BSc Neurosciences

Introduction to statistics for neuroscientists

Cynthia Sandor

Edmond J Safra Assistant Professor
UKRI Future Leader Fellow
UK Dementia Research Institute Group Leader

UK Dementia Research Institute
Imperial College London
THE MICHAEL J. FOX FOUNDATION FOR PARKINSON'S RESEARCH
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Agenda

Session 1 - 45 minutes
Introduction & Motivation
Descriptive statistics & visualization
Probability basics & expected value
Distributions & Central Limit Theorem

Break 10 minutes

Block 2 - 45 minutes
Hypothesis testing, p-values, and errors
Multiple testing & effect size
Parametric vs non-parametric tests, t-tests
Experimental design & power
Summary & wrap-up

Why Statistics?

•**Statistics =** science of data (collection, organization, analysis, interpretation)

•**Central in neuroscience:** separating *signal* from *noise*

•**Key questions:**

•What is the same/different between groups?
•What is significant vs. insignificant?
•How to design studies with power?

Motivating example: Is Tasty Beer cheating its customers?

- “Is Tasty Beer cheating its customers?”

- Small deviations in 0.5L bottles → suspicion of underfilling

- How can we test this?

What are Descriptive Statistics?

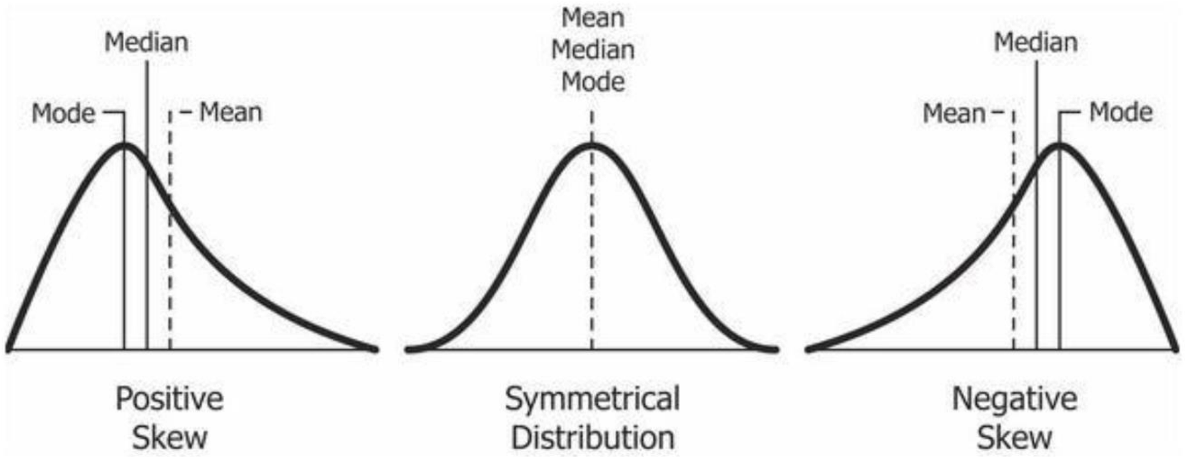
- Summarize and describe data
- First step before testing hypotheses
- Measures of central tendency & variability

Measures of Central Tendency

- **Mean:** Sum of all values divided by the total number of values

- **Median:** Middle value (when the data is arranged in order)

