UC Berkeley EECS
Lecturer Michael Ball

Computational Structures in Data Science

Lecture 6: Recursion (cont)

Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
  - indexing
- Data structures
- Tuple assignment
- Call Expressions
- Function Definition Statement
- Conditional Statement

- Iteration:
  - data-driven (list comprehension)
  - control-driven (for statement)
  - while statement
- Higher Order Functions
  - Functions as Values
  - Functions with functions as argument
  - Assignment of function values
- Recursion
- Lambda - function valued expressions

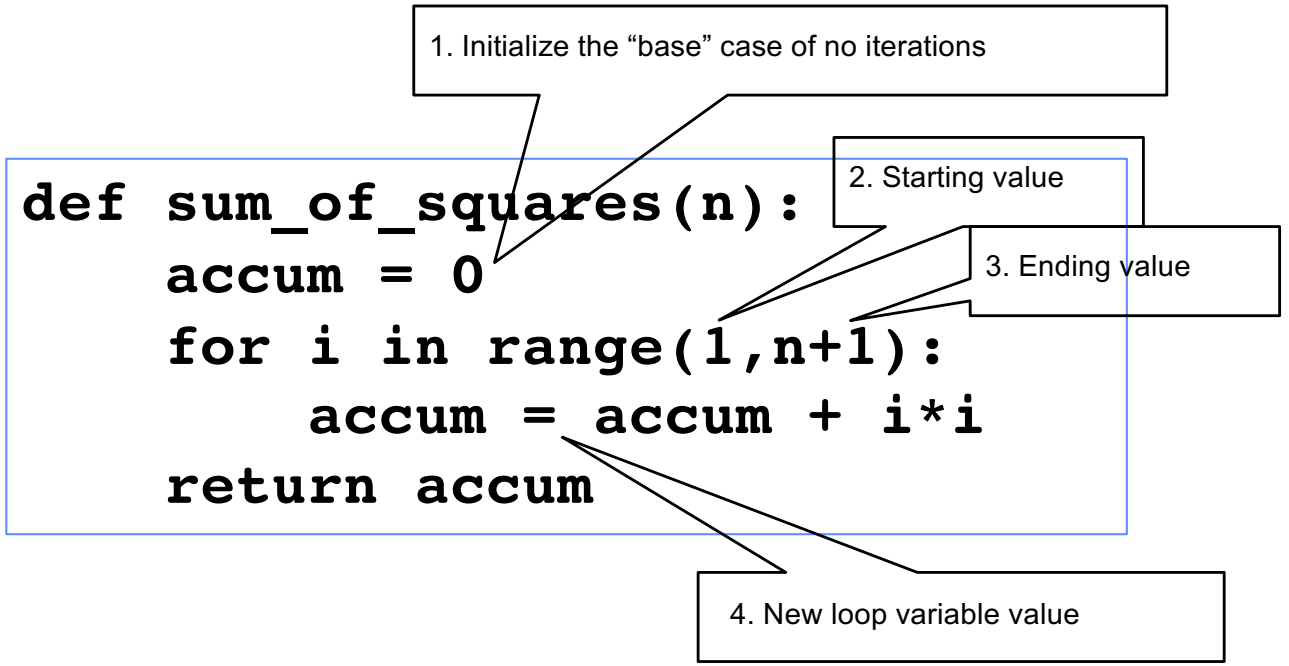
Today's Lecture

- **Recursion**
  - More practice
  - Some tips & tricks
- **Abstract Data Types**
  - More use of functions!
  - Value in documentation and clarity
  - Not on next Monday's midterm

