

INHERITANCE AND ASYMPTOTICS 8

DATA C88C

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1 Inheritance

1.1 Introduction

Python classes can implement a useful abstraction technique known as **inheritance**. To illustrate this concept, consider the following `Dog` and `Cat` classes.

```
class Dog():
    def __init__(self, name, owner):
        self.is_alive = True
        self.name = name
        self.owner = owner
    def eat(self, thing):
        print(self.name + " ate a " + str(thing) + "!")
    def talk(self):
        print(self.name + " says woof!")

class Cat():
    def __init__(self, name, owner, lives=9):
        self.is_alive = True
        self.name = name
        self.owner = owner
        self.lives = lives
    def eat(self, thing):
        print(self.name + " ate a " + str(thing) + "!")
    def talk(self):
        print(self.name + " says meow!")
```

Notice that because dogs and cats share a lot of similar qualities, there is a lot of repeated code! To avoid redefining attributes and methods for similar classes, we can write a single **superclass** from which the similar classes **inherit**. For example, we can write a class called `Pet` and redefine `Dog` as a **subclass** of `Pet`:

```
class Pet():
    def __init__(self, name, owner):
        self.is_alive = True      # It's alive!!!
        self.name = name
        self.owner = owner
    def eat(self, thing):
        print(self.name + " ate a " + str(thing) + "!")
    def talk(self):
        print(self.name)

class Dog(Pet):
    def talk(self):
        print(self.name + ' says woof!')
```

Inheritance represents a hierarchical relationship between two or more classes where one class *is a* more specific version of the other, e.g. a dog *is a* pet. Because `Dog` inherits from `Pet`, we didn't have to redefine `__init__` or `eat`. However, since we want `Dog` to talk in a way that is unique to dogs, we did **override** the `talk` method.

1.2 Questions

1. Assume these commands are entered in order. What would Python output?

```
class Foo:
    def __init__(self, a):
        self.a = a
    def garply(self):
        return self.baz(self.a)
```

```
class Bar(Foo):
    a = 1
    def baz(self, val):
        return val
```

```
>>> f = Foo(4)
>>> b = Bar(3)
>>> f.a
```

Solution: 4

```
>>> b.a
```

Solution: 3

```
>>> f.garply()
```

Solution: `AttributeError: 'Foo'object has no attribute 'baz'`

```
>>> b.garply()
```

Solution: 3

```
>>> b.a = 9
>>> b.garply()
```

Solution: 9

```
>>> f.baz = lambda val: val * val
>>> f.garply()
```

Solution: 16

2. Below is a skeleton for the `Cat` class, which inherits from the `Pet` class. To complete the implementation, override the `__init__` and `talk` methods and add a new `lose_life` method.

Hint: You can call the `__init__` method of `Pet` to set a cat's name and owner.

```
class Cat(Pet):
    def __init__(self, name, owner, lives=9):
```

Solution:

```
    Pet.__init__(self, name, owner)
    self.lives = lives
```

```
    def talk(self):
        """ Print out a cat's greeting.
        >>> Cat('Thomas', 'Tammy').talk()
        Thomas says meow!
        """
```

Solution:

```
        print(self.name + ' says meow!')
```

```
    def lose_life(self):
        """Decrements a cat's life by 1. When lives reaches
        zero, 'is_alive' becomes False.
        """
```

Solution:

```
        if self.lives > 0:
            self.lives -= 1
            if self.lives == 0:
                self.is_alive = False
        else:
            print("This cat has no more lives to lose :(")
```

[Video walkthrough](#)

3. More cats! Fill in this implementation of a class called `NoisyCat`, which is just like a normal `Cat`. However, `NoisyCat` talks a lot – twice as much as a regular `Cat`!

```
class _____: # Fill me in!
```

Solution:

```
class NoisyCat(Cat):
```

```
    """A Cat that repeats things twice."""
```

```
    def __init__(self, name, owner, lives=9):
```

```
        # Is this method necessary? Why or why not?
```

Solution:

```
        Cat.__init__(self, name, owner, lives)
```

No, this method is not necessary because NoisyCat already inherits Cat's `__init__` method

```
    def talk(self):
```

```
        """Talks twice as much as a regular cat.
```

```
        >>> NoisyCat('Magic', 'James').talk()
```

```
        Magic says meow!
```

```
        Magic says meow!
```

```
        """
```

Solution:

```
        Cat.talk(self)
```

```
        Cat.talk(self)
```

[Video walkthrough](#)

2 Asymptotics

2.1 Introduction

When we talk about the efficiency of a function, we are often interested in the following: as the size of the input grows, how does the runtime of the function change? And what do we mean by “runtime”?

