CS 5435

Cryptographic Hash Functions

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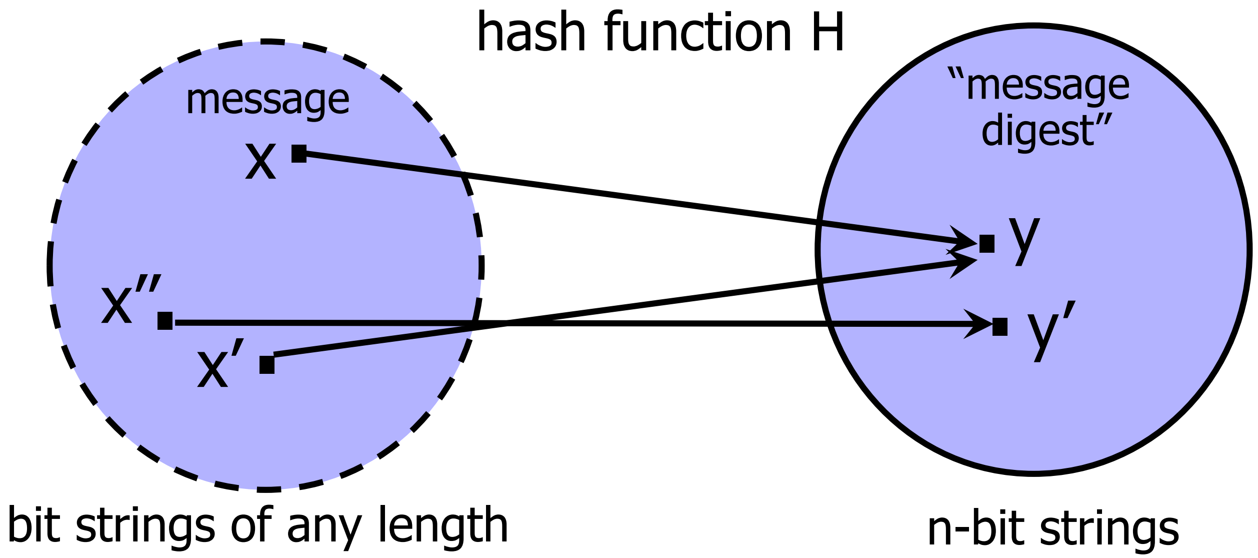
Hash Functions: Main Idea



Hash function H is a lossy compression function

- Collision: H(x)=H(x') for some inputs x≠x'

H(x) should look "random"

A cryptographic hash function must have certain properties

One-Way

Intuition: hash should be hard to invert

- "Preimage resistance"
- Given a random y, it should be hard to find any x such that h(x)=y
  - y is an n-bit string randomly chosen from the output space of the hash function, ie, y=h(x') for some x'

How hard?

- Brute-force: try every possible x, see if h(x)=y
- SHA-1 (a common hash function) has 160-bit output
  - Suppose we have hardware that can do 2^{30} trials a pop
  - Assuming 2^{34} trials per second, can do 2^{89} trials per year
  - Will take 2^{71} years to invert SHA-1 on a random image

Birthday Paradox

T people

Suppose each birthday is a random number taken from K days (K=365) – how many possibilities?

- K^T - samples with replacement

How many possibilities that are all different?

- $(K)_T = K(K-1)\dots(K-T+1)$ - samples without replacement

Probability of no repetition?

- $(K)_T/K^T \approx 1 - T(T-1)/2K$

Probability of repetition?

- $O(T^2)$

Collision Resistance

Should be hard to find $x \neq x'$ such that $h(x)=h(x')$

Birthday paradox

- Let T be the number of values x,x',x''… we need to look at before finding the first pair $x \neq x'$ s.t. $h(x)=h(x')$
- Assuming h is random, what is the probability that we find a repetition after looking at T values? $O(T^2)$
- Total number of pairs? $O(2^n)$
  - n = number of bits in the output of hash function
- Conclusion: $T \approx O(2^{n/2})$

Brute-force collision search is $O(2^{n/2})$, <u>not</u> $O(2^n)$

- For SHA-1, this means $O(2^{80})$ vs. $O(2^{160})$

One-Way vs. Collision Resistance

One-wayness does not imply collision resistance

- Suppose g() is one-way
- Define h(x) as g(x') where x' is x except the last bit
  - h is one-way (cannot invert h without inverting g)
  - Collisions for h are easy to find: for any x, h(x0)=h(x1)