Announcements

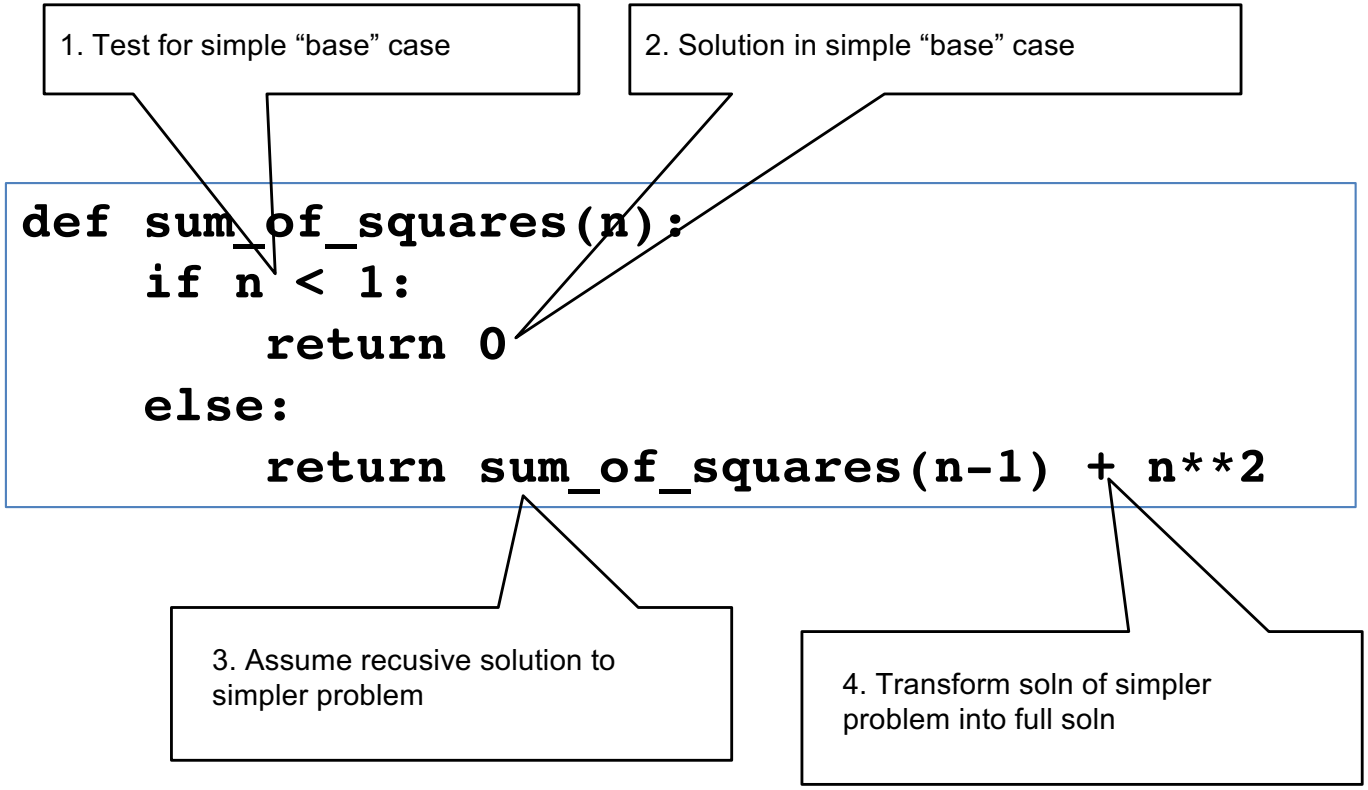
- **MIDTERM MOVED!**
- **Monday 10/21 7-9pm**
- **No Assignments Due this week**
- **Saturday October 19th 5 - 8:30 pm**

Recall: Iteration




```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

Recursion Key concepts – by example



Recursion

- **Base Case:**
  - What stops the recursion?
- **Recursive Case:**
  - Divide
  - Invoke
  - Combine2

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

Why does it work

```
sum_of_squares(3)

# sum_of_squares(3) => sum_of_squares(2) + 3**2
#                   => sum_of_squares(1) + 2**2 + 3**2
#                   => sum_of_squares(0) + 1**2 + 2**2 + 3**2
#                   => 0 + 1**2 + 2**2 + 3**2 = 14
```

How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case

- Reason about correctness “by induction”
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

Questions

- In what order do we sum the squares ?
- How does this compare to iterative approach ?