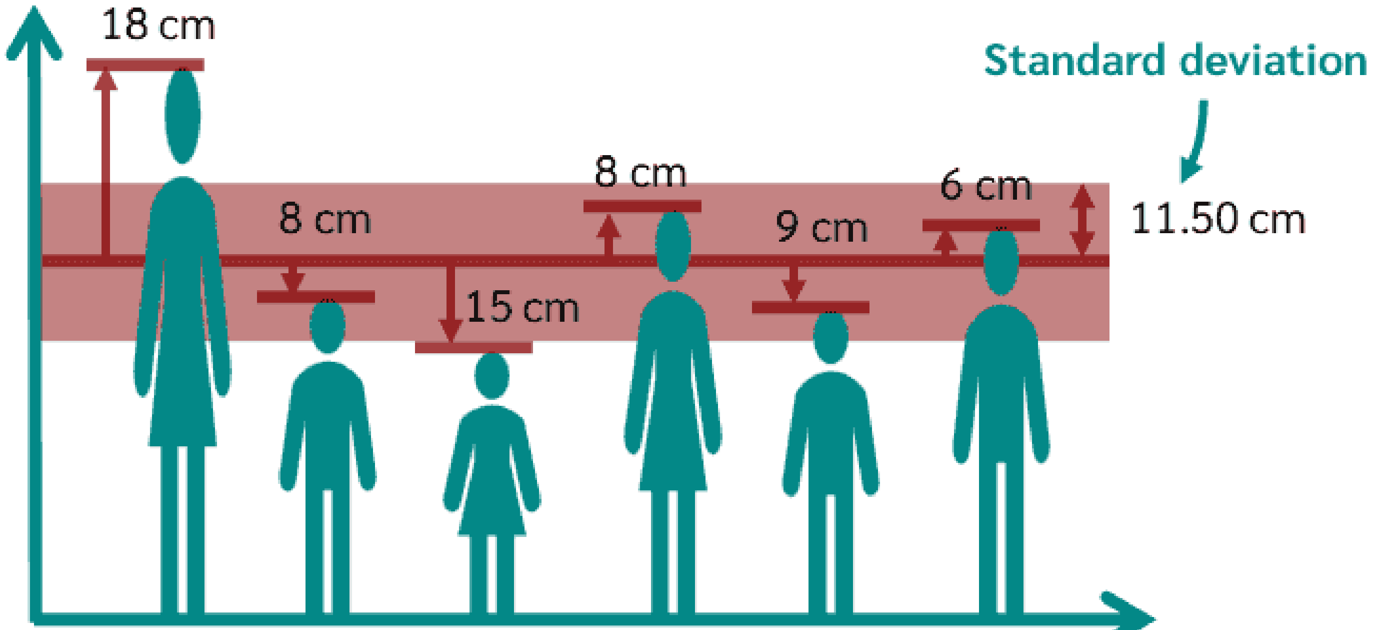
- **Mode:** Most common value (the value with the most frequency)



Variability

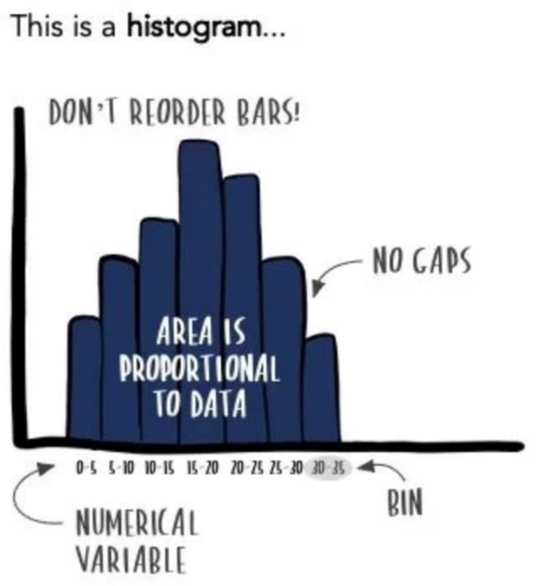
•**Range** = Highest value − Lowest value
→ Quick measure of spread
•**Variance (Var)** = Average squared distance from the mean

