Lecture 5

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March 20, 2023

1 Todos

- QUIZ Celebration of learning posted next weds DUE Tuesday 3/28 9am
- get these notes into Charlie Cruz's notation (or someone not me lol)
- HW5 due sometime after C2

2 2.4.2 e.g. == HW5 q1

Jeff deposits 100 at the end of each year for 13 years into a fund X. Jen deposits 100 at the end of each year for 13 years into a fund Y. Fund X earns an annual effective rate of 15% for the first five years and annual ERI of 6% for the remaining eight years. Fund Y earns an annual effective rate i. Both funds have the same accumulated value at the end of 13 years. What is the value of i?

$$\left(100S_{\overline{5}|.15}\right)(1+0.06)^8 + \left(100S_{\overline{8}|.06}\right) = 100S_{\overline{13}|.i}$$

$$i = ?$$

Remark: So far, our discussion has considered annuities for which frequency and interest conversion periods are the same. In general, this may not be the case.

3 2.4.3 e.g. == HW5 q2

- An annuity immediate has ten monthly payments of 1, and the quoted interest rate $i^{(4)} = 8\%$. Determine its PV and its FV
- If the quoted rate is an annual effective rate of 6%, find the PV and the fV of the annuity.

4 2.5: Increasing and Decreasing Annuities

4.1 2.5.1: Increasing Annuities

Consider an annuity in which successive payments follow an arithmetic progression, namely: 1, 2, 3, 4, 5, 6, 7, 8, 9, n.

Let the PV of this annuity be X. That is,

$$X = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + n$$

$PV ==$ Present Value

$$PV = (IA)_{\overline{n}|} = v + 2v^2 + \cdots + nv^n$$

Since $PV = X$, we have:
Eq1 (how to tack this to the side of the eqn and reference it later in tex?)

$$X = v + 2v^2 + \cdots + nv^n = X$$

Eq2

$$1 + 2v + 3v^2 + \cdots + nv^{n-1} = \frac{X}{v} = X\left(1+i\right)$$

$$\text{Eq1 - Eq2} \implies \left(1+i\right)X - X = 1 + v + v^2 + \cdots + v^{n-1} - nv^n$$

That is,

$$iX = 1 + v + 2v^2 + \cdots + v^{n-1} - nv^n$$

recall;

$$\ddot{a}_{\overline{n}|i} = 1 + v + 2v^2 + \cdots + v^{n-1}$$

$$iX \implies \ddot{a}_{\overline{n}|i} - nv^n$$

Therefore,

$$X = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

i.e.

$$PV = (IA)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

Similarly (proof is left as an exercise to the reader),

$$FV = (IS)_{\overline{n}|} = \frac{\ddot{s}_{\overline{n}|} - nv^n}{i}$$

Consequently, the PV of an increasing perpetuity is given by,

$$IA_{\overline{\infty}|} = \frac{1}{id} = \left(\frac{1}{i}\right)\left(\frac{1+i}{i}\right) = \frac{1}{i^2} + \frac{1}{i}$$