```
def sum_of_squares(n):
    accum = 0
    for i in range(1,n+1):
        accum = accum + i*i
    return accum
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return n**2 + sum_of_squares(n-1)
```

Local variables

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

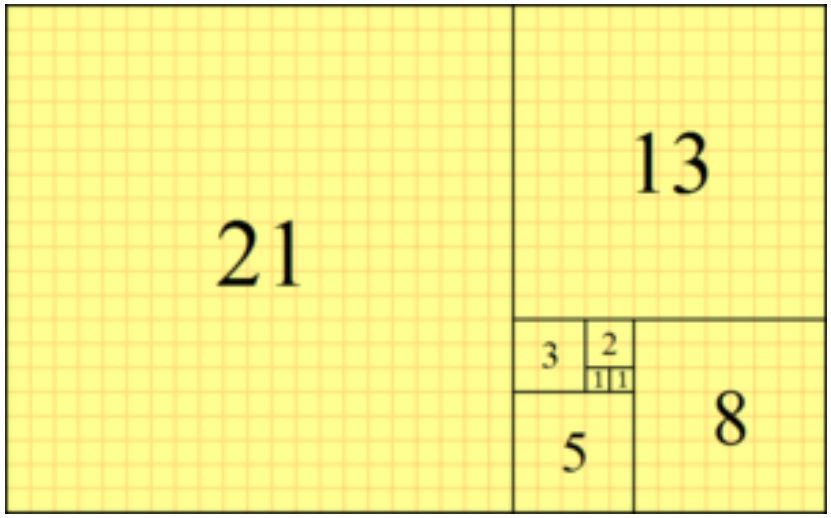