Collision resistance does not imply one-wayness

- Suppose g() is collision-resistant
- Define h(x) to be 0x if x is (n-1)-bit long, else 1g(x)
  - Collisions for h are hard to find: if y starts with 0, then there are no collisions; if y starts with 1, then must find collisions in g
  - h is not one way: half of all y's (those whose first bit is 0) are easy to invert (how?), thus random y is invertible with prob. 1/2

Weak Collision Resistance

Given a randomly chosen x, hard to find x′ such that h(x)=h(x′)

- Attacker must find collision for a <u>specific</u> x… by contrast, to break collision resistance, enough to find <u>any</u> collision
- Brute-force attack requires $O(2^n)$ time

Weak collision resistance does not imply collision resistance (why?)

Hashing vs. Encryption

Hashing is one-way. There is no “uh-hashing”!

- A ciphertext can be decrypted with a decryption key, hashes have no equivalent of “decryption”

Hash(x) looks “random”, but can be compared for equality with Hash(x’)

- Hash the same input twice $\rightarrow$ same hash value
- Encrypt the same input twice $\rightarrow$ different ciphertexts

Cryptographic hashes are also known as “cryptographic checksums”

Application: Password Hashing

Instead of user password, store hash(password)

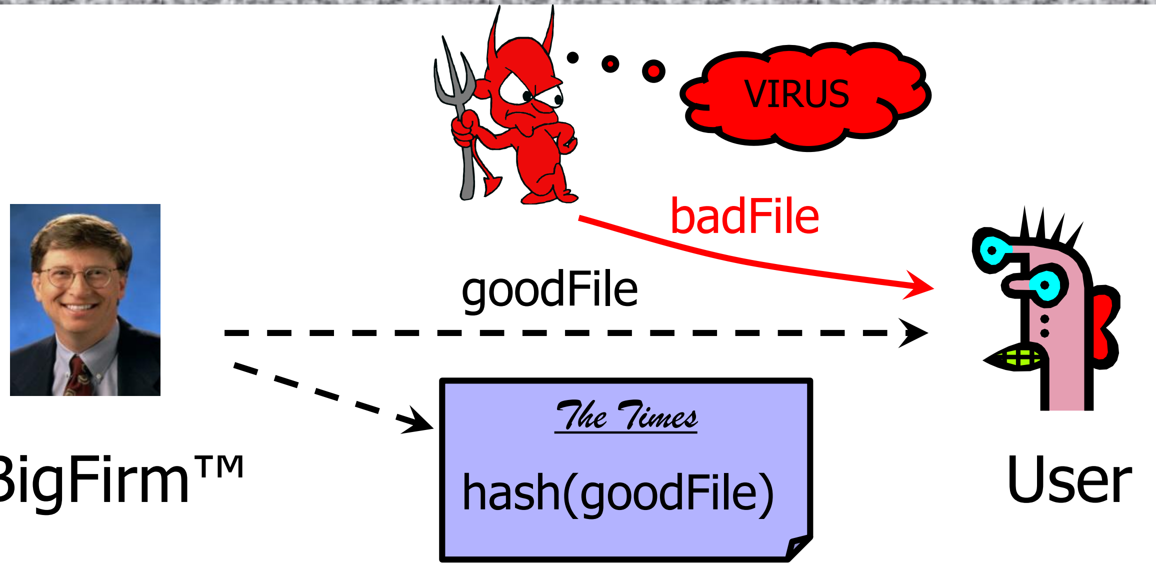
When user enters a password, compute its hash and compare with the entry in the password file

- System does not store actual passwords

Why is hashing better than encryption here?

Does hashing protect weak, easily guessable passwords?

Application: Software Integrity



Software manufacturer wants to ensure that the executable file is received by users without modification…

Sends out the file to users and publishes its hash in the NY Times

The goal is integrity, not secrecy

Idea: given goodFile and hash(goodFile), very hard to find badFile such that hash(goodFile)=hash(badFile)

GIFCT

Global Internet Forum to Counter Terrorism

Database of hashes of images and videos with "violent terrorist imagery and propaganda"

- Established by Facebook, Microsoft, Twitter, YouTube
- Multiple members, even more have access
- Advisory committee with government representatives

Facilitates fast content removal

- How? What properties of hash functions needed?

