Writing Assignment 5

Clark Saben
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Theorem 3.10. Suppose that A, B and C are sets. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.

Proof. Let A, B, and C be sets. Let $x \in A$. Since $A \subseteq B$ and $x \in A$, $x \in B$. Similarly, $B \subseteq C$ so $x \in C$. Therefore if $(A \subseteq B) \cap (B \subseteq C)$ then $A \subseteq C$. $\square$

Theorem 3.21b. If A and B are sets, then $(A \cap B)^c = A^c \cup B^c$.

Proof. Let A and B be sets. To show $(A \cap B)^c = A^c \cup B^c$, we must show that $(A \cap B)^c \subseteq A^c \cup B^c$ and $A^c \cup B^c \subseteq (A \cap B)^c$. Firstly, let $x \in (A \cap B)^c$, then $x \notin A$ and $x \notin B$. Therefore, by De-Morgan's law, $x \notin (A \cup B)$. If x is not a member of A or B by definition 3.14 it follows that $x \in A^c \cup B^c$. Secondly, let $x \in A^c \cup B^c$. Therefore, $x \notin (A \cup B)$. By De-Morgan's law, it follows that $x \notin (A \cap B)$. Hence, by definition 3.14, $x \in (A \cap B)^c$. Therefore, $(A \cap B)^c = A^c \cup B^c$. $\square$