Name: Net ID:

ORIE 3800: Information Systems and Analysis
Spring 2019 Prelim Exam

- This exam is closed notes and closed book. You **may not** use laptops, cellphones, tablets, calculators, any other electronic devices.
- The exam contains 3 questions on pages 2-10 (single-sided). You may use the backs of the pages for extra space if you need it. Pages 11-14 are blank for scratch work if you need it. You have 90 minutes.
- Make sure you answer the questions you find easy before spending a lot of time on the difficult ones.
- A correct answer does not guarantee full credit, and a wrong answer does not guarantee loss of credit. You should clearly but concisely indicate your reasoning and **show all relevant work**. Your grade on each problem will be based on our best assessment of your level of understanding as reflected by what you have written.
- Read and sign the pledge **before** beginning the exam.

Academic integrity is expected of all students of Cornell University at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare that I shall not give, use or receive unauthorized aid in this examination.

Signature:

Question	Score	Maximum
1		14
2		18
3		18
Total		50

Question 1 [18 points]

Alice proposes the following gamble to Bob: Bob pays $\frac{3}{2}$ dollars to Alice, and in return Alice will pay Bob $\bar{\omega}$ dollars, where $\bar{\omega}$ is a number (unknown to Bob) of the form

$$\bar{\omega} = n - \frac{1}{2^k},$$

for $n \in \{1, 2, 3\}$ and $k \in \{0, 1, 2\}$, with each (n, k) pair chosen with equal probability.

The problem that Bob faces is whether or not to accept this gamble.

(a) With no additional information, should Bob accept this gamble?

NO

Reasoning/work to be looked at:

Solution: Bob's expected reward if he chooses to accept the gamble is

$$\begin{aligned}
EV(\text{accept}) &= \mathbb{E}\left[\bar{\omega} - \frac{3}{2}\right] \\
&= -\frac{3}{2} + \sum_{i=1}^{3}\sum_{j=0}^{2} \mathbb{P}(n = i, k = j)\left(n - \frac{1}{2^k}\right) \\
&= -\frac{3}{2} + \frac{1}{9}\sum_{i=1}^{3}\sum_{j=0}^{2}\left(i - \frac{1}{2^j}\right) \\
&= -\frac{3}{2} + \frac{1}{9}\left(3\sum_{i=1}^{3} i - 3\sum_{j=0}^{2}\frac{1}{2^j}\right) \\
&= -\frac{3}{2} + \frac{1}{9}\left(3(1 + 2 + 3) - 3(1 + \frac{1}{2} + \frac{1}{4})\right) \\
&= -\frac{1}{12}
\end{aligned}$$

and Bob's expected reward if he chooses to reject the gamble is $EV(\text{reject}) = 0$.
As $EV(\text{accept}) < EV(\text{reject})$, with no additional information Bob should NOT accept this gamble.
□

Common mistakes: N/A

(b) Charlie knows the number k that Alice has chosen. Represent Charlie's information as an information partition $\mathcal{I}$. What is the value of Charlie's information to Bob?

- $\mathcal{I} = \{\{(1,0),(2,0),(3,0)\},\{(1,1),(2,1),(3,1)\},\{(1,2),(2,2),(3,2)\}\}$
- Value of Information = $\frac{1}{12}$

Reasoning/work to be looked at:

Solution: Charlie knows k but not n, therefore Charlie's information partition separates different values of k into different partitions, but puts different values of n for the same k into the same partition. If Bob knows Charlie's information (i.e. the value of k), then Bob's expected reward for accepting the gamble conditioned on the value of k is,

$$\begin{aligned}
EV(\textbf{accept}|k) &= \mathbb{E}\left[\bar{\omega} - \frac{3}{2} \mid k\right] \\
&= -\frac{3}{2} + \sum_{i=1}^{3} \mathbb{P}(n = i \mid k)\left(n - \frac{1}{2^k}\right) \\
&= -\frac{3}{2} + \frac{1}{3}\sum_{i=1}^{3}\left(i - \frac{1}{2^k}\right) \\
&= -\frac{3}{2} + \frac{1}{3}\left(6 - \frac{3}{2^k}\right) \\
&= \frac{1}{2} - \frac{1}{2^k},
\end{aligned}$$

