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EGSim - a C++ Toolkit for Analysis of Power Grid Systems

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ABSTRACT

We describe the load flow formulation and the solution algorithms available in the Electric power Grid Simulator (EGSim) software toolkit. EGSim contains tools aimed at simulating static load flow solutions for electric power grids. It parses power grid models described in IEEE Common Data Format, and generates solutions for the bus voltages and voltage angles, and real and reactive power values through the transmission lines. The software, written in C++, implements both Gauss-Seidel and Newton solution methods. Example results for the 118 bus models and 300 bus models are also presented.

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1. INTRODUCTION

The development of the Electric Grid Simulator (EGSim) toolkit was motivated by the need to have a canonical tool for the analysis of power flow in electric grids and that can be paired easily with a large set of heterogenous software frameworks. EGSim is a C++ software toolkit designed for load flow computations. Both Gauss-Seidel and Newton algorithms are implemented to solve for the static load flow solution [7]. The software can currently parse power grid setup files in IEEE Common Data Format [12].

This report has the following sections: Section 2 describes the models and equations implemented in this toolkit, Section 3 describes the numerical discretization, Section 4 presents results for canonical test cases, and Section 5 discusses the code architecture and installation instructions.

2. STEADY STATE EQUATIONS

During static load flow analyses, each bus i of the power system is characterized by 4 variables: the real power P_i, the reactive power Q_i, the voltage angle θ_i and magnitude $|V_i|$. Depending on the bus model, the values for some of these variables are set through input files while others are solutions of the system of equations described in this section. Three types of bus models are currently implemented in EGSim:

- **Load bus** - It is assumed that P and Q are known for load buses, and that voltage $V = |V|e^{\jmath\theta}$ needs to be determined. Here $\jmath = \sqrt{-1}$. Load buses are also called **PQ** buses.
- **Generator bus**. For generator buses it is assumed that P and $|V|$ are known and Q and θ are to be computed. Generator buses are also called **PV** buses.
- **Slack bus**. The slack bus, also named "swing" bus in some texts, is a generator bus with the voltage angle θ fixed to a reference value.

The power balance equations at each bus i are written as [7]

$$P_i + \jmath Q_i = \sum_k V_i \bar{V}_k \bar{Y}_{ki} \tag{1}$$

The real and imaginary parts of eq. (1) are expanded as

$$\begin{aligned} P_i &= \sum_k |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) && (2) \\ Q_i &= \sum_k |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) && (3) \end{aligned}$$

The sums in the right-hand sides *rhs* of Eqs. (1), (2) or (3) are over all lines $(i-k)$ connected to bus i. Here, the bar over some variables represents the complex conjugate, $\theta_{ik} = \theta_i - \theta_k$, and $Y_{ki} = G_{ik} + \jmath B_{jk}$ is the element (i,k) in the admittance matrix Y (see section 2.1).

In order to determine the unknown variables, two power balance equations are available at load buses and there is one equation for the real power at the generator buses. Since Q is not constrained for generator buses, the reactive power balance equation is not used for these buses. No equation is written for the slack bus for which the voltage magnitude and angle are set constant and equal to the initial condition.

2.1. Admittance matrix

The admittance matrix Y for a power grid system is computed as follows [7]:

$$Y_{ik} = -\sum_m y_{ik,m}, \text{ if } i \neq k \tag{4}$$

$$Y_{ii} = \sum_k \left(y_{ik} + \jmath \frac{l_{ch,ik}}{2} \right) + Gs_i + \jmath Bs_i. \tag{5}$$

Here $y_{ik,m}$ is the admittance for the line that connects busses i and k and is computed as $y_{ik,m} = 1/(r_{ik,m} + \jmath x_{ik,m})$, where $r_{ik,m}$ and $x_{ik,m}$ are the line resistance and reactance, respectively. The sum in eq. (4) is over all lines m between buses i and k, while the sum in eq. (5) is over all lines connected to bus i. The impedance, $r_{ik} + \jmath x_{ik}$, line charging susceptance, $l_{ch,ik}$, for each line $(i-k)$ and bus shunt admittance $Gs_i + \jmath Bs_i$ for each bus i are specified in the model setup.

