Appendix C

C/C++
Notes

Computer Number Systems

NOTE C.1

Number Systems For Computer Use And Conversions Between Them

"Never trust a man who can count to 1023 on his fingers"

Hexadecimal, Decimal, Octal, and Binary

The most common number systems used in computer environments are as follows:

System	Base/Radix	Character Set
HEXADECIMAL (HEX)	16	0-9, A-F/a-f
DECIMAL	10	0-9
OCTAL	8	0-7
BINARY	2	0-1

The terms *base* and *radix* refer to the number of unique digits available. For example, decimal numbers are expressed using combinations of the ten digits 0-9. Hex, on the other hand, uses 16 digits where the first six letters of the alphabet are used in addition to 0-9. To avoid confusion when working with multiple bases, numbers are commonly written with a subscript denoting their base. For example, 456.7_{10} is decimal while 456.7_{16} is hex.

The C and C++ programming languages permit numbers to be written in all of these systems except binary. The following table shows the equivalence of the first 16 numbers in all four systems:

Decimal	**Hexadecimal**	**Octal**	**Binary**
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
10	A	12	1010
11	B	13	1011
12	C	14	1100
13	D	15	1101
14	E	16	1110
15	F	17	1111

NOTE C.2

Converting Between Hex/Octal/Binary

The hex, octal, and binary bases are all powers of 2, that is, 2^4, 2^3, and 2^1 respectively. This results in easy conversions between them using regrouping techniques.

Octal or Hex to Binary

Any octal or hex digit can be represented by a unique combination of 3 or 4 binary bits, respectively. Substitute this combination in place of each digit in the number to create its binary equivalent. Be sure to always use 3 bits for octal and 4 for hex, padding with 0s when necessary. Some examples are:

$\text{abcd.ef}_{16} = 1010\ 1011\ 1100\ 1101\ .\ 1110\ 1111_2$

$7510.16_{16} = 0111\ 0101\ 0001\ 0000\ .\ 0001\ 0110_2$

$7510.16_8 = 111\ 101\ 001\ 000\ .\ 001\ 110_2$

Binary to Octal or Hex

Group the binary number into groups of 3 bits for octal and 4 bits for hex, starting at the radix point. Substitute the equivalent octal or hex digit in place of each binary group, as in:

$11011001100001.0011001100_2 = \mathit{00}11\ 0110\ 0110\ 0001\ .\ 0011\ 0011\ 00\mathit{00}_2 = 3661.330_{16}$
$= \mathit{0}11\ 011\ 001\ 100\ 001\ .\ 001\ 100\ 110\ 0\mathit{00}_2 = 33141.1460_8$

Hex to Octal

Convert the hex number to binary (in 4 bit groups). Regroup the binary into 3 bit groups and convert those groups to their octal equivalents. For example:

$\text{abcd.ef}_{16} = 1010\ 1011\ 1100\ 1101\ .\ 1110\ 1111_2$
$= \mathit{00}1\ 010\ 101\ 111\ 001\ 101\ .\ 111\ 011\ 11\mathit{0}_2$
$= 125715.736_8$

Octal to Hex

Convert the octal number to binary (in 3 bit groups). Regroup the binary into 4 bit groups and convert those groups to their hex equivalents. For example:

$12345670.012_8 = \mathit{00}1\ 010\ 011\ 100\ 101\ 110\ 111\ 000\ .\ 000\ 001\ 01\mathit{0}_2$
$= \mathit{00}10\ 1001\ 1100\ 1011\ 1011\ 1000\ .\ 0000\ 0101_2$
$= 29\text{cbb}8.05_{16}$

NOTE C.3

Converting A Number To Decimal

Hex, decimal, octal, and binary are all known as *positional* or *weighted* number systems meaning that there is a specific value associated with each digit position in the number, normally some power of the base. The decimal equivalent of any such number is obtained by merely evaluating each position value, multiplying by the value of the digit occupying that position, then adding the results from all positions. All math is performed in decimal. The following examples illustrate some conversions:

$$\begin{array}{lclclclcl}
456.7_{10} & = & (4 * 10^{2}) & + & (5 * 10^{1}) & + & (6 * 10^{0}) & + & (7 * 10^{-1}) \\
 & = & 4 * 100 & + & 5 * 10 & + & 6 * 1 & + & 7 * .1 \\
 & = & 400 & + & 50 & + & 6 & + & .7 \\
 & = & 456.7_{10} & & & & & &
\end{array}$$

$$\begin{array}{lclclclcl}
456.7_{16} & = & (4 * 16^{2}) & + & (5 * 16^{1}) & + & (6 * 16^{0}) & + & (7 * 16^{-1}) \\
 & = & 4 * 256 & + & 5 * 16 & + & 6 * 1 & + & 7 * .0625 \\
 & = & 1024 & + & 80 & + & 6 & + & .4375 \\
 & = & 1110.4375_{10} & & & & & &
\end{array}$$

