HW1

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Abstract Alegbra PSET

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Problem 1/1. Which of the following rules are operations on the indicated set? For each rule which is not an operation, explain why it is not.

Question A1. $a * b = \sqrt{|\ ab|}$ on $\mathbb{Q}$

Proof. Let $a = 1, b = 2$. Then $a * b = \sqrt{|\ 1 * 2|} = \sqrt{2} \notin \mathbb{Q}$. Therefore $*$ is not an operation on $\mathbb{Q}$. □

Question A2. $a * b = a \ln(b)$ on $\{x \in \mathbb{R} \mid x > 0\}$

Proof. Let $a = 1, b = 2$. Then $a * b = 1 \ln(2) = \ln(2) \notin \{x \in \mathbb{R} \mid x > 0\}$. Therefore $*$ is not an operation on $\{x \in \mathbb{R} \mid x > 0\}$.

□

Question A3. $a * b$ is a root of the equation $x^2 - a^2b^2 = 0$ on the set $\mathbb{R}$

Proof. This is not an operation on $\mathbb{R}$ because it is not well defined. For example, let $a = 1, b = 2$. Then $a * b$ is a root of the equation $x^2 - 4 = 0$. However, there are two roots to this equation, $x = 2$ and $x = -2$. Therefore $*$ is not an operation on $\mathbb{R}$.

□