Theory of Interest

Clark Saben

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1 3/8/23 Lecture

Homework and Logistics

- 3:30pm talk (student engagement)
- PS3: 2.2.7, 2.2.11, 2.2.13 (first two on last lecture notes, 3rd on today), Due: 6pm on Tuesday 3/14 with an extra problem given on monday. In total, 4 problems.

Remark:

In general,
$S_{\overline{n}|}$ $a_{\overline{n}|}$:= is the present value of " ".
Thus,
$v^k a_{\overline{n}|} := a_{\overline{n+k}|} - a_{\overline{k}|}$

Imagine a number line with 0 and k and $k+1$ on it.
Therefore,
$a_{\overline{n+k}|} = a_{\overline{k}|} + v^k a_{\overline{n}|}$

As we saw in the last lecture,
$S_{\overline{n+k}|} = S_{\overline{n}|}(1+i)^k + S_{\overline{n}|}$.
Here,
$v^k a_{\overline{n}|}$ is the PV of a k period deferred annuity immediate.

Example 2.2.13 a. Jim can make an investment of 10,000 in 2 ways:

1. Deposits into an account yielding an annual interest rate of i.
2. He can purchase an annuity immediate (payments occur at end of month) with 24 level payments (the amounts don't change) annually, at an annual rate of 10%. These payments are then deposited into a fund that yields an annual effective rate of 5%.

If both options produce the same accumulated value at the end of 24 years what is the value of i?

$\boxed{Solution:}$

Option A:

We will determine the respective accumuluated values. And equate them, to find i.

Accumulated value of the first option:
$S_{\overline{24}|} = 10,000\,(1+i)^{24}$

Accumulated value of the second option:

Let X be the level payment.
Then,
$10,000 = Xa_{\overline{24}|.1}$
Therefore,
$X = \frac{10{,}000}{a_{\overline{24}|.1}} = \frac{10000}{\frac{1-v^{24}}{i}}$
$i = 10$ in the actuarial angle%
What does X =??

Option B:

Finally,
The accumulated value of option B= $XS_{\overline{24}|.05}$
(How much Jim ends up with when Jim brings in X valued level payments for 24 years.)
Consequently,
$XS_{\overline{24}|.05} = 10,000\,(1+i)^{24}$
OR $AV(B) = AV(A)$