$$\begin{array}{lclclclcl}
456.7_{8} & = & (4 * 8^{2}) & + & (5 * 8^{1}) & + & (6 * 8^{0}) & + & (7 * 8^{-1}) \\
 & = & 4 * 64 & + & 5 * 8 & + & 6 * 1 & + & 7 * .125 \\
 & = & 256 & + & 40 & + & 6 & + & .875 \\
 & = & 302.875_{10} & & & & & &
\end{array}$$

$$\begin{array}{lclclclcl}
101.1_{2} & = & (1 * 2^{2}) & + & (0 * 2^{1}) & + & (1 * 2^{0}) & + & (1 * 2^{-1}) \\
 & = & 1 * 4 & + & 0 * 2 & + & 1 * 1 & + & 1 * .5 \\
 & = & 4 & + & 0 & + & 1 & + & .5 \\
 & = & 5.5_{10} & & & & & &
\end{array}$$

$$\begin{array}{lclclclcl}
abc.d_{16} & = & (a * 16^{2}) & + & (b * 16^{1}) & + & (c * 16^{0}) & + & (d * 16^{-1}) \\
 & = & (10 * 16^{2}) & + & (11 * 16^{1}) & + & (12 * 16^{0}) & + & (13 * 16^{-1}) \\
 & = & 10 * 256 & + & 11 * 16 & + & 12 * 1 & + & 13 * .0625 \\
 & = & 2560 & + & 176 & + & 12 & + & .8125 \\
 & = & 2748.8125_{10} & & & & & &
\end{array}$$

Converting A Decimal Number To Another Base

To convert a decimal number to another base, a series of divisions by that base must be performed on the integral part and a series of multiplications by that base must be performed on the fractional part. These two operations are performed separately, then the results of each are combined to form the converted number. All math is performed in decimal.

Example: Convert 61.61_{10} to binary

NOTE C.4

A. The integral part, $61._{10}$

1. Place the integral part of the number to be converted in the upper left position in the table calling it the dividend. Then place the desired base in the upper right, calling it the divisor.

Dividend	61	2	Divisor

2. Divide the dividend by the divisor. Place the quotient on the left side of the table below the dividend and place the remainder on the right side of the table beside the quotient.

Dividend	61	2	Divisor
Quotient	30	1	Remainder

3. The quotient then becomes the dividend for the next division, (however, the divisor remains the same). Repeat steps 1 and 2 until the quotient becomes zero.

	61	2	Divisor
Dividend	30	1	
Quotient	15	0	Remainder

	61	2	Divisor
	30	1	
Dividend	15	0	
Quotient	7	1	Remainder

	61	2	Divisor
	30	1	
	15	0	
Dividend	7	1	
Quotient	3	1	Remainder

	61	2	Divisor
	30	1	
	15	0	
	7	1	
Dividend	3	1	
Quotient	1	1	Remainder

	61	2	Divisor
	30	1	
	15	0	
	7	1	
	3	1	
Dividend	1	1	
Quotient	0	1	Remainder

4. The remainder found as a result of the very first division goes next to the radix point in the new number (that is, it is the LSD - least significant digit) and the rest of the remainders follow in order. So, the integral part of the new number is: $\mathbf{111101_2}$

NOTE C.5

B. The fractional part, $.61_{10}$

1. Place the fractional part of the number to be converted in the upper right position in the table, calling it the multiplicand. Then place the desired base in the upper left, calling it the multiplier.

Multiplier	2	.61	Multiplicand

2. Multiply the multiplicand by the multiplier. Place the fractional part of the product on the right side of the table below the multiplicand and place the integral part on the left side of the table beside the product.

Multiplier	2	.61	Multiplicand
Int Prod	1	.22	Fract Prod

3. The fractional part becomes the multiplicand for the next multiplication, (however, the multiplier remains the same). Repeat steps 1 and 2 until the fractional part becomes zero or until the desired fractional precision is obtained. This second condition is necessary because most decimal fractions cannot be exactly represented in a power of 2 base like binary, octal, and hex, and as a result, the fractional product will never become zero.

Multiplier	2	.61	
	1	.22	Multiplicand
Int Prod	0	.44	Fract Prod

Multiplier	2	.61	
	1	.22	
	0	.44	Multiplicand
Int Prod	0	.88	Fract Prod

Multiplier	2	.61	
	1	.22	
	0	.44	
	0	.88	Multiplicand
Int Prod	1	.76	Fract Prod

Multiplier	2	.61	
	1	.22	
	0	.44	
	0	.88	
	1	.76	Multiplicand
Int Prod	1	.52	Fract Prod

STOP

4. The integral product found as a result of the very first multiplication goes next to the radix point in the new number as was the case when the integral conversion of the original number was found. In this case, however, it is not the LSD but rather the MSD of the fractional part. So, the fractional part of the new number is: **approximately** $\mathbf{.10011_2}$

C. The entire converted number may now be found by concatenating the results of the previous two procedures:

$\mathbf{61.61_{10}}$ **approximately equals** $\mathbf{111101.10011_2}$