In realistic power grid systems, which contain thousands of buses, the admittance matrix is usually sparse as most buses are connected to a small subset of the bus network. While outside the scope of the current EGSim software development, in the future we will explore linear algebra methodologies that exploit the sparsity of the admittance matrix to increase the efficiency of *EGSim*.

3. NUMERICAL SOLUTION FOR THE LOAD FLOW PROBLEM

Several methodologies can be employed to solve for the non-linear system of equations that describes the power balance at each bus. Two iterative procedures are outlined below: (1) the Gauss-Seidel formulation and (2) the Newton formulation. The former algorithm has a smaller memory footprint, however it is slower to converge compared to the later option. The Newton approach however requires the evaluation of the Jacobian matrix and the solution of a linear system at each iteration. Typically it decrease the residual by several orders of magnitude per iteration, however it is more memory intensive.

3.1. Gauss-Seidel Formulation

Equation (1) can be also be written as

$$\frac{P_i + \jmath Q_i}{V_i} = \sum_k \bar{V}_k \bar{Y}_{ki} \rightarrow V_i = \frac{1}{Y_{ii}} \left(\overline{\left(\frac{P_i + \jmath Q_i}{V_i} \right)} - \sum_{k \neq i} V_k Y_{ki} \right) \tag{6}$$

Based on this expression, an iterative numerical approach similar to the Gauss-Seidel method used in the context of linear systems is outlined in algorithm (1).

Algorithm 1 Iterative algorithm for steady state power flow system

repeat
  increment iteration count n
  for $i = 1 \rightarrow$ number of buses **do**
    if bus i is not a slack bus **then**
      compute new V_i using eq. (6):
      $V_i^{(n)} \leftarrow \frac{1}{Y_{ii}} \left(\overline{\left(\frac{P_i + \jmath Q_i}{V_i^{(n-1)}} \right)} - \sum_{k<i} V_k^{(n)} Y_{ki} - \sum_{k>i} V_k^{(n-1)} Y_{ki} \right)$
      if bus i is a generator bus **then**
        reset $|V_i|$ to the specified value for bus i
        re-compute Q_i
      end if
    end if
  end for
until convergence

The iteration loop in algorithm (1) is stopped when the iteration count exceeds a certain ceiling, currently set at 100, or when the maximum change in voltage magnitude over all buses becomes smaller that a threshold value, currently set at 10^{-6}.

3.2. Newton Formulation

For the Newton iteration approach, Eq. (1) is rearranged as

$$\begin{aligned} g_i &= \mathrm{Re}(V_i \textstyle\sum_k \bar{V}_k \bar{Y}_{ki}) - P_i = 0 \\ g_{i+n} &= \mathrm{Im}(V_i \textstyle\sum_k \bar{V}_k \bar{Y}_{ki}) - Q_i = 0 \end{aligned} \tag{7}$$

where $V_i = |V_i| \exp(\jmath \theta_i)$. The unknowns for all buses $i = 1, 2, \ldots, n$, i.e. the voltage magnitude and angles, are organized as follows:

$$y_i = \theta_i \quad \& \quad y_{i+n} = |V_i|, i = 1, 2, \ldots, n \tag{8}$$

The system $g = 0$ is solved via Newton iteration as:

$$y^{p+1} = y^p - J^{-1} g^p \tag{9}$$

where $J = \partial g / \partial y$ is the Jacobian matrix. This matrix is computed as follows

1. Let A be an $n \times n$ matrix with elements A_{ik} computed as

$$A_{ik} = \bar{V}_i \left(\delta_{ik} \sum_{l=1}^{n} Y_{il} V_l - Y_{ik} V_k \right) \tag{10}$$