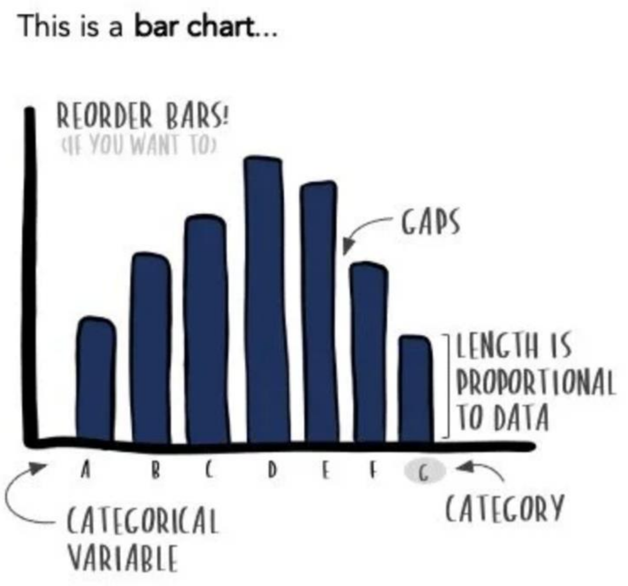
•**Standard Deviation (SD or σ)** = Square root of variance
→ "Average distance" of values from the mean

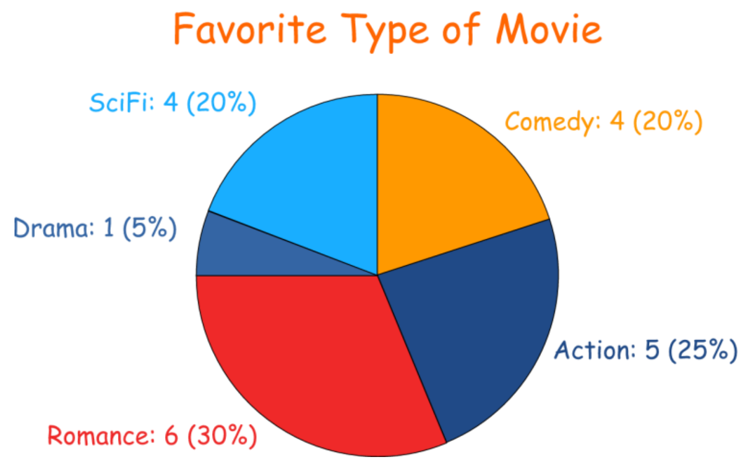


Visualisation Data

- **Histogram** → shows frequency of values in ranges
- **Bar chart** → compares categories
- **Pie chart** → shows proportions of categories







Visualisation Data

Boxplot (five-number summary):

