

CS202 HOMEWORK 3

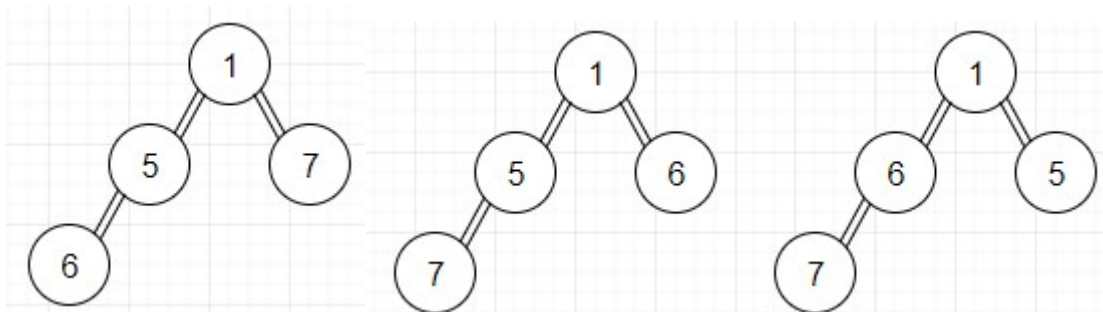
Ömer Fatih Çelik

21601387

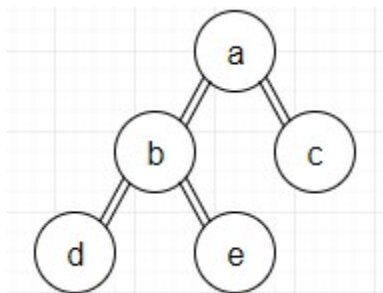
Section 1

QUESTION 1

Part a)



Part b)

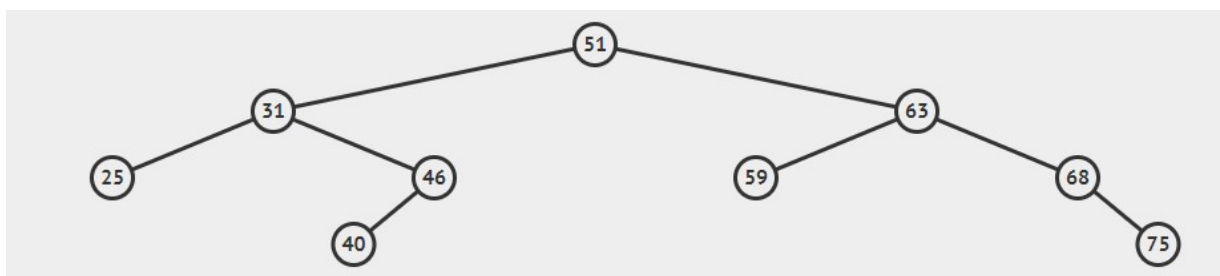


The general structure will be this, where a-b-c-d-e are one of the given numbers.

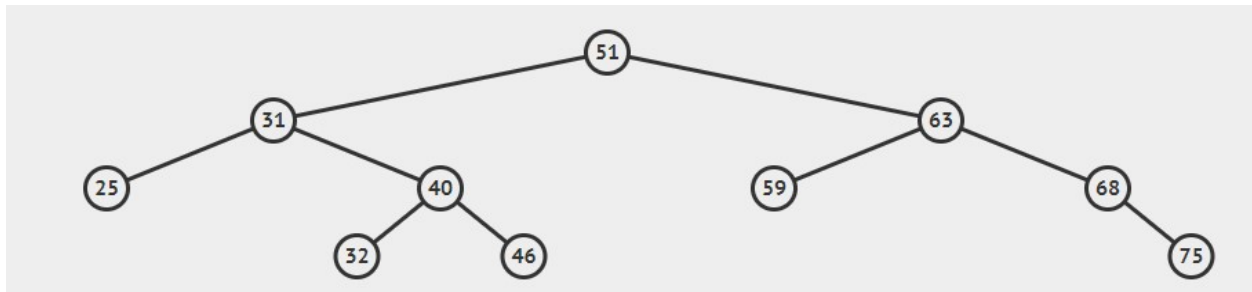
- a can **only** be the highest number of the initial 5
- c can be any of the **4** remaining numbers
- b can **only** be the highest of the remaining 3
- d and e can change between, resulting **2** solutions