and Bob's expected reward for rejecting the gamble conditioned on the value of k is $EV(\textbf{reject}|k) = 0$. Bob's optimal action is thus to accept if the above expected reward is positive and reject otherwise. Therefore Bob's optimal strategy given the value of k is to accept if $k = 2$, reject if $k = 0$. If $k = 1$, then Bob's expected reward for accepting is zero, so he gets the same expected reward whether he accepts or rejects the gamble thus either action is optimal. Therefore an optimal strategy is

$$a^*(k) = \begin{cases} \textbf{reject} & \text{if } k \in \{0,1\} \\ \textbf{accept} & \text{if } k = 2 \end{cases}$$

The expected value of this optimal strategy is

$$\begin{aligned}
EV^{(\text{with info})} &= \mathbb{P}(k=0)EV(a^*(0)|k=0) + \mathbb{P}(k=1)EV(a^*(1)|k=1) + \mathbb{P}(k=2)EV(a^*(2)|k=2) \\
&= \frac{1}{2}(0) + \frac{1}{2}(0) + \frac{1}{3}\left(\frac{1}{2} - \frac{1}{2^2}\right) = \frac{1}{12}.
\end{aligned}$$

Without information Bob's expected value is zero because rejecting is optimal

$$EV^{(\text{no info})} = \max(EV(\textbf{accept}), EV(\textbf{reject})) = \max(-\frac{1}{12}, 0) = 0$$

The value of information is thus the difference between Bob's expected value for playing his optimal strategy if he has the information minus the expected value if he

does not have the information, which is $\frac{1}{12} - 0 = \frac{1}{12}$.
□

Common mistakes:

- not computing expectation over k (only writing expression of expected reward for a specific value of k)
- incorrectly using $EV^{(\text{no info})} = EV(\text{accept}) = -\frac{1}{12}$ rather than realizing value with no information is zero because it is optimal to always reject
- not weighting for the probability of the signal, i.e. incorrectly using $EV^{(\text{with info})} = EV(a^*(2)|k = 2) = \frac{1}{4}$

(c) Find the value of information of the **perfect** information structure

$$\mathcal{I}_{\text{perfect}} = \{\{(1,0)\},\{(2,0)\},\{(3,0)\},\{(1,1)\},\{(2,1)\},\{(3,1)\},\{(1,2)\},\{(2,2)\},\{(3,2)\}\}.$$

Give an example of an information partition $\mathcal{J}$ that is not the perfect information partition, but whose value of information is equal to that of the perfect information partition.

- Value of Perfect Information = $\frac{1}{3}$
- $\mathcal{J} = \{\{(1,0),(1,1),(1,2),(2,0),(2,1)\},\{(2,2),(3,0),(3,1),(3,2)\}\}$

Reasoning/work to be looked at:

Solution: Given perfect knowledge of n and k, Bob's optimal strategy is to accept whenever $\bar{\omega} = n - \frac{1}{2^k} > \frac{3}{2}$. Therefore, the value of perfect information is

$$\begin{aligned}
EV^{(\text{with info})} - EV^{(\text{no info})} &= \mathbb{E}\left[\left(\bar{\omega} - \frac{3}{2}\right)\mathbb{I}\left(\bar{\omega} > \frac{3}{2}\right)\right] - 0 \\
&= \sum_{i=1}^{3}\sum_{j=0}^{2}\mathbb{P}(n=i,k=j)\mathbb{I}\left(\bar{\omega} > \frac{3}{2}\right)\left(\bar{\omega} - \frac{3}{2}\right) \\
&= \frac{1}{9}\sum_{i=1}^{3}\sum_{j=0}^{2}\mathbb{I}\left(i - \frac{1}{2^j} > \frac{3}{2}\right)\left(i - \frac{1}{2^j} - \frac{3}{2}\right) \\
&= \frac{1}{9}\left(\left(2 - \frac{1}{4} - \frac{3}{2}\right) + \left(3 - 1 - \frac{3}{2}\right) + \left(3 - \frac{1}{2} - \frac{3}{2}\right) + \left(3 - \frac{1}{4} - \frac{3}{2}\right)\right) \\
&= \frac{1}{9}\left(\frac{1}{4} + \frac{1}{2} + 1 + \frac{5}{4}\right) \\
&= \frac{1}{3}
\end{aligned}$$

