Queens College, CUNY, Department of Computer Science

Computational Finance

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The material in this homework assignment will be used for questions in exams.

7 Homework 7 (Lecture 12): Black–Scholes–Merton formula for options

- The symbols S, K, r, q, σ, t_0 and T have theit usual meanings.
- Define the variables d_1 and d_2 as follows:

$$\begin{aligned} d_1 &= \frac{\ln(S/K) + (r-q)(T-t_0)}{\sigma\sqrt{T-t_0}} + \tfrac{1}{2}\sigma\sqrt{T-t_0}\,, \\ d_2 &= d_1 - \sigma\sqrt{T-t_0}\,. \end{aligned} \tag{7.1}$$

- The Black–Scholes–Merton formula for the fair value of a European call c and a European put p, and the corresponding Delta Δ_c and Δ_p, is

$$\begin{aligned} c &= Se^{-q(T-t_0)}\,N(d_1) - Ke^{-r(T-t_0)}\,N(d_2)\,, \\ p &= Ke^{-r(T-t_0)}\,N(-d_2) - Se^{-q(T-t_0)}\,N(-d_1)\,, \\ \Delta_c &= e^{-q(T-t_0)}\,N(d_1)\,, \\ \Delta_p &= -e^{-q(T-t_0)}\,N(-d_1)\,. \end{aligned} \tag{7.2}$$

- The **cumulative normal function** $N(x)$ is given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\,du\,. \tag{7.3}$$

- Fortunately C++ provides a function `erf(x)` which can be used to compute $N(x)$:

$$N(x) = \frac{1 + \mathrm{erf}(x/\sqrt{2})}{2}\,. \tag{7.4}$$

- You may use the C++ function below to compute the value of $N(x)$.

```
double cum_norm(double x)
{
  const double root = sqrt(0.5);
  return 0.5*(1.0 + erf(x*root));
}
```

- **Write functions to calculate the fair value and Delta of European call and put options using the Black–Scholes–Merton formula.**
- You will require the above functions to answer questions in exams.

8 Black–Scholes–Merton valuations for examples in Lecture 17a

- Let us employ the parameter values for the examples in Lecture 17a.

$$\begin{aligned} S_0 &= 100\,, \\ K &= 100\,, \\ r &= 0.1\,, \\ q &= 0\,, \\ \sigma &= 0.5\,, \\ T &= 0.3\,, \\ t_0 &= 0\,. \end{aligned} \tag{8.1}$$

- We obtain

$$\begin{aligned} d_1 &\simeq 0.246475\,, \\ d_2 &\simeq -0.02739\,. \end{aligned} \tag{8.2}$$

- The Black–Scholes–Merton values for the European call and put are:

$$\begin{aligned} c_{\rm BSM} &\simeq 12.2721\,, \\ p_{\rm BSM} &\simeq 9.31668\,. \end{aligned} \tag{8.3}$$

- The Black–Scholes–Merton values for the Delta of the European call and put are:

$$\begin{aligned} \Delta_c &\simeq 0.597343\,, \\ \Delta_p &\simeq -0.402657\,. \end{aligned} \tag{8.4}$$

- Remember that the Delta of a put is negative.
- **Verify that the above values satisfy put-call parity**

$$c_{\rm BSM} - p_{\rm BSM} = Se^{-q(T-t_0)} - Ke^{-r(T-t_0)}\,. \tag{8.5}$$

- **Verify that the above values satisfy the relation for Delta**

$$\Delta_c - \Delta_p = e^{-q(T-t_0)}\,. \tag{8.6}$$

9 Black–Scholes–Merton valuations for examples in Lecture 19a

- In the worked examples in Lecture 19a, I calculated American call and put options.
- Nevertheless, we can employ the parameter values in Lecture 19a and calculate the fair values of the corresponding European call and put options.
- The parameter values are

$$\begin{aligned} S_0 &= 100\,, \\ K &= 100\,, \\ r &= 0.1\,, \\ q &= 0.1\,, \\ \sigma &= 0.5\,, \\ T &= 0.4\,, \\ t_0 &= 0\,. \end{aligned} \tag{9.1}$$

- The values of d_1 and d_2 are equal and opposite

$$\begin{aligned} d_1 &\simeq 0.158114\,, \\ d_2 &\simeq -0.158114\,. \end{aligned} \tag{9.2}$$

- The Black–Scholes–Merton values for the European call and put are equal:

$$\begin{aligned} c_{\rm BSM} &\simeq 12.07068\,, \\ p_{\rm BSM} &\simeq 12.07068\,. \end{aligned} \tag{9.3}$$

- **You should be able to work through the binomial tree in Lecture 19a and calculate the binomial model fair values of the corresponding European call and put options. The values should also be equal.**
- The Black–Scholes–Merton values for the Delta of the European call and put are:

$$\begin{aligned} \Delta_c &\simeq 0.540748\,, \\ \Delta_p &\simeq -0.420041\,. \end{aligned} \tag{9.4}$$

- Remember that the Delta of a put is negative.
- **Verify that the above values satisfy put-call parity**

$$c_{\rm BSM} - p_{\rm BSM} = Se^{-q(T-t_0)} - Ke^{-r(T-t_0)}\,. \tag{9.5}$$

- **Verify that the above values satisfy the relation for Delta**

$$\Delta_c - \Delta_p = e^{-q(T-t_0)}\,. \tag{9.6}$$