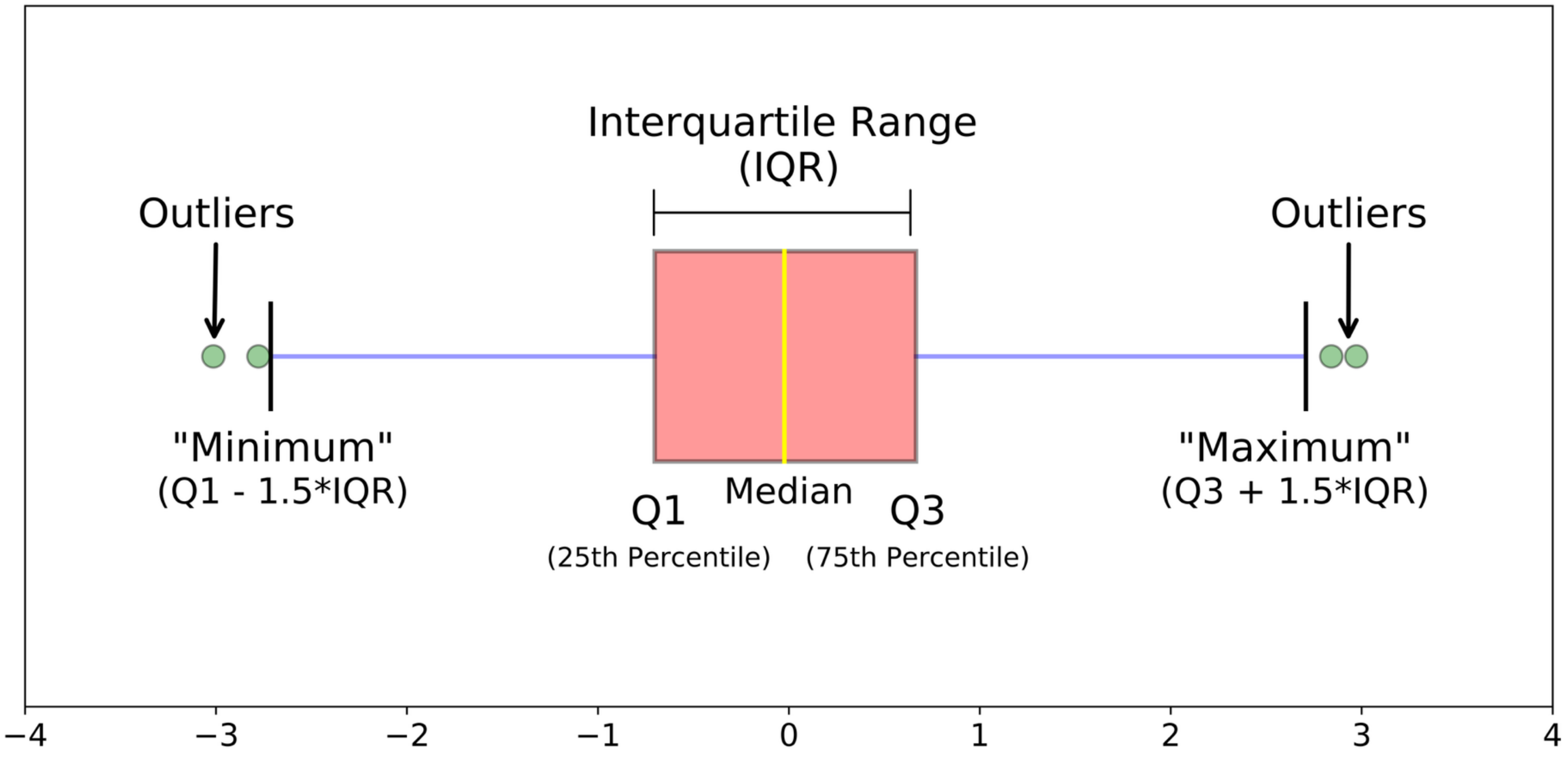


Table of summary statistics: Mean, Median, SD, etc.

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Probability Basics

•**Probability** = likelihood of an event (0 = impossible, 1 = certain)
Example The probability of getting heads in a coin toss

A probability is always between 0 (impossible) and 1 (certainty)

Certainty (Probability = 1):
A packed London Tube at rush hour (inevitable chaos!)

Impossibility (Probability = 0):
Snow falling in the Sahara

Somewhere in Between ($0 < p < 1$):
Probability that your favourite team will win