- Each call has its own "frame" of local variables
- What about globals?

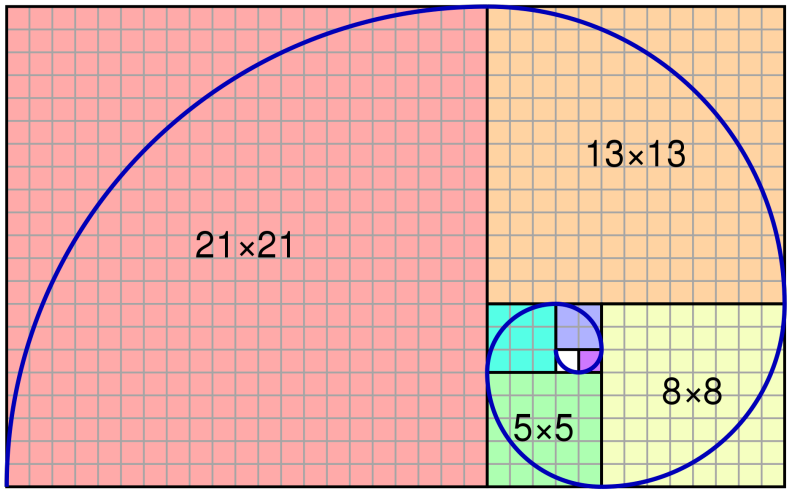
https://goo.gl/CiFaUJ

Fibonacci Sequence

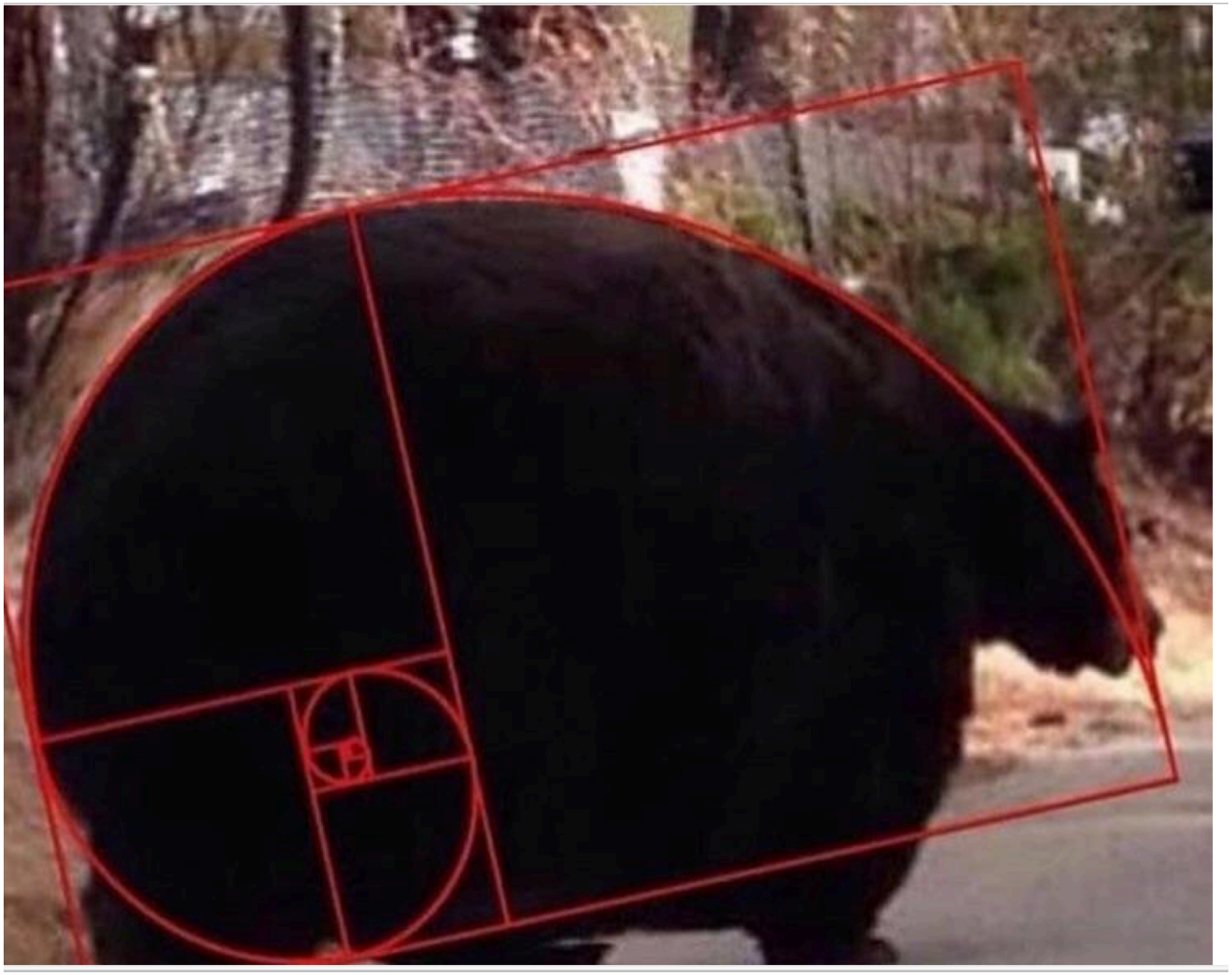
$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ….





Go Bears!



How many calls?

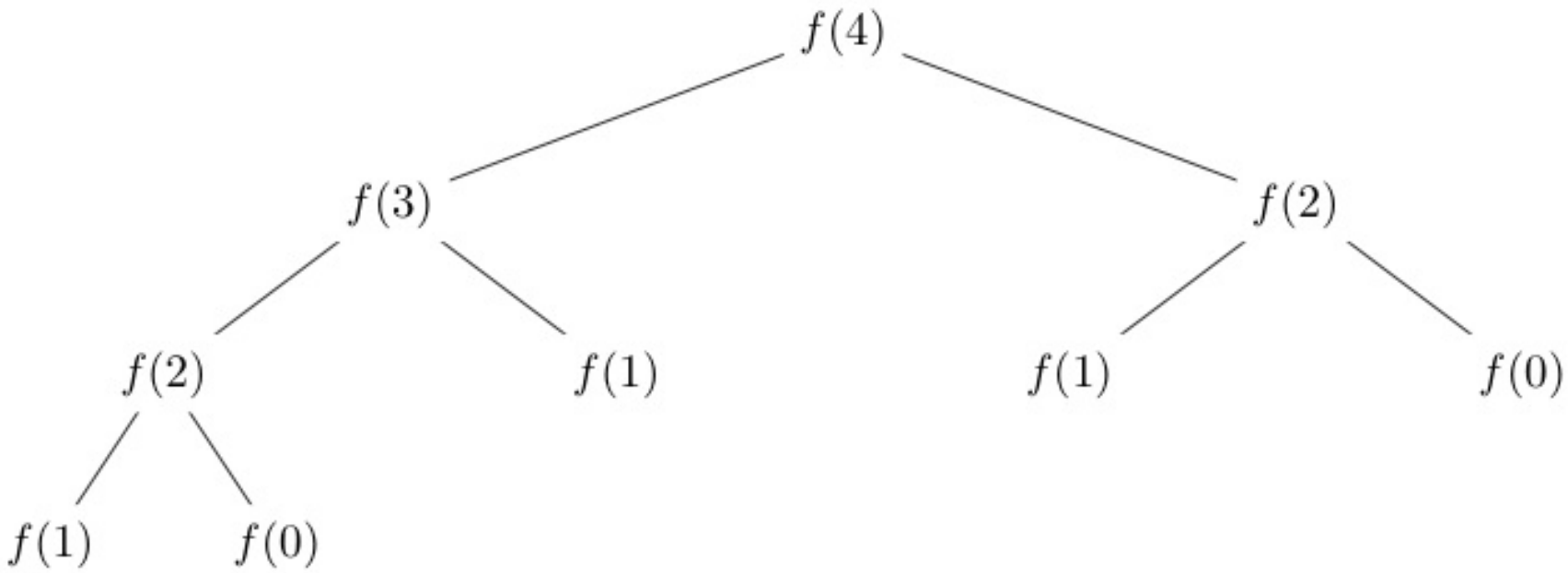