2. Let B be an $n \times n$ matrix with elements B_{ik} computed as

$$B_{ik} = V_i \overline{Y_{ik} \exp(\jmath\theta_k)} + \delta_{ik} \exp(\jmath\theta_i) \sum_{l=1}^{n} \overline{Y_{il} V_l} \tag{11}$$

3. The elements of the Jacobian matrix J are assembled from matrices A and B as follows:

$$\begin{aligned} J_{i,k} &= \operatorname{Im}\left(A_{i,k}\right) \\ J_{i,k+n} &= \operatorname{Re}\left(B_{i,k}\right) \\ J_{i+n,k} &= \operatorname{Re}\left(A_{i,k}\right) \\ J_{i+n,k+n} &= \operatorname{Im}\left(B_{i,k}\right) \quad i,k = 1,2,\ldots,n \end{aligned} \tag{12}$$

To facilitate specific modeling requirements for PV generators and swing buses, the following changes to the rhs vector g and Jacobian matrix J are implemented.

- For buses that are PV generators the active power and voltage magnitude are specified. To accomodate this setting, the components of g that correspond to this bus are set as follows:

  $$\begin{aligned} g_{i_{PV}} &= \operatorname{Re}(V_{i_{PV}} \sum_k \bar{V}_k \bar{Y}_{k i_{PV}}) - P_{i_{PV}} \\ g_{n_b+i_{PV}} &= 0 \end{aligned}$$

  The corresponding lines and columns of J are also set to zero, except the diagonal element which is set to 1.

  $$J_{n_b+i_{PV},n_b+i_{PV}} = 1 \;\;\&\;\; J_{n_b+i_{PV},i} = J_{i,n_b+i_{PV}} = 0, \quad i = 1,2,\ldots,n_b, i \neq i_{PV} \tag{13}$$

- Both the voltage magnitude and the angle are specified for the swing (SW) buses. For these buses, g entries corresponding to both voltage magnitude and angle are set to zero. Similar to the PV generators, the corresponding rows and columns of J are set to zero, except the diagonal elements which are set to 1.

  $$g_{i_{SW}} = g_{n_b+i_{SW}} = 0$$

  and

  $$J_{i_{SW},i_{SW}} = J_{n_b+i_{SW},n_b+i_{SW}} = 1 \tag{14}$$

  $$J_{i_{SW},i} = J_{i,i_{SW}} = J_{n_b+i_{SW},i} = J_{i,n_b+i_{SW}} = 0, \quad i = 1,2,\ldots,n_b, i \neq i_{SW} \tag{15}$$

The power and voltage values are usually solved for in “per unit” (p.u.) values. In other words these are normalized.

The Newton approach for solving the steady state power flow equations is summarized in Algorithm (2). The product $J^{-1}g$ in eq. (9) can be interpreted as the solution of the linear system $J \cdot \delta_y = g$ and it is solved numerically using the LAPACK [3] function *dgesv*.

Algorithm 2 Newton algorithm for steady state power flow system

repeat

Compute g using eq. (7)

Compute Jacobian matrix J using eqs. (12), (13), (14), (15)

Update system state y using eq. (9)

until $|J^{-1}g| < \epsilon$

4. NUMERICAL RESULTS

In this section we show numerical results generated with EGSim for two standard IEEE electric grid model problems downloaded from the Power Systems Test Case Archive [1]. Figure 4-1 shows the admittance matrices corresponding to these 2 problems. As expected these matrices are quite sparse since each bus is only connected with a small number of other neighbor buses. Additionally the clustering observed for the 300-bus case is indicative of islands formed by sparse connections between grid sections.

