Lecture 1

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1 2/21/23 lecture (Lakroski 4.5)

E. Infinite Well

The 'Program'

If is Time Independent: t—$\Psi(t=0)\rangle = e^{-i\hat{H}t\hbar}$
'Stationary' States: ——Psi$_n\rangle = E_n|\Psi_n\rangle$

So, —$\Psi(o)\rangle = \sum_{n=1}^{\infty}\langle\Psi n|\Psi_n\rangle$

$\implies \hat{U}(t)|\rangle = \sum_n e^{-i\hat{H}t/\hbar}$
General Solution of the TSDE if is Time indpendent:
All we need in are the —$\Psi_n\rangle$
Time independent shrodinger equation:
In matrix representation (homework, quiz, exam):
H$_n = E_n\vec{v}_n$
We do this if we don't know the eigenvectors associated with H and we want to know them.

In Eigenstate basis:
In another basis: Now you have stuff that are non-zero off diagnol elements.
The way you do this is you write the eigenvalues of the 2nd.
The other way to do it is in the The Wave Function Representation:
$\hat{H}(x)\Psi(x) = E_n\Psi_n(x)$
In 1D:

$\left(\frac{-\hbar}{2m}\frac{d^2}{dx^2} + V(x)\right)\Psi_n(x) = E_n\Psi_x(x)$

$->$ Second ODE
In class: Look for symmetries and/or conserverd quantities. (he makes a note about softwares that handle this being worth a lot)
$\implies$ [,] = 0;d¡¿/dt = 0
-¿$Isconserved.$

Advantage: We can identify conserved quantities using classical intuition.

Once we have identified a conseved quantity( ) then it has the same eigenstates as $\hat{H}$.

E.g. Infinite well

Q1: Which quantities are conseved in the region $x \in (0, a))$ for a particle in a infinite potential energy well of length a (assume elastic collisions):
answer: (D) Magnitude of Momentum and Energy

Explanation:
Angular Momentum:
$\vec{L} = \vec{r}x\vec{p}$

$|\hat{p}|$ and $\hat{H}$ are conserved, so:
$[\hat{H}, |\hat{p}|] = 0$
$\implies \hat{H}$ and $|\hat{p}|$ have simultaneous eigenstates.

Q2: What are appropiate eigenfunctions of the momentum operator?

answer: (B) $->$ $Ae^{-ipx/h}$)
Explanation:
For $|\hat{p}|$ we can make a superposition of $\pm$ Eigenstates:
$\Psi(x) = \alpha e^{\frac{ipx}{\hbar}} + \beta e^{-\frac{ipx}{\hbar}}$
' Another way to write this (SHO);
$= Asin(\frac{px}{\hbar}) + Bcos(\frac{px}{\hbar})$

Say, $k = \frac{p}{\hbar} \implies \Psi(x) = Asin(kx) + Bcos(kx)$

$\Psi(x)$ represents Stationary States!
Eigenstates of $\hat{H}$, because $[\hat{H}, \hat{p}] = 0$

(language note: state and wave fn == eigenstate)

For a time independent problem such as this you need the boundary conditions (i.c. is for time dependent problems).
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For $x > a$ or $x < 0$ the wave function is zero.

These are our boundary conditions. (see diagram in notes)
$\implies \Psi(x) = 0$
$\Psi(a) = 0$

Q3 TLDR: To satisfy the boundary condition $\Psi(0) = 0$, the constant B must

be zero.
Explanation: plug in $x = 0$ into $\Psi(x) = Asin(kx) + Bcos(kx)$

Moral: The wave function must be zero at the boundary.

Q4: To satisfy the boundary condition(b.c.) $\Psi(a) = 0$, the constant k in the wavefunction of a particle is an infinite well must be; (assume $B = 0$ from prev. b.c.)

answer: (A) $k = \frac{n\pi}{a}$ s.t. $n \in \mathbb{Z}$ but is not zero

a and/or n cannot simply be zero because otherwise the wave-fn is zero everywhere)

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$k_n = \frac{n\pi}{a}$ =pn$_{\overline{\hbar}}$

The boundary condition quantizes momentum.