Fictitious Play

Fictitious Play (FP) models the play of its associates and plays a *best response* with respect to this model. We consider here the most basic form of FP. We assume a two player game.

1. Algorithm Description

- Let player $-i$ be player i's associate (the player that is not player i).
- Let A_i and A_{-i} be the set of actions available to players i and $-i$ respectively.
- Let $a_i^t \in A_i$ be the action taken by player i at time t, and let $a_{-i}^t \in A_{-i}$ be the action taken by player $-i$ at time t.
- Let $r_i(a_i, a_{-i})$ be the payoff to player i when it plays a_i and its associate plays a_{-i}.
- $\kappa_i^t(b)$ is a weight function describing the # of times player i has observed player $-i$ play the action $b \in A_{-i}$ (as of time t). $\kappa_i^0(b)$ is a prior (perhaps obtained from observing player $-i$ interact with others).

After each round of the game, the weight function for each action $b \in A_{-i}$ is updated using:

$$\kappa_i^{t+1}(b) \leftarrow \kappa_i^t(b) + \begin{cases} 1 & \text{if } a_{-i}^t = b \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

FP forms an *assessment* $\gamma_i^t(b)$ (for all $b \in A_{-i}$) of its associate's play at each round of the game:

$$\gamma_i^t(b) = \frac{\kappa_i^t(b)}{\sum_{a_{-i} \in A_{-i}} \kappa_i^t(a_{-i})}. \tag{2}$$

Using this assessment and the payoff matrix, FP computes a *best response*:

$$a_i^t = \arg\max_{a \in A_i} \sum_{b \in A_{-i}} \gamma_i^t(b) r_i(a, b). \tag{3}$$

2. Formal Algorithm

FP for player i.

Initialize

(1) $t = 0$, form prior $\kappa_i^0(b)$ for all $b \in A_{-i}$.

Repeat

(2) Form assessment $\gamma_i^t(b)$ for all $b \in A_{-i}$ using Equation (2).
(3) Select action a_i^t using Equation (3).
(4) Observe a_{-i}^t and reward $r_i(a_i^t, a_{-i}^t)$.
(5) For all $b \in A_{-i}$, compute $\kappa_i^{t+1}(b)$ using Equation (1).
(6) $t \leftarrow t + 1$.