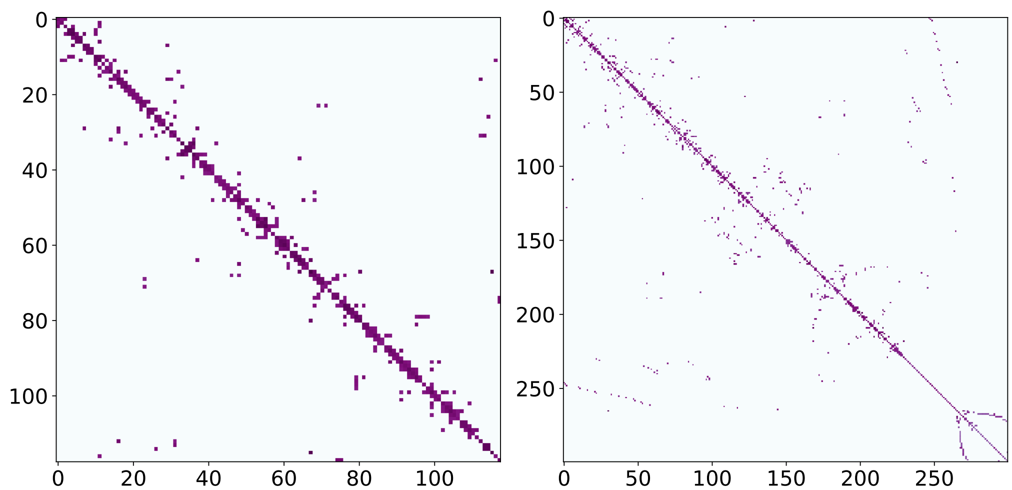

Figure 4-1. Magnitude of the admitance matrix components. Due to the larger nunber of matrix entries, the block size for the 300-bus model is smaller compared to the former model.

The toolkit outputs separate files for the bus voltages (in *statsol.dat*) as well as line power flows (in *linepow.dat*). Tables 4-1 and 4-2 show snippets corresponding to the results obtained for the 118 bus model.

Figures 4-2 and 4-3 display the bus voltages corresponding to the two models. In these figures, the reference bus angles are at 30° and 0°, respectively, for the two models.

Table 4-1. First few lines from file *statsol.dat* corresponding to 118 bus model. The table header is not present in the file.

Bus ID	Voltage Magnitude [p.u.]	Voltage Angle [deg]
1	0.9550	10.9921
2	0.9714	11.5298
3	0.9682	11.8672
4	0.9980	15.5857
5	1.0040	16.0041
...	...	...

Table 4-2. First few lines from file *linepow.dat* corresponding to 118 bus model. The table header is not present in the file.

Line ID	From Bus	To Bus	From Bus Injection P [p.u.]	From Bus Injection Q [p.u.]	To Bus Injection P [p.u.]	To Bus Injection Q [p.u.]
1	1	2	-0.1232	-0.1305	0.1242	0.1102
2	1	3	-0.3868	-0.1816	0.3893	0.1799
3	4	5	-1.0313	-0.5195	1.0336	0.5280
4	3	5	-0.6817	-0.1588	0.6942	0.1871
5	5	6	0.8847	0.0783	-0.8754	-0.0502
6	6	7	0.3554	-0.0477	-0.3548	0.0450
7	8	9	-4.4064	-0.8973	4.4525	0.2443
8	8	5	3.3872	1.1698	-3.3872	-0.8469
9	9	10	-4.4525	-0.2443	4.5000	-0.5104
...	...	...	...	...	...	...

