list', but this time, it should return an 'int option'
50 instead of an 'int'. Call it 'max_list_opt'. The 'None' return value
51 should be used when called on an empty list.
52
53 (Using the suffix '_opt' is a standard convention in OCaml for
54 functions that return an option type for this purpose. See, for
55 instance, the functions 'nth' and 'nth_opt' in the 'List' module.)
56 .....*)
57
58 let rec max_list_opt (lst : int list) : int option =
59   match lst with
60   | [] -> None
61   | head :: tail ->
62     match (max_list_opt tail) with
63     | None -> Some head
64     | Some max_tail -> Some (max head max_tail) ;;
65
66 (*.....
67 Exercise 2: Alternatively, we could have 'max_list' raise an exception
68 upon discovering the error condition. Reimplement 'max_list' so that it
69 does so. What exception should it raise? (See Section 10.3 in the
70 textbook for some advice.)
71 .....*)
72
73 let rec max_list (lst : int list) : int =
74   match lst with
75   | [] -> raise (Invalid_argument "max_list: empty list")
76   | [elt] -> elt
77   | head :: tail -> max head (max_list tail) ;;
78
79 (*.....
80 Exercise 3: Write a function 'min_option' to return the smaller of its
81 two 'int option' arguments, or 'None' if both are 'None'. If exactly one
82 argument is 'None', return the other. The built-in function 'min' from
83 the Stdlib module may be useful. You'll want to make sure that all
84 possible cases are handled; no nonexhaustive match warnings!
85 .....*)
86
87 let min_option (x : int option) (y : int option) : int option =
88   match x, y with
89   | None,      None      -> None
90   | None,      Some _right -> y

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91 | Some _left, None          -> x
92 | Some left, Some right -> Some (min left right) ;;
93
94 (*.....*)
95 Exercise 4: Write a function `plus_option` to return the sum of its two
96 `int option` arguments, or `None` if both are `None`. If exactly one
97 argument is `None`, return the other.
98 .....*)
99
100 let plus_option (x : int option) (y : int option) : int option =
101   match x, y with
102   | None,      None          -> None
103   | None,      Some _right -> y
104   | Some _left, None          -> x
105   | Some left, Some right -> Some (left + right) ;;
106
107 (=====
108 Part 2: Polymorphism practice
109
110 Do you see a pattern in your implementations of
111 `min_option` and `plus_option`? How can we factor out similar code?
112
113 .....*)
114 Exercise 5: Write a polymorphic higher-order function `lift_option` to
115 "lift" binary operations to operate on option type values, taking
116 three arguments in order: the binary operation (a curried function)
117 and its first and second arguments as option types. If both arguments
118 are `None`, return `None`. If one argument is `None`, the function
119 should return the other argument. If neither argument is `None`, the
120 binary operation should be applied to the argument values and the
121 result appropriately returned.
122
123 What is the type signature for `lift_option`? (If you're having
124 trouble figuring that out, call over a staff member, or check our
125 intended type at <https://url.cs51.io/lab4-1>.)
126
127 Now implement `lift_option`.
128 .....*)
129
130 (* SOLUTION: The type signature for `lift_option` is naturally
131    polymorphic:
132
133      ('a -> 'a -> 'a) -> 'a option -> 'a option -> 'a option
134
135    Notice the nice symmetry, which is perhaps made clearer when
136    parenthesized as

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137         ('a -> 'a -> 'a) -> ('a option -> 'a option -> 'a option)      .
138
139
140     To think about: Both the first and second argument of `f` must be
141     of the same type as the result type of `f` (and hence of each
142     other). Do you see why?
143
144     *)
145     let lift_option (f : 'a -> 'a -> 'a) (x : 'a option) (y : 'a option)
146       : 'a option =
147       match x, y with
148       | None,      None      -> None
149       | None,      Some _right -> y
150       | Some _left, None      -> x
151       | Some left,  Some right -> Some (f left right) ;;
152
153     (*.....
154     Exercise 6: Now rewrite `min_option` and `plus_option` using the
155     higher-order function `lift_option`. Call them `min_option_2` and
156     `plus_option_2`.
157     .....*)
158
159     let min_option_2 : int option -> int option -> int option =
160       lift_option min ;;
161
162     (* You may have not added in the specific type information in your
163     definition of `min_option_2`, and received an inscrutable warning
164     involving "weak type variables", and type problems when submitting
165     your code. Here's an example of that behavior:
166
167     # let min_option_2 =
168       lift_option min ;;
169     val min_option_2 : '_weak1 option -> '_weak1 option -> '_weak1 option = <fun>
170     # min_option_2 (Some 3) (Some 4) ;;
171     - : int option = Some 3
172     # min_option_2 (Some 4.2) (Some 4.1) ;;
173     Error: This expression [namely, the 4.2] has type float but an expression
174     was expected of type int
175
176     The type variables like `'_weak1` (with the underscore) are "weak
177     type variables", not true type variables. Weak type variables are
178     discussed briefly in Section 9.7 of the textbook. They arise
179     because in certain situations OCaml's type inference can't figure
180     out how to express the most general types and must resort to this
181     weak type variable approach.
182
183     When a function with these weak type variables is applied to

