Theorem 3.61a

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Theorem 3.61 a. Let A, B, C and D be sets. Prove if true or provide counterexample if false. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

Proof. First we will prove $(A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$. Let $(x, y) \in (A \cap B) \times (C \cap D)$. Then $x \in A \cap B$ and $y \in C \cap D$. Hence, $x \in A$ and $x \in B$ and $y \in C$ and $y \in D$. Therefore, $(x, y) \in (A \times C) \cap (B \times D)$. This proves $(A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$.

Next we will show that $(A \times C) \cap (B \times D) \subseteq (A \cap B) \times (C \cap D)$. Let $(x, y) \in (A \times C) \cap (B \times D)$. Then $(x, y) \in (A \times C)$ and $(x, y) \in (B \times D)$. Hence, $x \in A$ and $y \in C$ and $x \in B$ and $y \in D$. Therefore, $x \in A \cap B$ and $y \in C \cap D$. Then finally, $(x, y) \in (A \cap B) \times (C \cap D)$. This proves $(A \times C) \cap (B \times D) \subseteq (A \cap B) \times (C \cap D)$.

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