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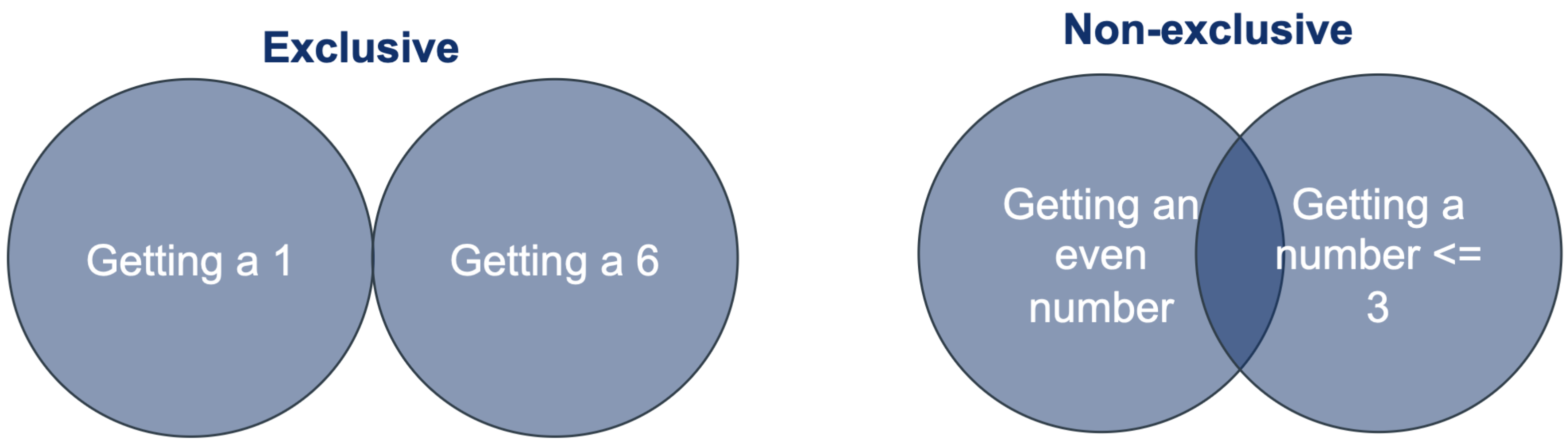
Probability Basics

Addition rule: The joint probability of two mutually exclusive events is the sum of their probabilities: P(A or B) = P(A) + P(B)

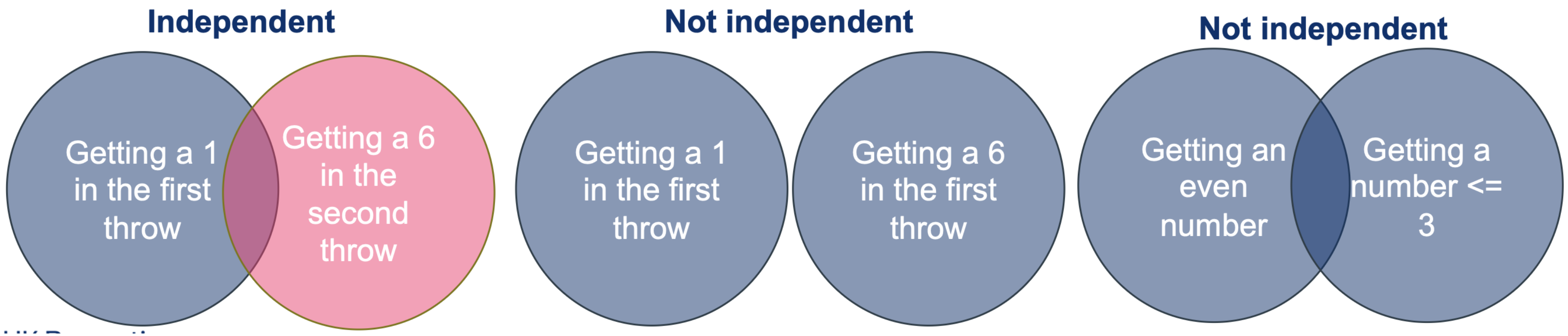
- **Mutually exclusive:** Only one of the events can occur



Probability Basics

Multiplication rule: The probability of two independent events both occurring is the product of their probabilities: P(A and B) = P(A) * P(B)

- **Independent =** the occurrence of one event does not affect the other
- Example: Getting heads in the first round and tails in the second round



Probability Basics

Calculating probability ! = Summing up events

- $\sum Events\ that\ lead\ to\ outcome / \sum All\ events$

- This only works for equally probable and mutually exclusive events

What is the probability of throwing an even number?

