Lecture 6

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1 Todos

- QUIZ Celebration of learning posted next weds DUE Tuesday 3/28 9am
- HW5 due sometime after C2
- get these notes into Charlie Cruz's notation (or someone not me lol)

2

Remark: For an increasing annuity,

$$PV = (Ia_{\overline{n}|}) = \frac{a_{\ddot{\overline{n}|}-nv^n}}{i}$$

$$FV = (IS_{\overline{n}|}) = \frac{n - a_{\overline{n}|}}{i}$$

for a Decreasing annuity,

$$PV = (Da_{\overline{n}|}) = \frac{n(i+1)^n - S_{\overline{n}|}}{i}$$

$$FV = (DS_{\overline{n}|}) = \frac{S_{\ddot{\overline{n}|}-nv^n}}{i}$$

3 2.5.2 Example == HW prob 3 of PSET 5:

A five year annuity has increasing monthly payment at the end of each month. The first payment us 600, and each subsequent payment is 10 learger than the previous payment. At a rate of 0.5% per month, find the PV of the annuity valued one month before the final payment.

Soln: There are at least two ways of approaching this problem.

1. We can think of the 5 year annuity as a level annuity of [BLANK] per month, and an increasing annuity annuity with additional payments of 10.

$$PV = 600a_{\overline{60}|i} + 10\left(Ia_{\overline{59}|i}\right)v$$

$$PV = 590a_{\overline{60}|i} + 10\left(Ia_{\overline{60}|i}\right)$$

2. We can consider the annuity as a combination of level payments of 1200, and decreasing payments starting with [BLANK=600] and going down by [BLANK=10] each month. In this case,

$$PV = 1200a_{\overline{60}|i} - 10\left(Da_{\overline{60}|i}\right)$$

4 2.5.3 Example == HW Problem 4 and last of PSET 5:

Jeff bought an increasing perpetuity-due (annuity due means its due at beginning of month immediate is at the end of the month) with annual payments starting at 5 and increasing by 5 each year until the payment amount reaches 100. Thereafter, the payments then remain at 100. If the annual effective rate is 7.5%, what is the PV of this perpetuity?

notes:

- Ia_{∞} will be used (but at the period when this happens you need to discount it)

$$Ia_{\overline{\infty}|}v^x$$

- our previous prob is almost same (different periods an such)

5 Chapter 3: Loan Repayment

note: excel sheets can handle a lot of this

5.1 3.1: Amortization:

see green book for numberline

5.2 3.1.1 Definition:

A loan L is amortized at interest i if the loan amount is equal to the PV of all the loan payments. $K_1, K_2, \ldots, K_n$. In other words (IOW),

$$L = K_1v + K_2v^2 + \cdots + K_nv^n$$

The outstanding balance of the loan at time t, denoted OB_t, is simply the sum of the unpaid principal and interest at time t. IOW OB_t is simply the PV of the remaining payments. For example, $OB_0 = L.$ and $OB_n = 0$

$$OB_1 = L(1+i) - K_1 \text{ (you have to accumulate the interest since t=0)}$$

$$OB_2 = L(1+i)^2 - K_1 - K_2 = OB_1(1+i) - K_2 = [L(1+i) - K_1](1+i) - K_2)]$$

$$OB_2 = L(1+i)^2 - K_1(1+i) - K_2$$

$$OB_3 = L(1+i)^3 - K_1(1+i)^2 - K_2(1+i) - K_3$$

In general,

$$OB_t = L(1+i)^t - \sum_{j=1}^{t} K_j(1+i)^{t-j}$$

Or,

$$OB_t = L(1+i)^t - K_1(1+i)^{t-1} - K_2(1+i)^{t-2} - \cdots - K_{t-1}(1+i)^1 - K_{t-0}$$

This is known as the retrospective view of OB_t.
On the other hand, OB_t can also be viewed in terms of the remaining loan repayments that is,

$$OB_t = \text{PV of all remaining payments}$$

$$OB_t = K_t + K_{t+1}v + K_{t+2}v^2 + \cdots + K_n v^{n-t}$$

This is called the prospective view of OB_t.