- **How many calls of fib() does executing fib(4) make?**

  E.g. Calling fib(1) makes 1 call.

- **A) 4**
- **B) 5**
- **C) 8**
- **D) 9**
- **E) 16**

Answer 9

- **Fib(4) → Fib(3), Fib(2)**
- **Fib(3) → Fib(2), Fib(1)**
- **Fib(2) → Fib(1), Fib(0)**

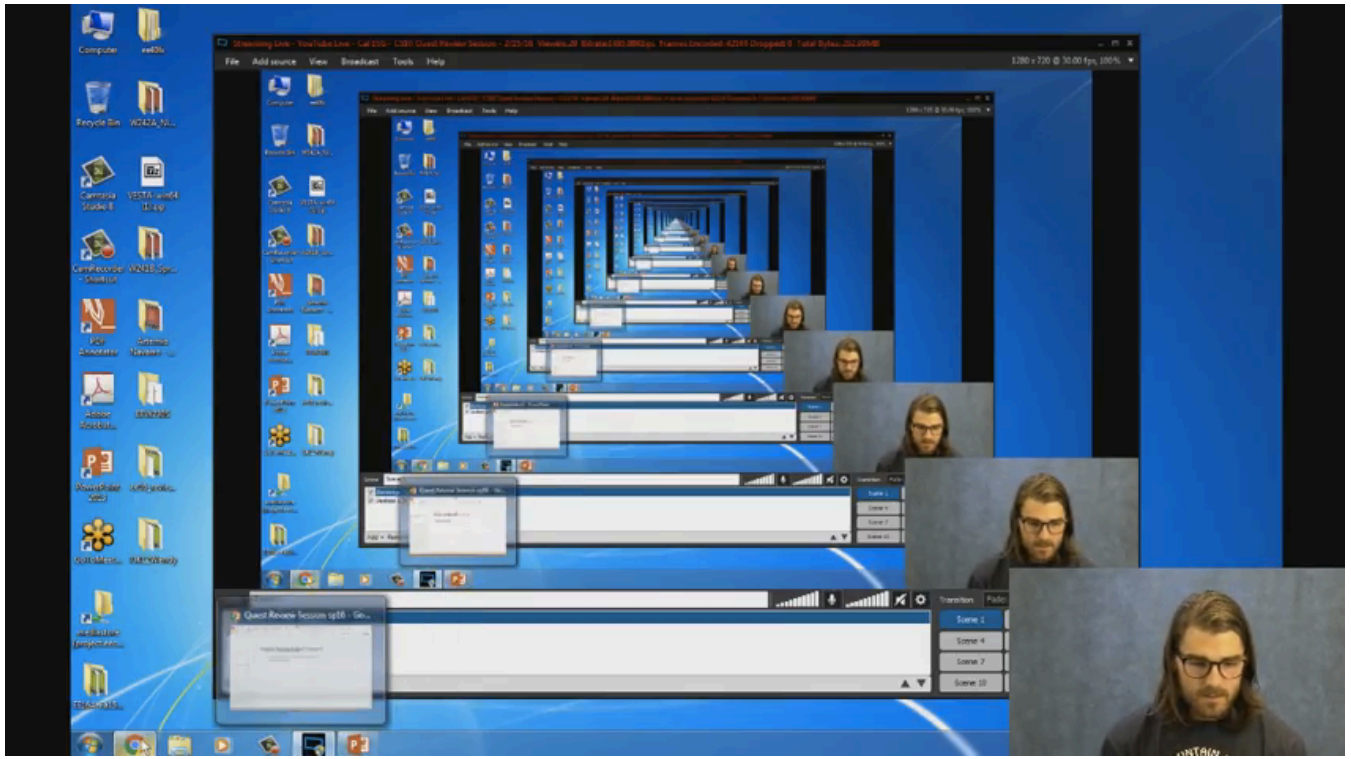


Trust …