An information partition that directly distinguishes all pairs (n,k) for which Bob should accept the bet from all pairs for which Bob should not accept the bet will achieve same value as perfect information. Set $\mathcal{J}_1 = \{(n,k)\ s.t.\ n - \frac{1}{2^k} \leq \frac{3}{2}\}$ and set $\mathcal{J}_2 = \{(n,k)\ s.t.\ n - \frac{1}{2^k} > \frac{3}{2}\}$, i.e.

$$\mathcal{J} = \{\{(1,0),(1,1),(1,2),(2,0),(2,1)\},\{(2,2),(3,0),(3,1),(3,2)\}\}$$

□

Common mistakes:

- not computing expectation over n, k (forgetting to weight/multiply by $\mathbb{P}(n = i, k = j)$)
- incorrectly using $EV^{(\text{no info})} = EV(\text{accept}) = -\frac{1}{12}$ rather than realizing value with no information is zero because it is optimal to always reject

Question 2 [18 points]

You are given a biased coin, for which the probability of heads is $\bar{\omega}$. Your prior on $\bar{\omega}$ is

$$\mathbf{P}\left(\bar{\omega} = \frac{1}{4}\right) = \mathbf{P}\left(\bar{\omega} = \frac{3}{4}\right) = \frac{1}{2}.$$

You flip the coin N times, observing the outcomes $\bar{s}_1, \cdots, \bar{s}_N$ of each flip. In particular, we have for each i,

$$\mathbf{P}(\bar{s}_i = H | \, \bar{\omega} = \omega) = \omega,$$
$$\mathbf{P}(\bar{s}_i = T | \, \bar{\omega} = \omega) = 1 - \omega,$$

for $\omega \in \{1/4, 3/4\}$. Conditional on $\bar{\omega}$, the coin tosses $\bar{s}_1, \cdots, \bar{s}_N$ are independent.

After seeing the outcomes of these coin tosses, you will choose whether to "play" or "reject". If you choose to play, then you will flip the coin one more time, earning \$80 if it comes up heads, and losing \$80 if it comes up tails. If you reject, then you earn \$0 dollars.

(a) What is the probability that $\bar{\omega} = 1/4$ given $\bar{s}_1 = H$?

$\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = H) = \frac{1}{4}$

Reasoning/work to be looked at:

Solution: By Bayes rule,

$$\begin{aligned}\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = H) &= \frac{\mathbb{P}(\bar{s}_1 = H \mid \bar{\omega} = \frac{1}{4})\mathbb{P}(\bar{\omega} = \frac{1}{4})}{\mathbb{P}(\bar{s}_1 = H \mid \bar{\omega} = \frac{1}{4})\mathbb{P}(\bar{\omega} = \frac{1}{4}) + \mathbb{P}(\bar{s}_1 = H \mid \bar{\omega} = \frac{3}{4})\mathbb{P}(\bar{\omega} = \frac{3}{4})} \\ &= \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} \cdot \frac{1}{2}} \\ &= \frac{1}{4}\end{aligned}$$

$\square$

Common mistakes: N/A

(b) Suppose $N = 10$. You have observed 6 heads and 4 tails. What is your posterior belief about $\bar{\omega}$? (specify posterior probabilities as a function of ω)

$$\mathbb{P}(\bar{\omega} = \omega \mid \text{6 Heads and 4 Tails}) = \begin{cases} \frac{1}{10} & \text{if } \omega = \frac{1}{4} \\ \frac{9}{10} & \text{if } \omega = \frac{3}{4} \\ 0 & \text{otherwise} \end{cases}$$

Reasoning/work to be looked at:

Solution:

$$\mathbb{P}(\text{6 Heads and 4 Tails} \mid \bar{\omega} = \frac{1}{4}) = \binom{10}{6}(\frac{1}{4})^6(\frac{3}{4})^4$$
$$\mathbb{P}(\text{6 Heads and 4 Tails} \mid \bar{\omega} = \frac{3}{4}) = \binom{10}{6}(\frac{3}{4})^6(\frac{1}{4})^4$$