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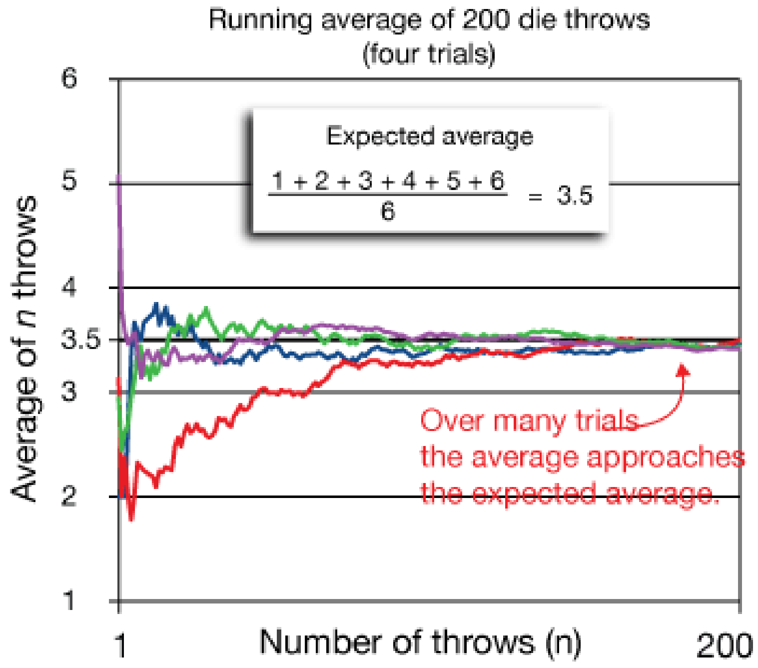
Expected Value & Law of Large Numbers

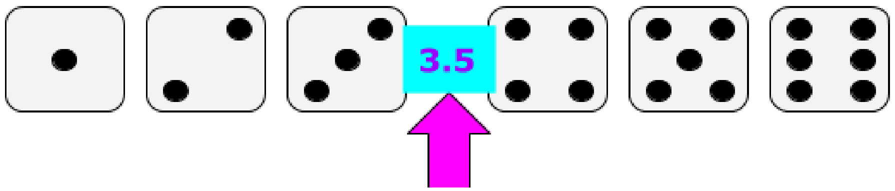
Expected Value (EV): long-run average outcome

Example (die roll): EV = (1+2+3+4+5+6)/6 = 3.5

Law of Large Numbers:

