Theory of Interest Homework

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Question 2.2.7 a. Find the monthly payments for a 30 year fixed loan of 200,000 with an APR of 4.5% compounded monthly + payments made at the end of each month.

Recall, as a FV problem we can state this as follows,

$$xS_{\overline{n}|i} = 200,000\,(1+i)^{360}$$

Note also, that our discount factor in this scenario is,

$$\begin{aligned} v &= \frac{1}{1+\frac{i}{12}} \\ v_{0.045} &= \frac{1}{1+\frac{.045}{12}} \\ v_{0.045} &= 0.9962 \end{aligned}$$

Keeping all this in mind to find the monthly payments given that we known 30 years is 360 months, we can use the following equivalent PV method,

$$\begin{aligned} xv\left(\frac{1-v^{360}}{1-v}\right) &= 200,000 \\ x\,(0.9962)\,(\frac{1-.9962^{360}}{1-.9962}) &= 200,000 \\ x\,(197.361) &= 200,000 \\ x &= 1013.371 \end{aligned}$$

Question 2.2.11 a. In the preceeding example, determine the amount of each monthly payment if no payment is made for the first 12 months.

Consider a new period of 360 months - 12 months and let's use our same method from the previous question,

$$\begin{aligned} xv\left(\frac{1-v^{348}}{1-v}\right) &= 200,000 \\ x\,(0.99626)\,(\frac{1-.99626^{348}}{1-.99626}) &= 200,000 \\ x\,(194.177) &= 200,000 \\ x &= 1029.986 \end{aligned}$$

Problem 2.2.13 a. Jim can make an investment of 10,000 in 2 ways:

1. Deposits into an account yielding an annual interest rate of i.
2. He can purchase an annuity immediate (payments occur at end of month) with 24 level payments (the amounts don't change) annually, at an annual rate of 10%. These payments are then deposited into a fund that yields an annual effective rate of 5%.

If both options produce the same accumulated value at the end of 24 years what is the value of i?

We will determine the respective accumulated values and equate them to find i.

Accumlated value of option 1:

$$10,000\,(1+i)^{24}$$

First, let x be the level payment, then $10,000 = xa_{\overline{24}|.1}$, therefore,

$$\begin{aligned} x &= \frac{10,000}{a_{\overline{24}|.1}} \\ x &= \frac{10,000}{\frac{1-v^{24}}{.1}} \text{ where i} = 0.1 \\ x &= \frac{10,000}{\frac{1-.1.015}{.1}} \\ x &= 1,112.9978 \end{aligned}$$

Finally, Accumlated value of option 2:

$$xS_{\overline{24|}.05}$$

Consequently,

$$\begin{aligned}
xS_{\overline{24|}.05} &= 10,000\,(1+i)^{24}\\
x\left(\frac{(1+i^n)-1}{i}\right) &= 10,000\,(1+i)^{24}\\
x\left(\frac{(1+.05^{24})-1}{.05}\right) &= 10,000\,(1+i)^{24}\\
1112.9978\,(44.502) &= 10,000\,(1+i)^{24}\\
49530.62524 &= 10,000\,(1+i)^{24}\\
(4.9530.62524)^{\frac{1}{24}} &= 1+i\\
i &= 6.89\%
\end{aligned}$$