Dartmouth CS87/287 Rendering Algorithms, Fall 2025
Reading Assignment 5

Sampling with the Inversion Method

Use the inversion method to generate samples according to the normalized Beckmann distribution:

$$D(\theta,\phi) = \underbrace{\frac{1}{2\pi}}_{\text{azimuthal part}} \cdot \underbrace{\frac{2e^{-\frac{\tan^2\theta}{\alpha^2}}}{\alpha^2\cos^3\theta}}_{\text{longitudinal part}} .$$

Note how the $D(\theta,\phi)$ is symmetric around the north pole (in other words: its spherical coordinate representation is separable). Sampling can thus be split into two steps:

1. Uniformly sampling the azimuth $\phi = 2\pi\varepsilon_1$ given a uniform variate ε_1.
2. Mapping a second uniform variate ε_2 through the inverse CDF of D's longitudinal part to obtain θ.

Show the details of the necessary steps and derivations in your submission.

Hint: You might find integration by substitution useful, e.g. using the mappings $x = \cos\theta$ and $\tan^2\theta = \frac{1-x^2}{x^2}$. In addition, this identity might come in handy:

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + C \text{ where } C \in \mathbb{R}.$$