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183 arguments with a specific type, the polymorphism of the function
184 disappears. Notice that the first time we apply min_option_2 above
185 to int options, things work fine. But the second time, applied to
186 float options, there's a type clash because the first use of
187 min_option_2 fixed the weak type variables to be ints. Since our
188 unit tests try using min_option_2 in certain ways inconsistent with
189 weak type variables, you'll get an error message saying that "The
190 type of this expression, '_weak1 option -> '_weak1 option ->
191 '_weak1 option, contains type variables that cannot be
192 generalized."
193
194 To correct the problem, you can add in specific typing information
195 (as we've done in the solution above) or make explicit the full
196 application of 'lift_option':
197
198     let min_option_2 x y =
199         lift_option min x y ;;
200
201 rather than the partial application we used. Either of these
202 approaches gives OCaml sufficient hints to infer types more
203 accurately.
204
205 For the curious, if you want to see what's going on in detail, you
206 can check out the discussion in the section "A function obtained
207 through partial application is not polymorphic enough" at
208 <https://v2.ocaml.org/learn/faq.html#Typing>. *)
209
210 let plus_option_2 : int option -> int option -> int option =
211     lift_option (+) ;;
212
213 (*.....*)
214 Exercise 7: Now that we have 'lift_option', we can use it in other
215 ways. Because 'lift_option' is polymorphic, it can work on things other
216 than 'int option's. Define a function 'and_option' to return the boolean
217 AND of two 'bool option's, or 'None' if both are 'None'. If exactly one
218 is 'None', return the other.
219 .....*)
220
221 let and_option : bool option -> bool option -> bool option =
222     lift_option (&&) ;;
223
224 (*.....*)
225 Exercise 8: In Lab 3, you implemented a polymorphic function 'zip' that
226 takes two lists and "zips" them together into a list of pairs. Here's
227 a possible implementation of 'zip':
228

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229     let rec zip (x : 'a list) (y : 'b list) : ('a * 'b) list =
230         match x, y with
231         | [], [] -> []
232         | xhd :: xtl, yhd :: ytl -> (xhd, yhd) :: (zip xtl ytl) ;;
233

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234 A problem with this implementation of `zip` is that, once again, its
 235 match is not exhaustive and it raises an exception when given lists of
 236 unequal length. How can you use option types to generate an alternate
 237 solution without this property?

238
 239 Do so below in a new definition of `zip` -- called `zip_opt` to make
 240 clear that its signature has changed -- which returns an appropriate
 241 option type in case it is called with lists of unequal length. Here
 242 are some examples:

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243     # zip_opt [1; 2] [true; false] ;;
244     - : (int * bool) list option = Some [(1, true); (2, false)]
245     # zip_opt [1; 2] [true; false; true] ;;
246     - : (int * bool) list option = None
247     .....*)
248

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249
250 let rec zip_opt (x : 'a list) (y : 'b list) : (('a * 'b) list) option =
251     match x, y with
252     | [], [] -> Some []
253     | xhd :: xtl, yhd :: ytl ->
254         (match zip_opt xtl ytl with
255          | None -> None
256          | Some ztl -> Some ((xhd, yhd) :: ztl))
257     | _, _ -> None ;;
258