- The recursive “leap of faith” works as long as we hit the base case eventually

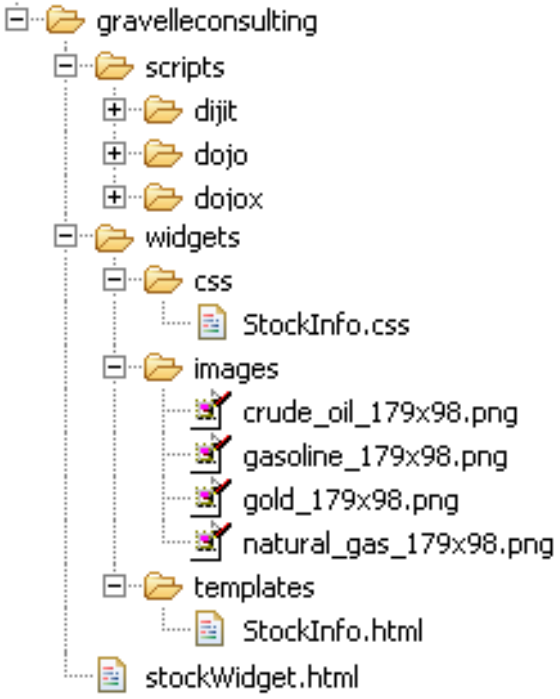
What happens if we don’t?

Recursion (unwanted)



Example I

List all items on your hard disk



- Files
- Folders contain
  - Files
  - Folders

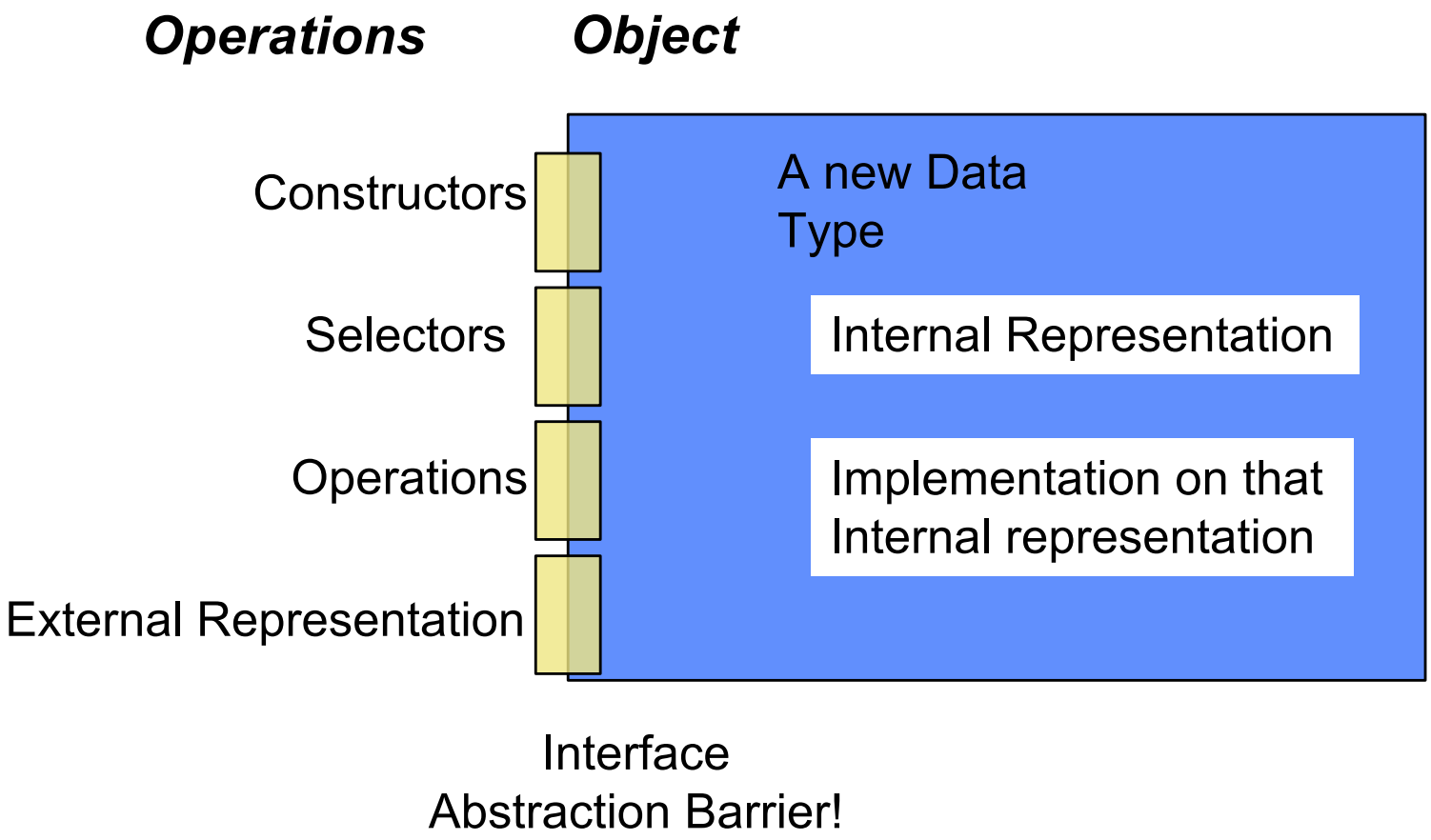
Recursion!

Extra: List Files in Python