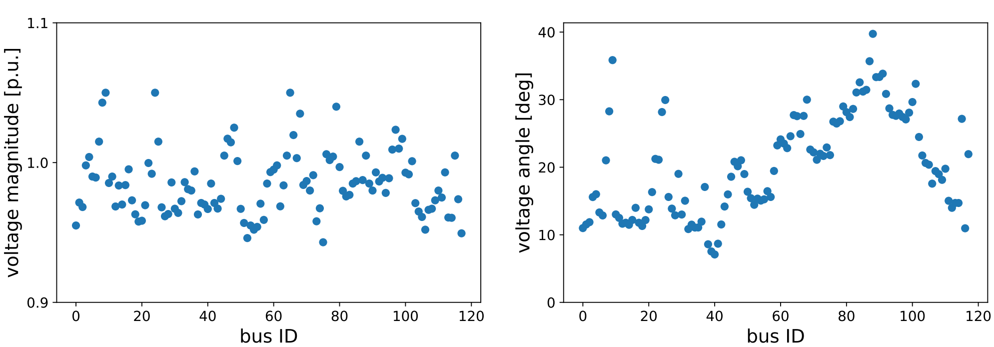

Figure 4-2. Voltage magnitude and angles for the 118 bus load flow solution. The swing bus reference angle is at $30°$.

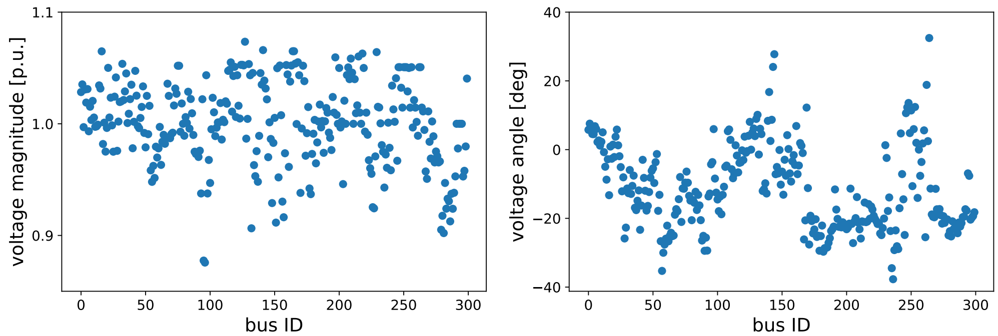

Figure 4-3. Voltage magnitude and angles for the 118 bus load flow solution. The swing bus reference angle is at $0°$.

5. SIMULATOR ARCHITECTURE AND EXECUTION

EGSim software is written in the *C++* language. The algorithm based on iterative Newton steps require links to the NetLib libraries *blas* and *lapack* [3].

The electric grid components are organized in C++ classes as follows

- **bus** - (in *egsim_bus.hpp*) contains information about buses. Currently 3 types of buses are implemented: swing, static PV generator, and PQ load.
- **line** - (in *egsim_line.hpp*) contains information about transmission lines. Whenever line data changes, i.e. lines are added to the system, the admittance matrix of the electric grid is recomputed.

The software currently can read power grid setup files in IEEE Common Data Format [12].

5.1. Installation Notes

The following installation instructions are also captured in *README.md* located under the EGSim root directory. Please refer to this file as it will be updated with up-to-date information more frequently compared to this report.

After downloading EGSim, create a *build* directory, preferably outside the directory where the source term was unpacked. A template configuration script is provided under the *config* directory. This script needs to be updated with local compiler paths and *install* destination. The software requires *cmake* to configure the build. A set of compatible Fortran 90 and C++ compilers are also required. This package has been been built and tested with GNU Compilers versions **5.5** through **7.2** on OSX 10.10.X – 10.13.X. An example *cmake* command with options is provided below.

```
cmake -DCMAKE_INSTALL_PREFIX:PATH=path-to-install-directory \
      -DCMAKE_Fortran_COMPILER=f90-compiler \
      -DCMAKE_CXX_COMPILER=c++-compiler \
      path-to-egsim-source
```

After the software is successfully configured, change to the *examples* directory. The enclosed bash script *run.sh* computes load flow solutions for two cases, the IEEE's 118 bus model and the 300 bus model, respectively. The output files are checked against saved solutions for bus-voltages and line power flows. If the user has python with numpy and matplotlib packages installed on the system it uses for EGSim, the test case will also produce the PDF files for the frames in Figures 4-1 to 4-3.

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