Option Pricing, Hedging, and Risk Analysis under Rough Volatility Models: Executive Summary

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Recent research has shown that the logarithm of volatility is well modeled using a fractional Brownian motion process with Hurst parameter of 0.1 and has led to the development of so called rough volatility models . The paths of fractional Brownian motion are rougher than that of standard Brownian motion and are more consistent with realized time series data. It is generally believed that in equity markets the overall shape of the implied volatility surface does not change but rather the level and orientation of the surface thus suggesting the surface to be modeled as a time homogenous process.

Naturally it is desirable to use some type of stochastic volatility model when pricing derivatives as opposed to the simple Black-Sholes model where volatility is assumed to be constant. Using a stochastic volatility model allows for the modelling of the implied volatility smile. However, most stochastic volatility models are not able to capture the term structure of at-the-money volatility skew particularly close to maturity. The term structure of ATM volatility skew is empirically observed as proportional to $1/\tau^{H-1/2}$, where H is the Hurst parameter, for a wide range of maturities. The rough Bergomi model is a is a stochastic volatility model that implements fractional Brownian motion in order to achieve this effect of modeling $\psi(\tau)$. The rBergomi model is still arbitrage free because fractional Brownian motion is not used on modeling the return of the stock but that of the instantaneous volatility.

This work begins by building a simulation framework capable of executing Monte Carlo simulation to generate sample paths of the stock price and variance processes. This implementation utilizes a hybrid scheme of convolution and Riemann sums to generate fractional Brownian motion and is also capable of setting the full initial forward variance curve. The problem of calibration is then considered and a neural network is trained to represent the pricing map from model parameters to implied volatilities with an average relative error ranging from -0.6% to 0.2%. Next a sensitivity analysis of the model parameters is performed to gain insight into what effect each has on changing the volatility surface. Finally, a vega risk analysis of several exotic options by analyzed by hedging implied volatility dynamics with a static hedge of vanilla calls.