

**Recursive description:**

- A **tree** has a **root label** and a list of **branches**
- Each branch is a **tree**
- A tree with zero branches is called a **leaf**

Relative description:

- Each location is a **node**
- Each **node** has a **label**
- One node can be the **parent/child** of another

```
class Tree:
```

```
    def __init__(self, label, branches=[]):
        self.label = label
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
```

```
    def is_leaf(self):
        return not self.branches
```

Built-in `isinstance` function: returns True if `branch` has a class that *is or inherits from* `Tree`

```
def leaves(tree):
    "The leaf values in a tree."
    if tree.is_leaf():
        return [tree.label]
    else:
        return sum([leaves(b) for b in tree.branches], [])
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_tree(n-2)
        right = fib_tree(n-1)
        fib_n = left.label+right.label
        return Tree(fib_n, [left, right])
```

Exponential growth. E.g., recursive `fib`
Incrementing n multiplies *time* by a constant

$$O(b^n)$$

Quadratic growth. E.g., `overlap`
Incrementing n increases *time* by n times a constant

$$O(n^2)$$

Linear growth. E.g., slow `exp`
Incrementing n increases *time* by a constant

$$O(n)$$

Logarithmic growth. E.g., `exp_fast`
Doubling n only increments *time* by a constant

$$O(\log n)$$

Constant growth. Increasing n doesn't affect *time*

$$O(1)$$

A table has columns and rows

A column has a name and a type

Latitude	Longitude	Name
38	122	Berkeley
42	71	Cambridge
45	93	Minneapolis

A row has a value for each column

```
SELECT [expression] AS [name], [expression] AS [name], ... ;
SELECT [columns] FROM [table] WHERE [condition] ORDER BY [order];
```

```
CREATE TABLE parents AS
SELECT "daisy" AS parent, "hank" AS child UNION
SELECT "ace" AS parent, "bella" AS child UNION
SELECT "ace" AS parent, "charlie" AS child UNION
SELECT "finn" AS parent, "ace" AS child UNION
SELECT "finn" AS parent, "dixie" AS child UNION
SELECT "finn" AS parent, "ginger" AS child UNION
SELECT "ellie" AS parent, "finn" AS child;
```

```
CREATE TABLE dogs AS
SELECT "ace" AS name, "long" AS fur UNION
SELECT "bella" AS name, "short" AS fur UNION
SELECT "charlie" AS name, "long" AS fur UNION
SELECT "daisy" AS name, "long" AS fur UNION
SELECT "ellie" AS name, "short" AS fur UNION
SELECT "finn" AS name, "curly" AS fur UNION
SELECT "ginger" AS name, "short" AS fur UNION
SELECT "hank" AS name, "curly" AS fur;
```

```
SELECT a.child AS first, b.child AS second
FROM parents AS a, parents AS b
WHERE a.parent = b.parent AND a.child < b.child;
```

First	Second
bella	charlie
ace	dixie
ace	ginger
daisy	ginger

```
CREATE TABLE lift AS
SELECT 101 AS chair, 2 AS single, 2 AS pair UNION
SELECT 102 AS chair, 0 AS single, 3 AS pair UNION
SELECT 103 AS chair, 4 AS single, 1 AS pair;
```

```
SELECT chair, single + 2 * pair AS total FROM lift;
```

String values can be combined to form longer strings

```
sqlite> SELECT "hello," || " world";
hello, world
```

Basic string manipulation is built into SQL

```
sqlite> CREATE TABLE phrase AS SELECT "hello, world" AS s;
sqlite> SELECT substr(s, 4, 2) || substr(s, instr(s, " ") + 1, 1)
FROM phrase;
low
```

The number of groups is the number of unique values of an expression
A **having** clause filters the set of groups that are aggregated

```
SELECT weight/legs, count(*) FROM animals
GROUP BY weight/legs
HAVING COUNT(*) > 1;
```

weight/ legs	count(*)
5	2
2	2

weight/legs=5
weight/legs=2
weight/legs=2
weight/legs=3
weight/legs=5
weight/legs=6000

kind	legs	weight
dog	4	20
cat	4	10
ferret	4	10
parrot	2	6
penguin	2	10
t-rex	2	12000

An aggregate function in the `[columns]` clause computes a value from a group of rows:

- `MAX([expression])` evaluates to the largest value of `[expression]` for any row in a group
- `COUNT(*)` evaluates to the number of rows in a group
- `MIN`, `SUM`, & `AVG` are also aggregate functions similar to `MAX`

With no `GROUP BY` clause, aggregation is performed over all rows:

```
select max(legs) from animals;
```

max(legs)
4