5-minute discussion: Problems? Risks?

Better ways to suppress problematic content?

Which Property Is Needed?

Passwords stored as hash(password)

- One-wayness: hard to recover entire password

  … but passwords are not random and thus guessable

Integrity of software distribution

- Weak collision resistance?

  … but software images are not random - maybe need full collision resistance?

Auctions: to bid B, send H(B), later reveal B

- One-wayness… but does not protect B from guessing
- Collision resistance: bidder should not be able to find two bids B and B′ such that H(B)=H(B′)

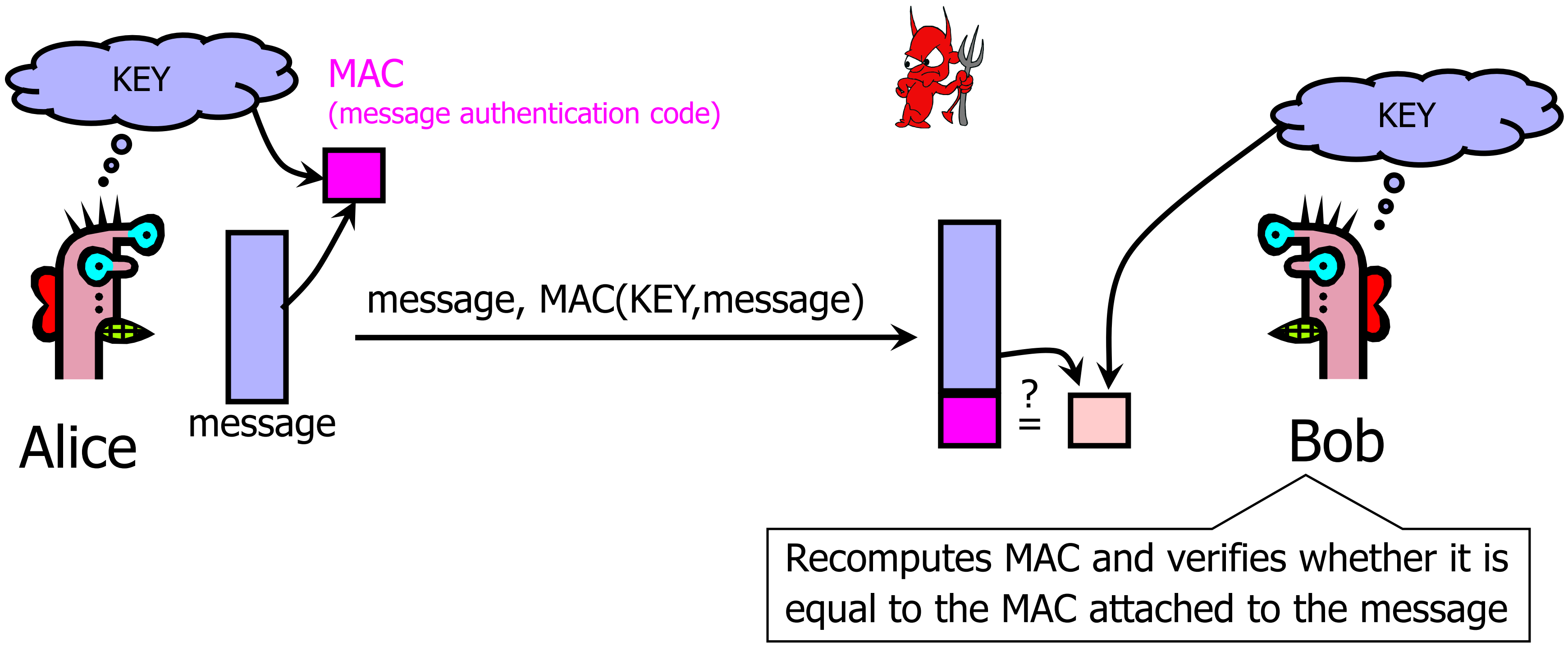
Common Hash Functions

SHA256, SHA512, SHA-3

Do not use MD5, SHA-1

- Still very common in implementations

Integrity and Authentication


KEY
MAC
(message authentication code)
KEY
message, MAC(KEY,message)
?
=
Alice
message
Bob
Recomputes MAC and verifies whether it is equal to the MAC attached to the message

Integrity and authentication: only someone who knows KEY can compute correct MAC for a given message

