Theory of Interest

Clark Saben

March 6, 2023

1 3/6/23 Lecture

Homework and Logistics

- wednesday 4pm math meeting
- illustrations are in green small handbook associated with this day
- Get a credit card plan
- get a rider such that after your life insurance expires it keeps building up. you can have a rider to pull money out 60% if you are terminally ill. having a will makes things better.
- Make a master tex file for this folder
- fix errors and make equations look nice

Example 2.2.6 a. consider and annuity as in e.g. 2.2.4 with the following adjustments. suppose that the interest rate is 12% per annun for the first 10 months with payments of X each, and the rate doubles to 24% for the rest of the term with payments of $2X$ each. Determine the level payment for each period.

$\boxed{Solution:}$

Recall that the accumulated value of this annuity (see e.g. 2.2.4) is 10000, consequently (keep in mind we accumulate the 10 years),

$$\begin{aligned} XS_{\overline{10}|i_1}(1+i_2)^{10} + 2XS_{\overline{10}|i_2} &= 10000 \\ X\frac{[(1.01)^{10}-1]}{0.01} + 2X\frac{[(1.02)^{10}-1]}{0.02} &= 10000 \\ X &= 288.58 \end{aligned}$$

Example 2.2.7 b. Find the monthly payment for a 30 year fixed loan of 200,000 with APR of 4.5% compounded monthly, and payments made at the end of each month.

$\boxed{Solution:}$

This problem can be approached in two ways:

1. Future value of a loan (number lie from 1 to 360 where loan is given at 1).
   X(Accum Value) should = Accum value of the Loan
   $XS\overline{n|}i = 200,000(1+i)$

2. Present value of a loan (number lie from 1 to 360 where loan is given at 360).
   PV discussion happens at time=0

Let v be the discount factor, where $v = \frac{1}{1+i}$, and i is the interest rate.
Then the sum of all discounts should = the PV of the loan.

$Xv + Xv^2 + ... + Xv^{360} = 200,000$
$Xv\,(1 + v + v^2 + ... + v^{359}) = 200,000$ (notice we have the geometriic series)
$Xv\frac{[1-v^{360}]}{[1-v]} = 200,000$

Now, find X

2.2.8 Remark: In general, we denote
$v + v^2 + .. + v^n = a\overline{n|}i$, where $v = \frac{1}{1+i}$

2.2.9 Definition:
$a\overline{n|}i$ is the present value of an annuity-immediate with level payments of 1 occuring at the end of each month for n months at a rate of i where a$\overline{n|}i = v + v^2 + .. + v^n$

Theorem 2.2.10 1. $a_{\overline{n|}i} = v + v^2 + ... + v^n = \frac{1-v^n}{i}$

Proof. Observe that,
$a_{\overline{n|}i} = v + v^2 + ... + v^n = v[1 + v + v^2 + ... + v^{n-1}]$
$= v\frac{[1-v^n]}{[1-v]}$

It suffices to show that $\frac{v}{1-v} = \frac{1}{i}$

To this end, since $v = \frac{1}{1+i}$, we have
$1 - v = 1 - \frac{1}{1+i} = \frac{1+i}{1+i} - \frac{1}{1+i} = \frac{i}{1+i}$
$\frac{v}{1-v} = \frac{1}{1+i} / \frac{i}{1+i} = \frac{1+i}{i} * \frac{1}{1+i} = \frac{1}{i}$
$a_{\overline{n|}i} = \frac{v}{1-v^n}\left(1 - v^n\right) = \frac{1}{i}\left(1 - v^n\right) = \frac{1-v^n}{i} \quad \square$

2.1.1 Example a. In the preceding example, determine the amount of each monthly payment if no payment is made for the frist 12 months.