A4-Draft_CSaben

Clark Saben

November 8, 2023

1. (6 points) Let $\langle G_1, \cdot\rangle$ and $\langle G_2, *\rangle$ be groups with identities e_1 and e_2, respectively. Suppose that $f : G_1 \to G_2$ is an isomorphism. Prove that $f(e_1) = e_2$. Write - when you multiply elements of G_1, and $*$ when you multiply elements of G_2.

Proof. 1 Note since that $f : G_1 \to G_2$ is an isomorphism, that it preserves the operation the for any elements $a, b \in G_1$, we have $f(a \cdot b) = f(a) * fb$. Note that by definition, $f : G_1 \to G_2$ is also bijective and surjective. So, $b \in G_2$ is the image of some $a \in G_1$ since f is surjective. Thus $b = f(a)$ for some $a \in G_1$. It follows that if $e_1 \in G_1$ and $e_2 \in G_2$ are the identities of each respectve group that,

$$\begin{aligned} f(e_1) * f(a) &= f(e_1 \cdot a) \\ &= f(a) \\ &= b \end{aligned}$$

and thus,

$$\begin{aligned} f(e_1) * b &= b \\ &= e_2 * b. \end{aligned}$$

Therefore, $f(e_1) = e_2$. $\square$

2. (6 points) Let G be a group and let $a \in G$ be an element of order 6 . Construct a Cayley table (written in LTEXX) for the cyclic subgroup $\langle a\rangle$ of G generated by a. Each element in your table must be written as a^r, where $r \in \{0, 1, \ldots, 5\}$. If you wish, you can write a^0 as e, and a^1 as a.

$\cdot$	e	a	a^2	a^3	a^4	a^5
e	e	a	a^2	a^3	a^4	a^5
a	a	a^2	a^3	a^4	a^5	e
a^2	a^2	a^3	a^4	a^5	e	a
a^3	a^3	a^4	a^5	e	a	a^2
a^4	a^4	a^5	e	a	a^2	a^3
a^5	a^5	e	a	a^2	a^3	a^4

3. (8 points) Let D_4 be the group of symmetries of the square.

(a) List all distinct cyclic subgroups of D_4. Write each cyclic subgroup as $\langle a \rangle = \{\ldots\}$, using the symbols from Class Notes for Chapter 7: $R_0, R_{90}, R_{180}, R_{270}, \rho_A, \rho_B, \rho_H$, and ρ_V. For example, $\langle \rho_A \rangle = \{R_{90}, R_{180}, \rho_V\}$ (that is incorrect). Do not repeat.

(b) Find a subgroup H of D_4 such that $H \neq D_4$ and H is not cyclic. Explain why your H is not cyclic.

Hint: See Assignment 2, but be careful if your table for D_4 was not correct.

(a)

- $\langle R_0 \rangle = \{R_0\}$ since it is the identity.
- $\langle R_{90} \rangle = \{R_0, R_{90}, R_{180}, R_{270}\}$ because each is a 90 degree rotation
- $\langle R_{180} \rangle = \{R_0, R_{180}\}$ because there are only two outcomes of 180 degree rotations
- $\langle R_{270} \rangle = \{R_0, R_{270}\}$ is the same group as 90 degree since it is 90 degrees thrice.
- $\langle \rho_A \rangle = \{R_0, \rho_A\}$ b/c only can be the identity and the reflection across A.
- $\langle \rho_B \rangle = \{R_0, \rho_B\}$ b/c only can be the identity and the reflection across B.
- $\langle \rho_H \rangle = \{R_0, \rho_H\}$ b/c only can be the identity and the reflection across the horizontal axis
- $\langle \rho_V \rangle = \{R_0, \rho_V\}$ b/c it consist of the identity and the vertical axis

(b)

$H = \{R_0, \rho_H, \rho_V, \rho_A\}$ since there is no element in H that generates all of D_4.