Number of distinct heaps buildable = $1 * 4 * 1 * 2 = 8$

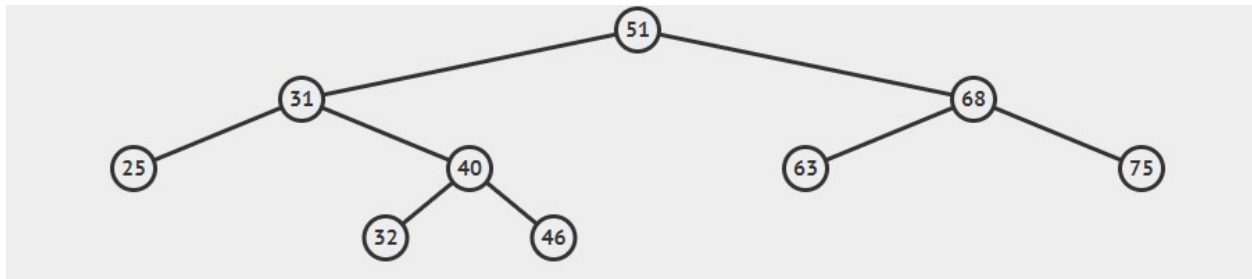
Part c)



After inserting 32:



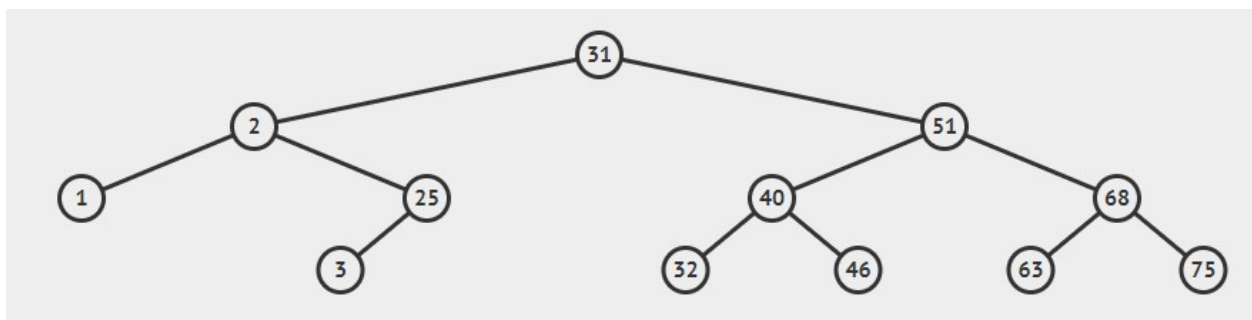
After deleting 59:



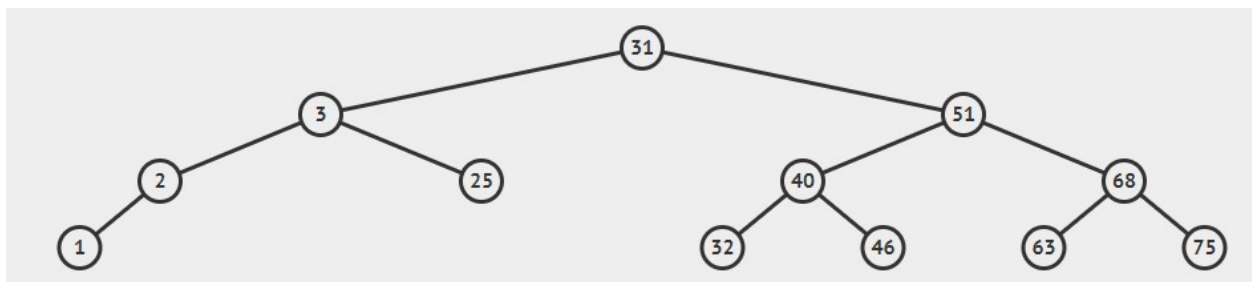
Part d)

Answer is NO.

An example can be illustrated on the previous AVL tree, using {1,2,3} as the set of numbers to insert. When the numbers are inserted in 1/2/3 order the following tree results:



However when the numbers are inserted in 3/2/1 order the following tree results:



QUESTION 2

Array Size	Random	Ascending	Descending
1000	656	990	990
2000	1313	1989	1989
3000	1949	2988	2988
4000	2550	3988	3988
5000	3156	4987	4987
6000	3755	5987	5987
7000	4322	6987	6987
8000	4837	7987	7987
9000	5417	8986	8986
10000	5902	9986	9986

Above are my test results.

The theoretical expected results for random insertions would be correlated with the size linearly but with a negative acceleration as seen in the test results. As AVL tree increase in size the empty leaf slots which do not break the balance increase in ratio against other empty leaf slots. The theoretical expected results are taken from an article wrote by "Society for Industrial and Applied Mathematics"¹ (1986).

Test results for Ascending and Descending insertions on the other hand, are same with the theoretical expected results, " $\text{size} - \log(\text{size}) - 1$ ". The reasoning of this theoretical expected result is that every new insertion tries to link to tree from it's edge, resulting a rotation to balance the tree. However, after a rotation resulting full tree there is no need to rotate for the next insertion.

The random number range plays a role in the rotation count. Wider ranges result more balanced insertions in general, decreasing the need for rotations.

¹ <https://people.mpi-inf.mpg.de/~mehlhorn/ftp/AmortizedAnalysisAVL.pdf>