"will eval to"
or
"has type"

$$e ::= n \mid \texttt{true} \mid \texttt{false}$$
$$\mid (\texttt{let } (x\ e)\ e)$$
$$\mid (\texttt{+ } e\ e) \mid (\texttt{< } e\ e)$$
$$\mid (\texttt{if } e\ e\ e)$$

$$n ::= \text{number literals}$$
$$x ::= \text{variable names}$$
$$\tau ::= \text{Num} \mid \text{Bool}$$
$$\Gamma ::= \{x : \tau, \cdots\}$$

$\rightarrow \mid \tau \cup \tau$

$5 : \text{Num}$

$\text{true} : \text{Bool}$

$\text{false} : \text{Bool}$

$3 : \text{Num}$

TR-NUM $\quad \Gamma \vdash n : \text{Num}$ $\qquad$ TR-TRUE $\quad \Gamma \vdash \texttt{true} : \text{Bool}$ $\qquad$ TR-FALSE $\quad \texttt{false} : \text{Bool}$

general rule for typing number expressions

$(\texttt{+ } 3\ 7) : \underline{\text{Num}}$ $\qquad$ because $3 : \text{Num}$ and $7 : \text{Num}$ and + produces Num when that's true

$(\texttt{+ } \text{true}\ \text{false}) : \underline{\text{HAS NO TYPE!}}$

$(\texttt{< } 3\ 7) : \underline{\text{Bool}}$

$(\texttt{+ } (\texttt{+ } 3\ 7)\ 4) : \underline{\text{Num}}$ $\qquad$ because $(\texttt{+ } 3\ 7) : \text{Num}$ and $4 : \text{Num}$ and + produces Num ....

TR-PLUS $\quad \dfrac{\Gamma \vdash e_1 : \text{Num} \qquad \Gamma \vdash e_2 : \text{Num}}{\Gamma \vdash (\texttt{+ } e_1\ e_2) : \boxed{\text{Num}}}$ $\qquad$ TR-LESS $\quad \dfrac{\Gamma \vdash e_1 : \text{Num} \qquad \Gamma \vdash e_2 : \text{Num}}{\Gamma \vdash (\texttt{< } e_1\ e_2) : \boxed{\text{Bool}}}$

$(e_1 : \text{Num}) \wedge (e_2 : \text{Num}) \rightarrow (\texttt{+ } e_1\ e_2) : \text{Num}$

TR-Plus $\quad$ TR-Less $\quad \dfrac{\dfrac{3 : \text{Num} \qquad 7 : \text{Num}}{(\texttt{+ } 3\ 7) : \text{Num}} \qquad 4 : \text{Num} \quad \checkmark \text{TR-Num}}{(\texttt{< } (\texttt{+ } 3\ 7)\ 4) : \underline{\text{Bool}}}$

"look up a in env Γ"

$$\frac{\Gamma[a] = \tau}{\Gamma \vdash a : \underline{\tau}}$$

$\overset{\curvearrowright}{\text{"in environment Gamma"}}$

$$\frac{\Gamma[a] = Num}{\Gamma \vdash a : \underline{Num}}$$

$$\frac{\Gamma[a] = Bool}{\Gamma \vdash a : \underline{Bool}}$$

$\Gamma := \{x : \tau, \cdots\}$

$\Gamma[x:\tau]$ A
$\Gamma, (x,\tau)$ B
$(x,\tau) :: \Gamma$ C

"add x env Γ"

TR-ID
$$\frac{\boxed{}}{\Gamma \vdash x : \boxed{}}$$

TR-LET
$$\frac{\boxed{\Gamma \vdash e_1 : \tau \qquad (x,\tau) :: \Gamma \vdash e_2 : \tau_2}}{\Gamma \vdash (\texttt{let } (x\ e_1)\ e_2) : \boxed{\tau_2}}$$

$(\texttt{let } (x\ 5)\ (< 5\ 7))$

T-If
$$\boxed{\frac{\Gamma \vdash e_1 : Bool \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash (\texttt{if } e_1\ e_2\ e_3) : \tau}}$$

FOR SOMEONE ELSE'S IF

TR-IFA $\dfrac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma \vdash e_2 : \tau_2 \qquad \Gamma \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{if } e_1\ e_2\ e_3) : \tau_1}$

TR-IFB $\dfrac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : \tau_2 \qquad \Gamma \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{if } e_1\ e_2\ e_3) : \underline{\tau_2}}$

TR-IFC $\dfrac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : \tau_2 \qquad \Gamma \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{if } e_1\ e_2\ e_3) : \underline{\tau_3}}$

TR-IFD $\dfrac{\boxed{\Gamma \vdash e_1 : \tau_1} \qquad \Gamma \vdash e_2 : \tau_2 \qquad \Gamma \vdash e_3 : \tau_3}{\Gamma \vdash (\texttt{if } e_1\ e_2\ e_3) : \tau_2}$

TR-IfE: None of the above

$\overbrace{}^{Num}$

$(\texttt{let } (a\ (\texttt{if } (< 1\ 2)\ 3\ 4))$
$\quad a)$

— Give result $\tau_2 \cup \tau_3$

— Force $\tau_2 = \tau_3$

— Special rule for true/false

$$\frac{\Gamma \vdash e_1 : \tau_1, \quad \Gamma \vdash e_2 : \tau_2}{\Gamma \vdash (\texttt{if false } e_1\ e_2) : \tau_2}$$

Typed Racket

Sam Tobin-Hochstadt

$$\boxed{\Gamma \vdash e : \tau}$$

Occurrence Typing
If-splitting
Flow Typing

TR-Id $\dfrac{(x, Num) :: \{\}\,[x] = Num}{(x, Num) :: \{\} \vdash x : Num}$ $7 : Num$ ✓TR-Num

$\boxed{\text{TR·Less}}$

TR-Num ✓

$5 : Num$ $(x, Num) :: \{\} \vdash (< x\ 7) : Bool$

TR-Let $\dfrac{\qquad\qquad\qquad\qquad\qquad}{\{\} \vdash (let\ (x\ 5)\ (< x\ 7)) : \underline{Bool}}$

input
/ \
"true"   "....number lit...."
"false"

$\dfrac{\qquad\qquad\qquad}{(input, Num) :: \{\} \vdash (+\ input\ 1) : \underline{\qquad\qquad}}$

$\boxed{(if\ input\ 3\ false)}$ Works w/o types

Can't work with these types