By Bayes rule,

$$\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \text{6 Heads and 4 Tails})$$
$$= \frac{\mathbb{P}(\text{6 Heads and 4 Tails} \mid \bar{\omega} = \frac{1}{4})\mathbb{P}(\bar{\omega} = \frac{1}{4})}{\mathbb{P}(\text{6 Heads and 4 Tails} \mid \bar{\omega} = \frac{1}{4})\mathbb{P}(\bar{\omega} = \frac{1}{4}) + \mathbb{P}(\text{6 Heads and 4 Tails} \mid \bar{\omega} = \frac{3}{4})\mathbb{P}(\bar{\omega} = \frac{3}{4})}$$
$$= \frac{\binom{10}{6}(\frac{1}{4})^6(\frac{3}{4})^4 \cdot \frac{1}{2}}{\binom{10}{6}(\frac{1}{4})^6(\frac{3}{4})^4 \cdot \frac{1}{2} + \binom{10}{6}(\frac{3}{4})^6(\frac{1}{4})^4 \cdot \frac{1}{2}}$$
$$= \frac{3^4}{3^4 + 3^6} = \frac{1}{1 + 3^2} = \frac{1}{10}$$

As $\bar{\omega}$ can only take values either $\frac{1}{4}$ or $\frac{3}{4}$ we only need to compute these two probabilities, and

$$\mathbb{P}(\bar{\omega} = \frac{3}{4} \mid \bar{s} = s) = 1 - \mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s} = s) = \frac{9}{10}$$

□

Common mistakes:

- Not applying Bayes Rule/Conditional Probability
- Not recalling binomal distribution (e.g. square values, not multiply)
- Not computing the posterior belief for specific ω
- Only calculating numerator, not denominator

(c) Suppose $N = 1$. What is your optimal strategy? Compute the expected value of the optimal strategy.

$a^*(\bar{s}_1 = H) =$ "play"
$a^*(\bar{s}_1 = T) =$ "reject"
Expected value of strategy $a^*(s)$ = 10

Reasoning/work to be looked at:

Solution: Using similar calculations as part (a), we compute the posterior probabilities,

$$\begin{aligned}
\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = H) &= \frac{1}{4} \\
\mathbb{P}(\bar{\omega} = \frac{3}{4} \mid \bar{s}_1 = H) &= \frac{3}{4} \\
\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = T) &= \frac{3}{4} \\
\mathbb{P}(\bar{\omega} = \frac{3}{4} \mid \bar{s}_1 = T) &= \frac{1}{4}
\end{aligned}$$

If we observe $\bar{s}_1 = H$, then the expected reward for playing conditioned on this observation is

$$\begin{aligned}
&(80(\frac{1}{4}) - 80(\frac{3}{4}))\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = H) + (80(\frac{3}{4}) - 80(\frac{1}{4}))\mathbb{P}(\bar{\omega} = \frac{3}{4} \mid \bar{s}_1 = H) \\
&= -40(\frac{1}{4}) + 40(\frac{3}{4}) = 20
\end{aligned}$$

Since this is larger than 0, then it is optimal to "play" if $\bar{s}_1 = H$.
If we observe $\bar{s}_1 = T$, then the expected reward for playing conditioned on this observation is

$$\begin{aligned}
&(80(\frac{1}{4}) - 80(\frac{3}{4}))\mathbb{P}(\bar{\omega} = \frac{1}{4} \mid \bar{s}_1 = T) + (80(\frac{3}{4}) - 80(\frac{1}{4}))\mathbb{P}(\bar{\omega} = \frac{3}{4} \mid \bar{s}_1 = T) \\
&= -40(\frac{3}{4}) + 40(\frac{1}{4}) = -20
\end{aligned}$$

Since this is less than 0, then it is optimal to "reject" if $\bar{s}_1 = T$.
The expected value of the optimal strategy is thus

$$\begin{aligned}
\mathbb{P}(\bar{s} = H)20 + 0 &= 20\mathbb{P}(\bar{\omega} = \frac{1}{4}, \bar{s}_1 = H) + 20\mathbb{P}(\bar{\omega} = \frac{3}{4}, \bar{s}_1 = H) \\
&= 20((\frac{1}{4})(\frac{1}{2}) + (\frac{3}{4})(\frac{1}{2})) \\
&= 10
\end{aligned}$$