```
def listfiles(directory):
    content = [os.path.join(directory, x) for x in os.listdir(directory)]

    dirs = sorted([x for x in content if os.path.isdir(x)])
    files = sorted([x for x in content if os.path.isfile(x)])

    for d in dirs:
          print d
          listfiles(d)

    for f in files:
          print f
```

Iterative version about twice as much code and much harder to think about.

Abstract Data Type



Why ADTs?

- **"Self-Documenting"**
  - **contact_name(contact)**
    - **» Vs contact[0]**
  - **"0" may seem clear now, but what about in a week? 3 months?**
- **Change your implementation**
  - **Maybe today it's just a Python List**
  - **Tomorrow: It could be a file on your computer; a database in web**

Examples Data Types You have seen

- **Lists**
  - Constructors:
    - `list( … )`
    - `[ <exps>,… ]`
    - `[<exp> for <var> in <list> [ if <exp> ] ]`
  - Selectors: `<list> [ <index or slice> ]`
  - Operations: `in, not in, +, *, len, min, max`
    - Mutable ones too (but not yet)
- **Tuples**
  - Constructors:
    - `tuple( … )`
    - `( <exps>,… )`
  - Selectors: `<tuple> [ <index or slice> ]`
  - Operations: `in, not in, +, *, len, min, max`

An Abstract Data Type: Key-Value Pair

- **Collection of key-Value bindings**
  - **Key : Value**
- **Many real-world examples**
  - **Dictionary, Directory, Phone book, Course Schedule, Facebook Friends, Movie listings, …**

Given some Key, What is the value associated with it?

Key-Value ADT

- **Constructors**
  - kv_empty: create an empty KV
  - kv_add: add a key:value binding to a KV
  - kv_create: create a KV from a list of key,value tuples
- **Selectors**
  - kv_items: list of (key,value) tuple in KV
  - kv_keys: list of keys in KV
  - kv_values: list of values in KV
- **Operations**
  - kv_len: number of bindings
  - kv_in: presence of a binding with a key
  - kv_display: external representation of KV

A little application

```
phone_book_data = [
    ("Christine Strauch", "510-842-9235"),
    ("Frances Catal Buloan", "932-567-3241"),
    ("Jack Chow", "617-547-0923"),
    ("Joy De Rosario", "310-912-6483"),
    ("Casey Casem", "415-432-9292"),
    ("Lydia Lu", "707-341-1254")
]


phone_book = pb_create(phone_book_data)

print("Jack Chows's Number: ", pb_get(phone_book, "Jack Chow"))

print("Area codes")
area_codes = list(map(lambda x:x[0:3], pb_numbers(phone_book)))
print(area_codes)
```

A Layered Design Process

- **Build the application based entirely on the ADT interface**
  - **Operations, Constructors and Selectors**
- **Build the operations in ADT on Constructors and Selectors**
  - **Not the implementation representation**
- **Build the constructors and selectors on some concrete representation**

Example 1

- **KV represented as list of (key, value) pairs**

Dictionaries

- Lists, Tuples, Strings, Range
- Dictionaries
  - Constructors:
    - `dict( <list of 2-tuples> )`
    - `dict( <key>=<val>, ...) # like kwargs`
    - `{ <key exp>:<val exp>, … }`
    - `{ <key>:<val> for <iteration expression> }`
  - Selectors: `<dict> [ <key> ]`
    - `<dict>.keys(), .items(), .values()`
    - `<dict>.get(key [, default] )`
  - Operations:
    - `Key in, not in, len, min, max`
    - `<dict>[ <key> ] = <val>`

Dictionary Example

```
In [1]: text = "Once upon a time"
        d = {word : len(word) for word in text.split()}
        d

Out[1]: {'Once': 4, 'a': 1, 'time': 4, 'upon': 4}

In [2]: d['Once']

Out[2]: 4

In [3]: d.items()

Out[3]: [('a', 1), ('time', 4), ('upon', 4), ('Once', 4)]

In [4]: for (k,v) in d.items():
            print(k,"=>",v)

        ('a', '=>', 1)
        ('time', '=>', 4)
        ('upon', '=>', 4)
        ('Once', '=>', 4)

In [5]: d.keys()

Out[5]: ['a', 'time', 'upon', 'Once']

In [6]: d.values()

