

Theory of Interest

Clark Saben

April 17, 2023

1 3/6/23 Lecture

Homework and Logistics

- wednesday 4pm math meeting
 - illustrations are in green small handbook associated with this day
 - Get a credit card plan
 - get a rider such that after your life insurance expires it keeps building up. you can have a rider to pull money out 60% if you are terminally ill. having a will makes things better.
 - Make a master tex file for this folder
 - fix errors and make equations look nice
-

Example 2.2.6 a. consider an annuity as in e.g. 2.2.4 with the following adjustments. suppose that the interest rate is 12% per annum for the first 10 months with payments of X each, and the rate doubles to 24% for the rest of the term with payments of $2X$ each. Determine the level payment for each period.

Solution :

Recall that the accumulated value of this annuity (see e.g. 2.2.4) is 10000, consequently (keep in mind we accumulate the 10 years),

$$\begin{aligned}XS_{10i_1}(1+i_2)^{10} + 2XS_{10i_2} &= 10000 \\X \frac{[(1.01)^{10} - 1]}{0.01} + 2X \frac{[(1.02)^{10} - 1]}{0.02} &= 10000 \\X &= 288.58\end{aligned}$$

Example 2.2.7 b. Find the monthly payment for a 30 year fixed loan of 200,000 with APR of 4.5% compounded monthly, and payments made at the end of each month.

Solution :

This problem can be approached in two ways:

1. Future value of a loan (number lie from 1 to 360 where loan is given at 1).
 $X(\text{Accum Value}) \text{ should} = \text{Accum value of the Loan}$
 $XSn_i = 200,000(1+i)$
2. Present value of a loan (number lie from 1 to 360 where loan is given at 360).
 PV discussion happens at time=0

Let v be the discount factor, where $v = \frac{1}{1+i}$, and i is the interest rate.
 Then the sum of all discounts should = the PV of the loan.

$$Xv + Xv^2 + \dots + Xv^{360} = 200,000$$

$$Xv(1 + v + v^2 + \dots + v^{359}) = 200,000 \text{ (notice we have the geometriic series)}$$

$$Xv \frac{[1-v^{360}]}{[1-v]} = 200,000$$

Now, find X

2.2.8 Remark: In general, we denote
 $v + v^2 + \dots + v^n = ani$, where $v = \frac{1}{1+i}$

2.2.9 Definition:
 ani is the present value of an annuity-immediate with level payments of 1 occuring at the end of each month for n months at a rate of i where $ani = v + v^2 + \dots + v^n$

Theorem 2.2.10 1. $a_{ni} = v + v^2 + \dots + v^n = \frac{1-v^n}{i}$

Proof. Observe that,
 $a_{ni} = v + v^2 + \dots + v^n = v[1 + v + v^2 + \dots + v^{n-1}]$
 $= v \frac{[1-v^n]}{[1-v]}$

It suffices to show that $\frac{v}{1-v} = \frac{1}{i}$

To this end, since $v = \frac{1}{1+i}$, we have
 $1 - v = 1 - \frac{1}{1+i} = \frac{1+i}{1+i} - \frac{1}{1+i} = \frac{i}{1+i}$
 $\frac{v}{1-v} = \frac{\frac{1}{1+i}}{\frac{i}{1+i}} = \frac{1+i}{i} * \frac{1}{1+i} = \frac{1}{i}$
 $a_{ni} = \frac{v}{1-v^n} (1 - v^n) = \frac{1}{i} (1 - v^n) = \frac{1-v^n}{i} \quad \square$

2.1.11 Example a. In the preceding example, determine the amount of each monthly payment if no payment is made for the first 12 months.