Assignment 1 (Due: Nov. 9, 2025)
(Send your solutions to TA: 13893138834@163.com)

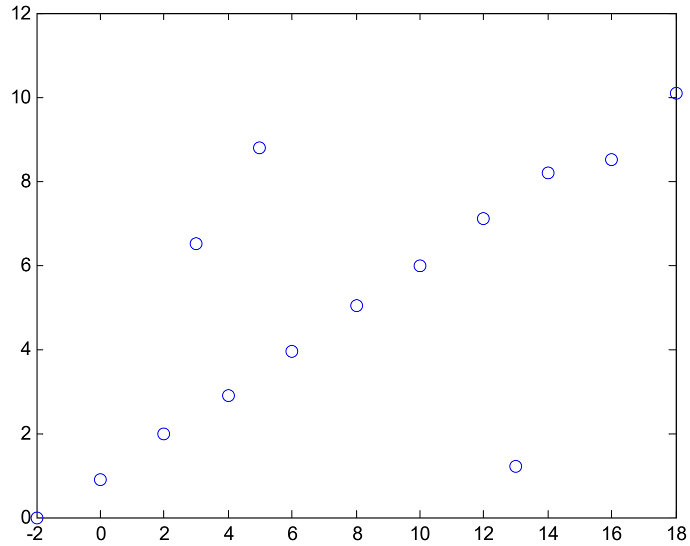
1. (**Math**) In our lectures, we mentioned that matrices that can represent Euclidean transformations can form a group. Specifically, in 3D space, the set comprising matrices $\{M_i\}$ is actually a group, where
$M_i = \begin{bmatrix} R_i & \mathbf{t}_i \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$, $R_i \in \mathbb{R}^{3\times 3}$ is an orthonormal matrix, det($\mathbf{R}_i$)=1, and $\mathbf{t}_i \in \mathbb{R}^{3\times 1}$ is a vector.
Please prove that the set $\{M_i\}$ forms a group.
Hint: You need to prove that $\{M_i\}$ satisfies the four properties of a group, i.e., the closure, the associativity, the existence of an identity element, and the existence of an inverse element for each group element.

2. (**Math**) When deriving the Harris corner detector, we get the following matrix M composed of first-order partial derivatives in a local image patch w,
$$M = \begin{bmatrix} \sum\limits_{(x_i,y_i)\in w} (I_x)^2 & \sum\limits_{(x_i,y_i)\in w} (I_x I_y) \\ \sum\limits_{(x_i,y_i)\in w} (I_x I_y) & \sum\limits_{(x_i,y_i)\in w} (I_y)^2 \end{bmatrix}$$
   a) Please prove that M is positive semi-definite.
   b) In practice, M is usually positive definite. If M is positive definite, prove that in the Cartesian coordinate system, $[x, y] M \begin{bmatrix} x \\ y \end{bmatrix} = 1$ represents an ellipse.
   c) Suppose that M is positive definite and its two eigen-values are λ_1 and λ_2 and $\lambda_1 > \lambda_2 > 0$. For the ellipse defined by $[x, y] M \begin{bmatrix} x \\ y \end{bmatrix} = 1$, prove that the length of its semi-major axis is $\frac{1}{\sqrt{\lambda_2}}$ while the length of its semi-minor axis is $\frac{1}{\sqrt{\lambda_1}}$.

3. (**Math**) In the lecture, we talked about the least square method to solve an over-determined linear system $A\mathbf{x} = b, A \in \mathbb{R}^{m\times n}, \mathbf{x} \in \mathbb{R}^{n\times 1}, m > n, rank(A) = n$. The closed form solution is $\mathbf{x} = \left(A^T A\right)^{-1} A^T b$. Try to prove that $A^T A$ is non-singular (or in other words, it is invertible).

4. (**Programming**) RANSAC is widely used in fitting models from sample points with outliers. Please implement a program to fit a straight 2D line using RANSAC

from the following sample points:
(-2, 0), (0, 0.9), (2, 2.0), (3, 6.5), (4, 2.9), (5, 8.8), (6, 3.95), (8, 5.03), (10, 5.97), (12, 7.1), (13, 1.2), (14, 8.2), (16, 8.5) (18, 10.1). Please show your result graphically.



5. (**Programming**) Get two images I_1 and I_2 of our campus and make sure that the major parts of I_1 and I_2 are from the same physical plane. Stitch I_1 and I_2 together to get a panorama view. There are additional requirements:
   1) The program should be implemented with C++ and OpenCV;
   2) For key point detection and matching, ORB features should be used;
   3) For solving linear equation systems, Moore-Penrose inverse should be explicitly used.