□
Common mistakes:

- Calculating expected value of strategy as 20 (forgetting to consider weighting probability of H, T.
- Forgetting if expectation is negative (i.e. when $\bar{s}_1$ = H, reject, implying that value is 0, not negative)
- Only completing part of the calculation, resulting in value = 40 or 80

Question 3 [14 points]

The state $\bar{\omega}$ takes values in $\{A, B, C\}$, with each outcome equally likely:

$$\mathbf{P}(\bar{\omega} = A) = \frac{1}{3}, \quad \mathbf{P}(\bar{\omega} = B) = \frac{1}{3}, \quad \mathbf{P}(\bar{\omega} = C) = \frac{1}{3}.$$

Without knowing the value of $\bar{\omega}$, you have to choose an action $a \in \{0, 1\}$. Each action a provides a reward $r(\bar{\omega}, a)$ as follows:

$$r(\bar{\omega}, a = 0) = \begin{cases} +5 & \text{if } \bar{\omega} = A; \\ -1 & \text{if } \bar{\omega} = B; \\ +2 & \text{if } \bar{\omega} = C. \end{cases} \qquad r(\bar{\omega}, a = 1) = \begin{cases} -3 & \text{if } \bar{\omega} = A; \\ +1 & \text{if } \bar{\omega} = B; \\ +2 & \text{if } \bar{\omega} = C. \end{cases}$$

Before you make the decision, you obtain a signal $\bar{s} \in [0, 1]$ with the following conditional density:

$$\begin{aligned} f(\bar{s} = x | \bar{\omega} = A) &= 3x^2, \\ f(\bar{s} = x | \bar{\omega} = B) &= 3(1 - x)^2, \\ f(\bar{s} = x | \bar{\omega} = C) &= 6x(1 - x), \end{aligned}$$

for $x \in [0, 1]$.

(a) Without access to the signal, what is your optimal action?

$$a^* = 0$$

What is your optimal expected reward?

$$\mathbb{E}[r(\bar{\omega}, a^*)] = 2$$

Reasoning/work to be looked at:

Solution: The expected reward for choosing action $a = 0$ (with no signal) is

$$\frac{5}{3} - \frac{1}{3} + \frac{2}{3} = 2$$

The expected reward for choosing action $a = 1$ (with no signal) is

$$-\frac{3}{3} + \frac{1}{3} + \frac{2}{3} = 0$$

Therefore, the optimal action with no signal is $a = 0$. The optimal expected reward is 2.

□

Common mistakes: N/A

(b) Suppose you observe the signal $\bar{s} = \frac{1}{2}$. What is your optimal action?

$$a^*(\bar{s} = \frac{1}{2}) = 0$$

Reasoning/work to be looked at:

Solution: We need to compute the posterior probabilities $\mathbb{P}(\bar{\omega} = \omega \mid \bar{s} = \frac{1}{2})$. We use Bayes rule,

$$\mathbb{P}\left(\bar{\omega} = \omega \mid \bar{s} = \frac{1}{2}\right) = \frac{f(\bar{s} = \frac{1}{2} \mid \bar{\omega} = \omega)\mathbb{P}(\bar{\omega} = \omega)}{\sum_{\omega' \in \{A,B,C\}} f(\bar{s} = \frac{1}{2} \mid \bar{\omega} = \omega')\mathbb{P}(\bar{\omega} = \omega')}.$$

Let's compute the numerator for each value of ω.

$$\begin{aligned} f(\bar{s} = \frac{1}{2} \mid \bar{\omega} = A)\mathbb{P}(\bar{\omega} = A) &= 3\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{4} \\ f(\bar{s} = \frac{1}{2} \mid \bar{\omega} = B)\mathbb{P}(\bar{\omega} = B) &= 3\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{4} \\ f(\bar{s} = \frac{1}{2} \mid \bar{\omega} = C)\mathbb{P}(\bar{\omega} = C) &= 6\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{2}. \end{aligned}$$

