HW1

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Abstract Alegbra PSET

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Problem 1/1. Which of the following rules are operations on the indicated set? For each rule which is not an operation, explain why it is not.

Question A1. $a * b = \sqrt{|\ ab|}$ on the set $\mathbb{Q}$

Let $a = 1, b = 2$. Then $a * b = \sqrt{|\ 1 * 2|} = \sqrt{2} \notin \mathbb{Q}$. Therefore $*$ is not an operation on $\mathbb{Q}$.

Question A2. $a * b = a\ln(b)$ on $\{x \in \mathbb{R} \mid x > 0\}$

The operation, $a * b = a\ln(b)$, is an operation on $\{x \in \mathbb{R} \mid x > 0\}$, since for all $a, b \in \{x \in \mathbb{R} \mid x > 0\}$, a will always be real and $ln(b)$ will always be real, since $b > 0$. Thus, $a * b = a\ln(b) \in \{x \in \mathbb{R} \mid x > 0\}$.

Question A3. $a * b$ is a root of the equation $x^2 - a^2b^2 = 0$ on the set $\mathbb{R}$

The predefined $a * b$ is not an operation on $\mathbb{R}$, since $a * b$ is not uniquely defined for all $a, b \in \mathbb{R}$, $a \neq 0$, and $b \neq 0$. If $a \neq 0$ and $b \neq 0$, then $a * b = \pm ab$ has two roots, $a * b = ab$ and $a * b = -ab$.

Question A4. Subtraction, on the set $\mathbb{Z}$

Subtraction is an operation on $\mathbb{Z}$, since for all $a, b \in \mathbb{Z}$, $a - b \in \mathbb{Z}$.