Dartmouth CS87/287 Rendering Algorithms, Fall 2025
Reading Assignment 7

Next-flight Estimator for Transmittance

In PBR 11.2.1, Equation 11.13, we have transmittance defined as a recursive integral:

$$T_{\mathrm{r}}(\mathrm{p} \to \mathrm{p}') = \mathrm{e}^{-\sigma_{\mathrm{maj}}|\mathrm{p}-\mathrm{p}'|} + \int_0^{|\mathrm{p}-\mathrm{p}'|} \mathrm{e}^{-\sigma_{\mathrm{maj}}t}\sigma_{\mathrm{n}}(\mathrm{p}+t\omega)T_{\mathrm{r}}(\mathrm{p}+t\omega \to \mathrm{p}')\mathrm{d}t. \tag{1}$$

(a) Please show how to derive Equation 11.15, i.e., the Monte Carlo next-flight estimator:

$$T_{\mathrm{r}}(\mathrm{p} \to \mathrm{p}') \approx \mathrm{e}^{-\sigma_{\mathrm{maj}}|\mathrm{p}-\mathrm{p}'|} + \begin{cases} \frac{\sigma_{\mathrm{n}}(\mathrm{p}+t'\omega)}{\sigma_{\mathrm{maj}}} T_{\mathrm{r}}(\mathrm{p}+t'\omega \to \mathrm{p}')t' & t' < |\mathrm{p}-\mathrm{p}'|, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

(b) Could you get rid of the recursive transmittance term $T_r(p + t'\omega \to p')$ by expanding the recursion?