The denominator is the sum of the above conditional pdf's which is 1. Therefore,

$$\mathbb{P}(\bar{\omega} = \omega \mid \bar{s} = \frac{1}{2}) = \begin{cases} \frac{1}{4} & \text{if } \omega = A \\ \frac{1}{4} & \text{if } \omega = B \\ \frac{1}{2} & \text{if } \omega = C \end{cases}.$$

Then we compute $\mathbb{E}[r(\bar{\omega}, a) \mid \bar{s} = \frac{1}{2}]$ and choose the action that maximizes the expected reward conditioned on $\bar{s} = \frac{1}{2}$.

$$\begin{aligned} \mathbb{E}\left[r(\bar{\omega}, 0) \mid \bar{s} = \frac{1}{2}\right] &= \frac{5}{4} - \frac{1}{4} + \frac{2}{2} = 2 \\ \mathbb{E}\left[r(\bar{\omega}, 1) \mid \bar{s} = \frac{1}{2}\right] &= -\frac{3}{4} + \frac{1}{4} + \frac{2}{2} = \frac{1}{2}. \end{aligned}$$

Therefore $a^*(\bar{s} = \frac{1}{2}) = 0$.

□

Common mistakes:

- Did not use Bayes rule to compute the posterior probabilities;
- Did not include / compute the denominator $f(\bar{s} = \frac{1}{2})$ in the Bayes rule;
- Computed the denominator using an integral over s.

(c) **(Bonus/Extra Credit)** Find the optimal strategy with the signal (specify for what values of $\bar{s} = s$ that you should choose either action 0 or 1).

$$a^*(s) = \begin{cases} 0 & \text{if } s > \frac{1}{3} \\ 1 & \text{otherwise} \end{cases}$$

Reasoning/work to be looked at:

Solution: By Bayes rule,

$$\mathbb{P}(\bar{\omega} = \omega \mid \bar{s} = s) \propto f(\bar{s} = s \mid \bar{\omega} = \omega)\mathbb{P}(\bar{\omega} = \omega).$$

For each value of $\omega \in \{A, B, C\}$,

$$f(\bar{s} = s \mid \bar{\omega} = A)\mathbb{P}(\bar{\omega} = A) = 3s^2 \left(\frac{1}{3}\right) = s^2$$
$$f(\bar{s} = s \mid \bar{\omega} = B)\mathbb{P}(\bar{\omega} = B) = 3(1-s)^2 \left(\frac{1}{3}\right) = (1-s)^2$$
$$f(\bar{s} = s \mid \bar{\omega} = C)\mathbb{P}(\bar{\omega} = C) = 6s(1-s) \left(\frac{1}{3}\right) = 2s(1-s).$$

Therefore the marginal density of s is the sum of these,

$$f(\bar{s} = s) = s^2 + (1-s)^2 + 2s(1-s) = (s + (1-s))^2 = 1,$$

which implies

$$\mathbb{P}(\bar{\omega} = \omega \mid \bar{s} = s) = f(\bar{s} = s \mid \bar{\omega} = \omega)\mathbb{P}(\bar{\omega} = \omega).$$

Next we compute the expected reward for choosing each action, conditioned on the observed signal,

$$\mathbb{E}[r(\bar{\omega}, 0) \mid \bar{s} = s] = 5s^2 - (1-s)^2 + 4s(1-s) = 6s - 1$$
$$\mathbb{E}[r(\bar{\omega}, 1) \mid \bar{s} = s] = -3s^2 + (1-s)^2 + 4s(1-s) = -6s^2 + 2s + 1.$$

Since $s \in [0, 1]$, action 0 is better than action 1 in expectation iff

$$6s - 1 > -6s^2 + 2s + 1 \Rightarrow s > \frac{1}{3}.$$

Therefore the optimal strategy is

$$a^*(s) = \begin{cases} 0 & \text{if } s > \frac{1}{3} \\ 1 & \text{otherwise} \end{cases}.$$

□

Common mistakes:

- Did not use Bayes rule to compute the posterior probabilities;
- Did not include / compute the denominator $f(\bar{s} = s)$ in the Bayes rule;
- Computed the denominator using an integral over s;
- Failed to compare the expected rewards of action $a = 0, 1$ / picked a single action for all s.