- `square(1)` requires one primitive operation: `*` (multiplication). `square(100)` also requires one. No matter what input `n` we pass into `square`, it always takes one operation.

input	function call	return value	number of operations
1	<code>square(1)</code>	$1 \cdot 1$	1
2	<code>square(2)</code>	$2 \cdot 2$	1
\vdots	\vdots	\vdots	\vdots
100	<code>square(100)</code>	$100 \cdot 100$	1
\vdots	\vdots	\vdots	\vdots
n	<code>square(n)</code>	$n \cdot n$	1

- `factorial(1)` requires one multiplication, but `factorial(100)` requires 100 multiplications. As we increase the input size of `n`, the runtime (number of operations) increases linearly proportional to the input.

input	function call	return value	number of operations
1	<code>factorial(1)</code>	$1 \cdot 1$	1
2	<code>factorial(2)</code>	$2 \cdot 1 \cdot 1$	2
\vdots	\vdots	\vdots	\vdots
100	<code>factorial(100)</code>	$100 \cdot 99 \cdots 1 \cdot 1$	100
\vdots	\vdots	\vdots	\vdots
n	<code>factorial(n)</code>	$n \cdot (n - 1) \cdots 1 \cdot 1$	n

2.2 Guidelines

Here are some general guidelines for finding the order of growth for the runtime of a function:

- If the function is recursive or iterative, you can subdivide the problem as seen above:
 - Count the number of recursive calls/iterations that will be made in terms of input size n .
 - Find how much work is done per recursive call or iteration in terms of input size n .

The answer is usually the product of the above two, but be sure to pay attention to control flow!

- If the function calls helper functions that are not constant-time, you need to take the runtime of the helper functions into consideration.
- We can ignore constant factors. For example, $\Theta(1000000n) = \Theta(n)$.
- We can also ignore lower-order terms. For example, $\Theta(n^3 + n^2 + 4n + 399) = \Theta(n^3)$. This is because the n^3 term dominates as n gets larger.

2.3 Questions

1. What is the runtime of the following function?

```
def one(n):  
    if 1 == 1:  
        return None  
    return n
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: $\Theta(1)$ - the function always returns None, because $1 == 1$ is always True. And even if it was a false statement, the function would just return n . So since the runtime of the function doesn't change with respect to the size of the input, it is constant time.

2. What is the runtime of the following function?

```
def two(n):  
    for i in range(n):  
        print(n)
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: $\Theta(n)$ - the function iterates n times; if n increases by 1, the function loops 1 additional time. Therefore there is a linear relationship between the input size and runtime.

3. What is the runtime of the following function?

```
def three(n):
    while n > 0:
        n = n // 2
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: $\Theta(\log n)$ - The function continues to loop as long as $n > 0$. Inside the while loop, we divide n by 2 every loop. So to get the function to loop one additional time, we need to double our original input size. This is a logarithmic relationship between input size and runtime.

4. What is the runtime of the following function?

```
def four(n):
    for i in range(n):
        for j in range(i):
            print(str(i), str(j))
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: d. $\Theta(n^2)$ - The outer loop loops through every number from 0 to n . The inner loop loops corresponding to the outer loop. So the total number of loops from the inner loop looks like this: $0 + 1 + 2 + 3 + 4 \dots + n$. This is the summation of the first n natural numbers $= n(n + 1)/2$, which asymptotically is $\Theta(n^2)$.

5. What is the runtime of the following function?

```
def five(n):
    if n <= 0:
        return 1
    return five(n - 1) + five(n - 2)
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: e. $\Theta(2^n)$ - Draw out the tree of recursive calls. You should see that every node branches out into 2 more nodes. Since the base case returns when $n \leq 0$, and each recursive call subtracts 1 or 2 from n , the height of our tree is n . We're

branching out by a factor of 2 each layer for n layers – that means we'll have 2^n nodes in our tree of recursive calls. Each 'node' represents 1 'unit of work' as all the function does is return something. So 1 unit of work across 2^n nodes is 2^n total.

6. What is the runtime of the following function?

```
def six(n):  
    if n <= 0:  
        return 1  
    return six(n//2) + six(n//2)
```

- a. $\Theta(1)$ b. $\Theta(\log n)$ c. $\Theta(n)$ d. $\Theta(n^2)$ e. $\Theta(2^n)$

Solution: c. $\Theta(n)$ - Draw out the tree of recursive calls. You should see that every node branches out into 2 more nodes. Since the base case returns when $n \leq 0$, and each recursive call divides n by 2, the height of our tree is $\log n$ (by the same logic as `three(n)`: if we want one additional layer in our tree, our original input has to be doubled, which is a logarithmic relationship). We're branching out by a factor of 2 each layer for $\log n$ layers – that means we'll have $2^{\log n} = n$ nodes in our tree of recursive calls. Each 'node' represents 1 'unit of work' as all the function does is return something. So 1 unit of work across n nodes is n total.