Out[6]: [1, 4, 4, 4]
```

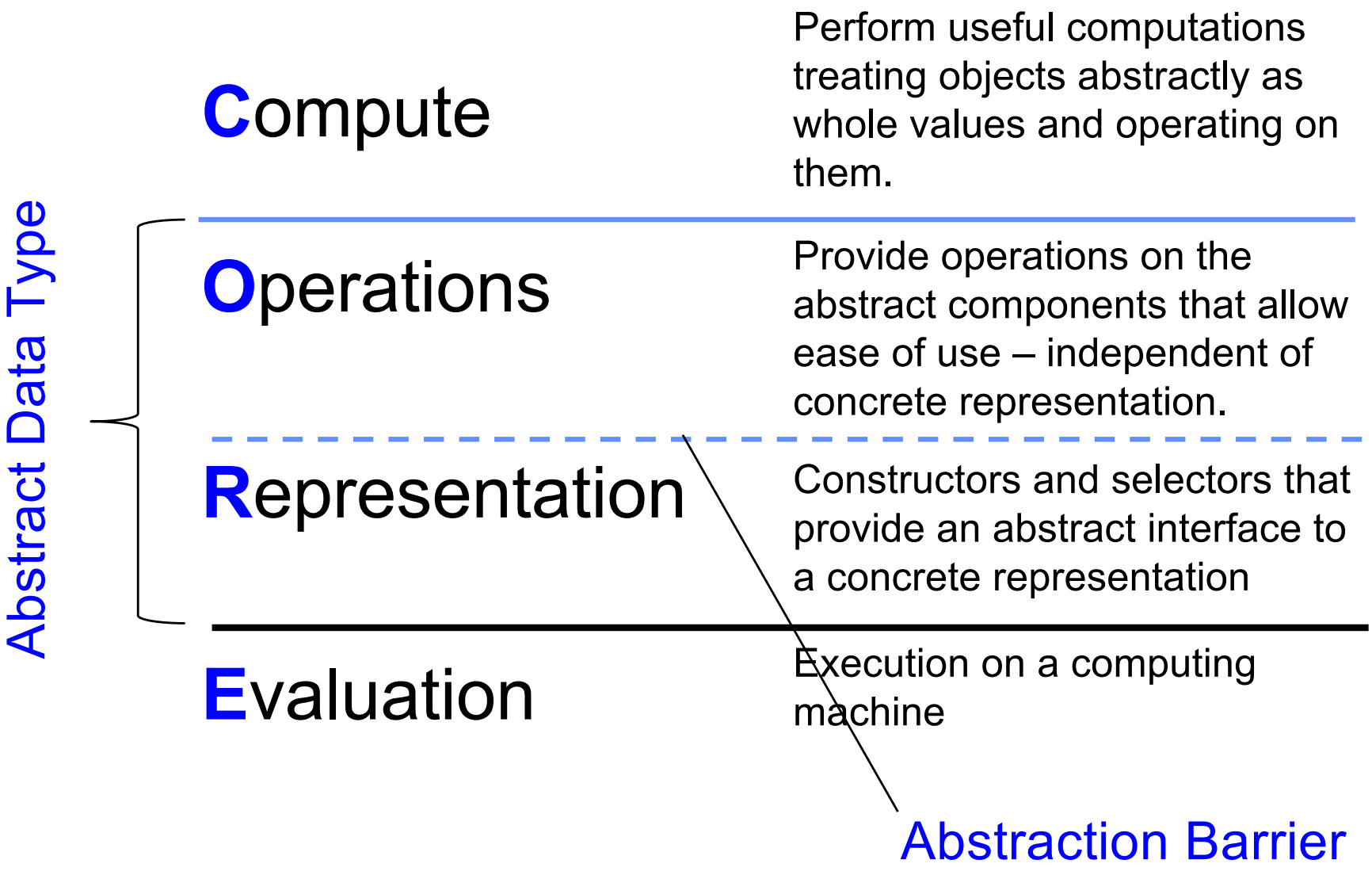
Beware

- **Built-in data type `dict` relies on mutation**
  - **Clobbers the object, rather than "functional" – creating a new one**
- **Throws an errors of key is not present**
- **We will learn about mutation shortly**

Example 3

- **KV represented as dict**

C.O.R.E concepts



Creating an Abtract Data Type

- Constructors & Selectors
- Operations
  - Express the behavior of objects, invariants, etc
  - Implemented (abstractly) in terms of Constructors and Selectors for the object
- Representation
  - Implement the structure of the object

- An *abstraction barrier violation* occurs when a part of the program that can use the higher level functions uses lower level ones instead
  - At either layer of abstraction
- Abstraction barriers make programs easier to get right, maintain, and modify
  - Few changes when representation changes

Building Apps over KV ADT

```
friend_data = [
    ("Christine Strauch", "Jack Chow"),
    ("Christine Strauch", "Lydia Lu"),
    ("Jack Chow", "Christine Strauch"),
    ("Casey Casem", "Christine Strauch"),
    ("Casey Casem", "Jack Chow"),
    ("Casey Casem", "Frances Catal Buloan"),
    ("Casey Casem", "Joy De Rosario"),
    ("Casey Casem", "Casey Casem"),
    ("Frances Catal Buloan", "Jack Chow"),
    ("Jack Chow", "Frances Catal Buloan"),
    ("Joy De Rosario", "Lydia Lu"),
    ("Joy De Lydia", "Jack Chow")
    ]
```

- **Construct a table of the friend list for each person**

Example: make_friends

```
def make_friends(friendships):
    friends = kv_empty()
    for (der, dee) in friendships:
        if not kv_in(friends, der):
            friends = kv_add(friends, der, [dee])
        else:
            der_friends = kv_get(friends, der)
            friends = kv_add(kv_delete(friends, der),
                             der, [dee] + der_friends)
    return friends
```