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259 (*=====

260 Part 3: Factoring out None-handling

261
 262 Recall the definition of `dotprod` from Lab 2. Here it is, adjusted to
 263 an option type:

```

264
265     let dotprod_opt (a : int list) (b : int list) : int option =
266         let pairs_opt = zip_opt a b in
267         match pairs_opt with
268         | None -> None
269         | Some pairs -> Some (sum (prods pairs)) ;;
270

```

271 It uses `zip_opt` from Exercise 8, `prods` from Lab 3, and a function
 272 `sum` to sum up all the integers in a list. The `sum` function is
 273 simply *)

274

```

275 let sum : int list -> int =
276   List.fold_left (+) 0 ;;
277
278 (* and a version of `prods` is *)
279
280 let prods =
281   List.map (fun (x, y) -> x * y) ;;
282
283 (* Notice how in `dotprod_opt` and other option-manipulating functions
284 we frequently and annoyingly have to test if a value of option type is
285 `None`; this requires a separate match, and passing on the `None`
286 value in the "bad" branch and introducing a `Some` in the "good"
287 branch. This is something we're likely to be doing a lot of. Let's
288 factor that out to simplify the implementation.
289
290 .....
291 Exercise 9: Define a function called `maybe` that takes a first
292 argument, function of type `'arg -> 'result`, and a second argument,
293 of type `'arg option`, and "maybe" applies the first (the function) to
294 the second (the argument), depending on whether its argument is a
295 `None` or a `Some`. The `maybe` function either passes on the `None`
296 if its second argument is `None`, or if its second argument is `Some
297 v`, it applies its first argument to that `v` and returns the result,
298 appropriately adjusted for the result type.
299
300 What should the type of the `maybe` function be?
301
302 Now implement the `maybe` function.
303 .....*)
304
305 let maybe (f : 'arg -> 'result) (x : 'arg option) : 'result option =
306   match x with
307   | None -> None
308   | Some v -> Some (f v) ;;
309
310 (*.....
311 Exercise 10: Now reimplement `dotprod_opt` to use the `maybe`
312 function. (The previous implementation makes use of functions `sum`
313 and `prods`, which we've provided for you above.) Your new solution
314 for `dotprod` should be much simpler than the version we provided
315 above at the top of Part 3.
316 .....*)
317
318 let dotprod_opt (a : int list) (b : int list) : int option =
319   maybe (fun pairs -> sum (prods pairs))
320         (zip_opt a b) ;;

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321
322 (*.....
323 Exercise 11: Reimplement `zip_opt` using the `maybe` function, as
324 `zip_opt_2` below.
325 .....*)
326
327 (* We remove the embedded match using a maybe: *)
328
329 let rec zip_opt_2 (x : 'a list) (y : 'b list) : (('a * 'b) list) option =
330   match x, y with
331   | [], [] -> Some []
332   | xhd :: xtl, yhd :: ytl ->
333     maybe (fun ztl -> ((xhd, yhd) :: ztl))
334           (zip_opt_2 xtl ytl)
335   | _, _ -> None ;;
336
337 (*.....
338 Exercise 12: [Optional] For the energetic, reimplement `max_list_opt`
339 as `max_list_opt_2` along the same lines. There's likely to be a
340 subtle issue here, since the `maybe` function always passes along the
341 `None`.
342 .....*)
343
344 let rec max_list_opt_2 (lst : int list) : int option =
345   match lst with
346   | [] -> None
347   | [single] -> Some single
348   | head :: tail ->
349     maybe (fun max_tail -> max head max_tail)
350           (max_list_opt_2 tail) ;;
351
352 (* The subtle issue is this. Recall the previous definition of
353 `max_list_opt` above:
354
355     let rec max_list_opt (lst : int list) : int option =
356       match lst with
357       | [] -> None
358       | head :: tail ->
359         match (max_list_opt tail) with
360         | None -> Some head
361         | Some max_tail -> Some (max head max_tail) ;;
362
363 In this version, no special match case is needed for the case of a
364 singleton list. Instead, we can recur all the way to the empty list
365 case, and handle the singleton case in the first case in the
366 embedded match, where the `None` from the recursive call becomes

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367 `Some head`. However, when using the `maybe` in the corresponding
368 case in `max_list_opt_2`, we can't allow the recursion to proceed
369 all the way to the empty list, where `None` would be returned,
370 because `maybe` always preserves `None`s; we can't "promote" the
371 `None` to a `Some`. Consequently, we need to handle the singleton
372 case explicitly. *)