HMAC

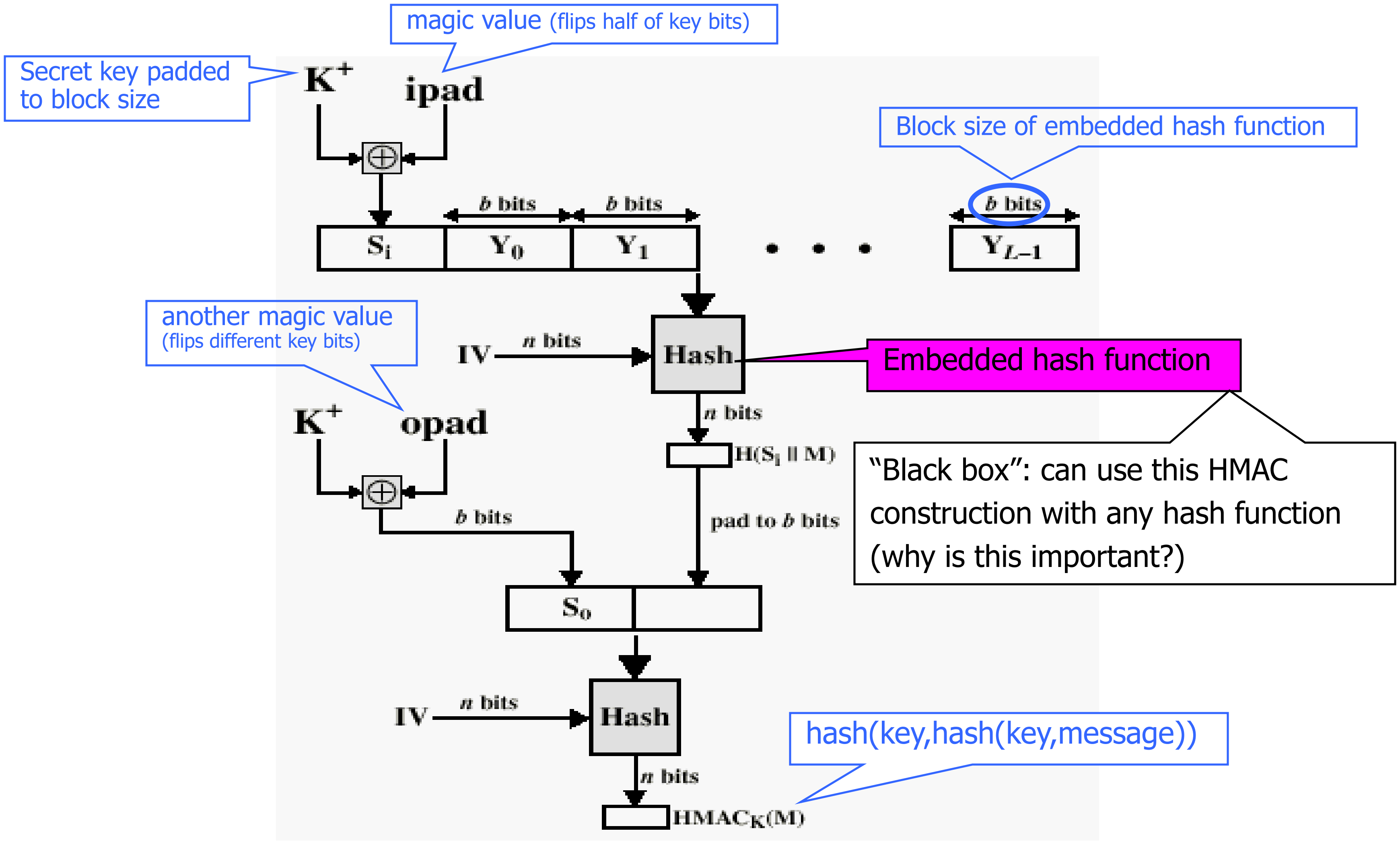
Construct MAC from a cryptographic hash function

- Invented by Bellare, Canetti, and Krawczyk (1996)
- Used in SSL/TLS, mandatory for IPsec

Why not encryption?

- Hashing is faster than encryption
- Library code for hash functions widely available
- Can easily replace one hash function with another
- There used to be US export restrictions on encryption

Structure of HMAC



"Black box": can use this HMAC construction with any hash function (why is this important?)