Assignment 3 (Due: Jan. 04, 2026)

1. (**Math**) Nonlinear least-squares. Suppose that $\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})\right): \mathbb{R}^n \to \mathbb{R}^m$, $\mathbf{x} \in \mathbb{R}^n, \mathbf{f} \in \mathbb{R}^m$ and some $f_i(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}$ is a (are) non-linear function(s). Then, the problem,

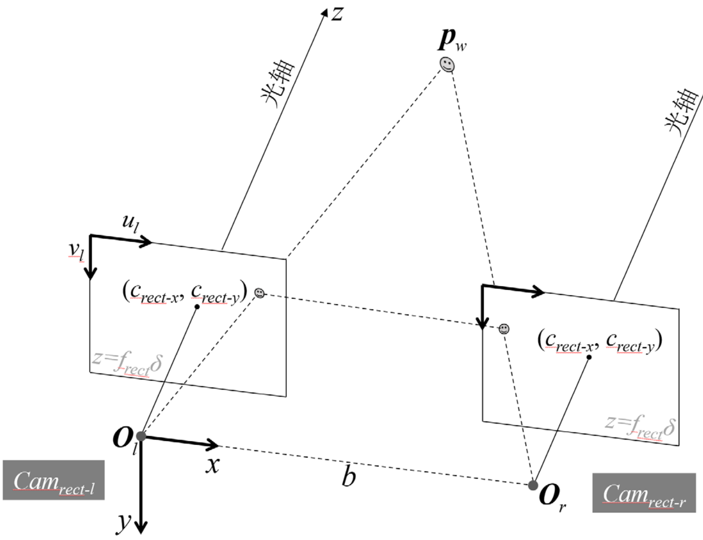
$$\mathbf{x}^* = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{f}(\mathbf{x})\|_2^2 = \arg\min_{\mathbf{x}} \frac{1}{2}(\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x})$$

is a nonlinear least-squares problem. In our lecture, we mentioned that Levenberg-Marquardt algorithm is a typical method to solve this problem. In L-M algorithm, for each updating step, at the current **x**, a local approximation model is constructed as,

$$\begin{aligned} L(\mathbf{h}) &= \frac{1}{2}\left(\mathbf{f}\left(\mathbf{x}+\mathbf{h}\right)\right)^T \mathbf{f}\left(\mathbf{x}+\mathbf{h}\right) + \frac{1}{2}\mu\mathbf{h}^T\mathbf{h} \\ &= \frac{1}{2}(\mathbf{f}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T \mathbf{f}(\mathbf{x}) + \frac{1}{2}\mathbf{h}^T(\mathbf{J}(\mathbf{x}))^T \mathbf{J}(\mathbf{x})\mathbf{h} + \frac{1}{2}\mu\mathbf{h}^T\mathbf{h} \end{aligned}$$

where **J**(**x**) is **f**(**x**)'s Jacobian matrix, and $\mu > 0$ is the damped coefficient. Please prove that L(**h**) is a strictly convex function. (Hint: If a function L(**h**) is differentiable up to at least second order, L is strictly convex if its Hessian matrix is positive definite.)

2. (**Math**) Rectified binocular system.



- f_{rect} is the focal length (unit: *pixel*) of the intrinsics matrix
- δ is the physical length of each pixel (unit: *mm/pixel*)
- $(c_{rect\text{-}x}, c_{rect\text{-}y})$ is the position of the principal point on the imaging plane (unit: *pixel*)
- b (unit: *mm*) is the distance between the two camera centers

For the rectified binocular system, the distortion coefficient vectors for the two cameras are **0**s

- Rectified binocular system is a "**virtual**" system; it can simplify the depth estimation
- By moving $\boldsymbol{O}_l$-xyz b(*mm*) along the x-axis, we get the $\boldsymbol{O}_r$-xyz coordinate system
- $Cam_{rect\text{-}l}$ and $Cam_{rect\text{-}r}$ have the same intrinsics matrix,

$$\boldsymbol{K}_{rect} = \begin{bmatrix} f_{rect} & 0 & c_{rect-x} \\ 0 & f_{rect} & c_{rect-y} \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The two images $\boldsymbol{I}_{rect\text{-}l}$ and $\boldsymbol{I}_{rect\text{-}r}$ of a rectified binocular system are row-aligned. That means, if $\boldsymbol{u}_l(u_l, v_l)$ and $\boldsymbol{u}_r(u_r, v_r)$ are the images of the same spatial point on $\boldsymbol{I}_{rect\text{-}l}$ and $\boldsymbol{I}_{rect\text{-}r}$, we should have $v_l=v_r$

In the lecture, we have introduced the rectified binocular system. Please prove that in such an ideal binocular system, the left and right two images are row-aligned.

3. (**Experiment**) Instant-NGP. Please refer to the files on the course website.

4. (**Experiment**) 3D face scan and editing. Please refer to the files on the course website.