

Foundations Test 2

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Foundations of Mathematics

April 18, 2023

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Theorem 1. If $n \in \mathbb{Z}$ and $n \geq 0$, then $\sum_{i=0}^n i \cdot i! = (n+1)! - 1$.

Theorem 2. The inequality $2^n \leq 2n^{n-1} - 1$ holds for all $n \in \mathbb{N}$.

Theorem 3. Define the relation R on \mathbb{Z} such that xRy if and only if $3x - 5y$ is even. Then R is an equivalence relation.

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Theorem 4. Let $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$. If $a \equiv_n b$ then $a^2 \equiv_n b^2$. (recall the definition of \equiv_n from Definition 7.79)

Theorem 5. For an $n \geq 4$, one can obtain n dollars using only \$2 and \$5 bills.

Theorem 6. (Need to finish transitive case) Define $\Psi = \{(a, b) : a, b \in \mathbb{Z}, b \neq 0\}$ and define a relation \sim on Ψ via $(a, b) \sim (c, d)$ if and only if $ad = bc$. Then \sim is an equivalence relation.

Let $\Psi = \{(a, b) : a, b \in \mathbb{Z}, b \neq 0\}$ and define a relation \sim on Ψ via $(a, b) \sim (c, d)$ if and only if $ad = bc$. To show that \sim is an equivalence relation, we must show that \sim is reflexive, symmetric, and transitive.

To show that \sim is reflexive, we must show that for all $(a, b) \in \Psi$, $(a, b) \sim (a, b)$. It can be shown that,

$$\begin{aligned}(a, b) \sim (a, b) &\Leftrightarrow ab = ba \\ &\Leftrightarrow ab = ab \text{ (commutative property)}\end{aligned}$$

Therefore, \sim is reflexive.

To show that \sim is symmetric, we must show that for all $(a, b) \in \Psi$ and $(c, d) \in \Psi$, if $(a, b) \sim (c, d)$ then $(c, d) \sim (a, b)$.
It can be shown that,

$$(a, b) \sim (c, d) \Leftrightarrow ad = bc$$

and that,

$$\begin{aligned}(c, d) \sim (a, b) &\Leftrightarrow cb = da \\ &\Leftrightarrow ad = bc \text{ (commutative property)}\end{aligned}$$

Therefore, \sim is symmetric.

3 (complete)

Problem 1. Consider the following relation:

$$R = \{(a, a), (a, b), (a, d), (b, d), (c, c), (d, b), (d, c)\}$$

Problem 1. .1 What elements must be add to R to make it reflexive?

To be reflexive, (b, b) and (d, d) must be added to R .

Problem 1. .1 What elements must be add to R to make it symmetric?

To be symmetric, (b, a) , (d, a) and (c, d) must be added to R .