As n → ∞, the sample average → expected value





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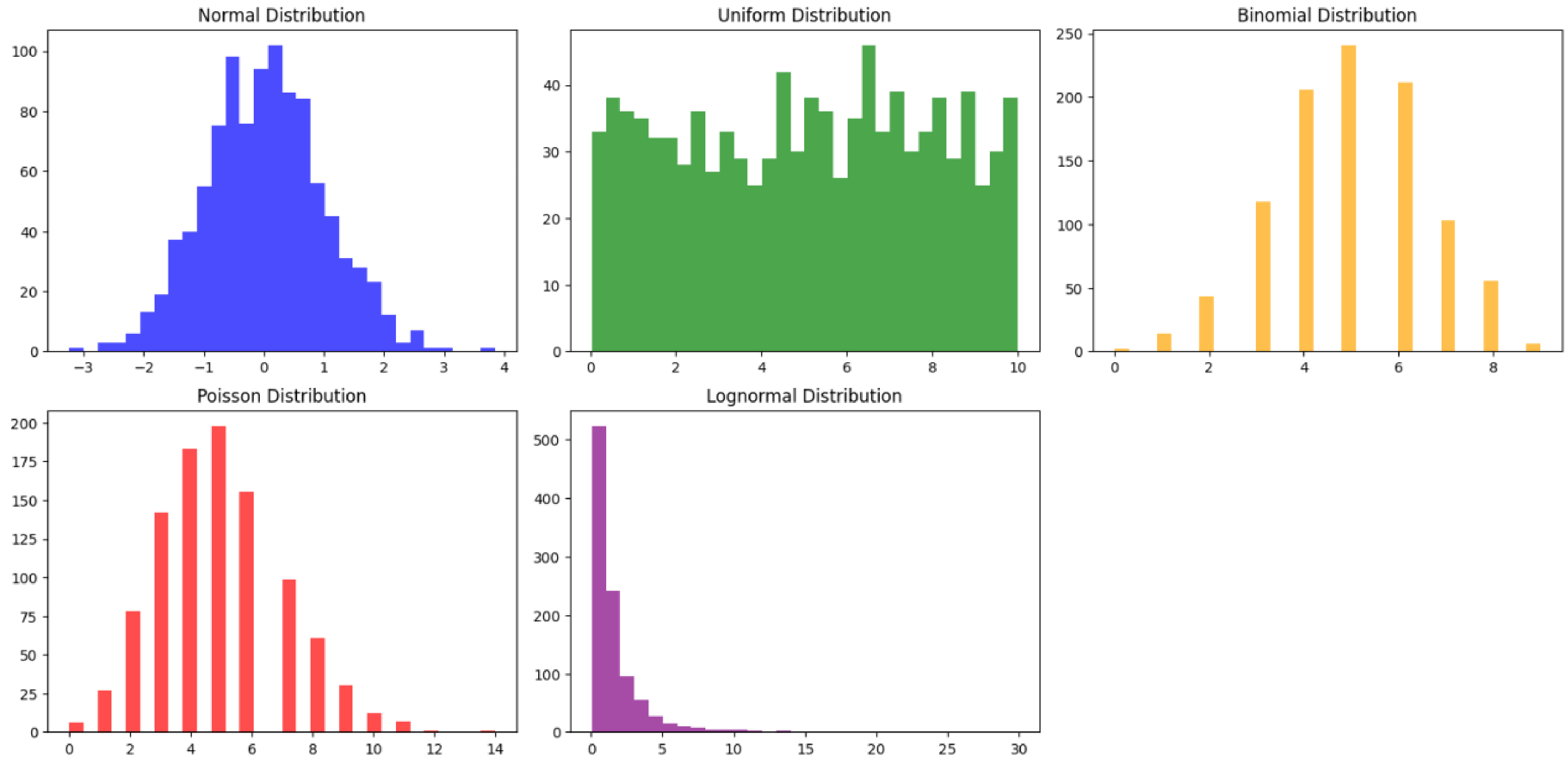
Probability Distributions

Discrete: only certain values (coin toss, number of goals a player has scored this season)

Continuous: any value in a range (height, car speed,)

Probability Distributions

- A **probability distribution** tells us how likely different outcomes are



Probability Distributions: common

- **Binomial distribution**: "yes/no" outcomes in repeated trials e.g. number of heads in 10 coin tosses

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

- **Poisson distribution: counts of events in a fixed time or space e.g. number of spikes in 1 second**

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

- **Normal distribution**: continuous "bell curve" e.g. eight, reaction time, blood pressure

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

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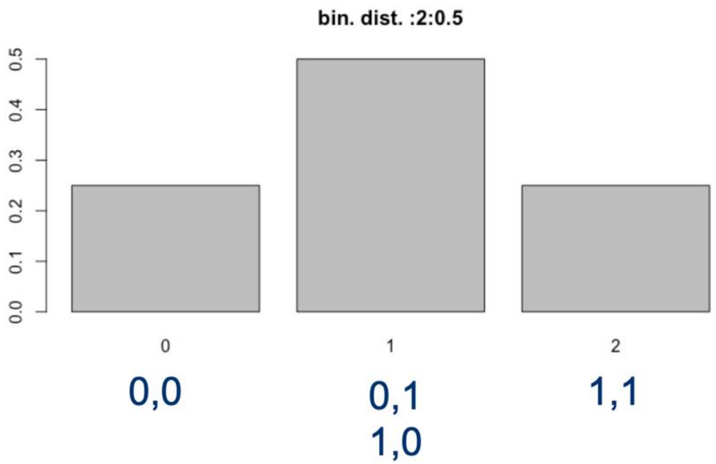

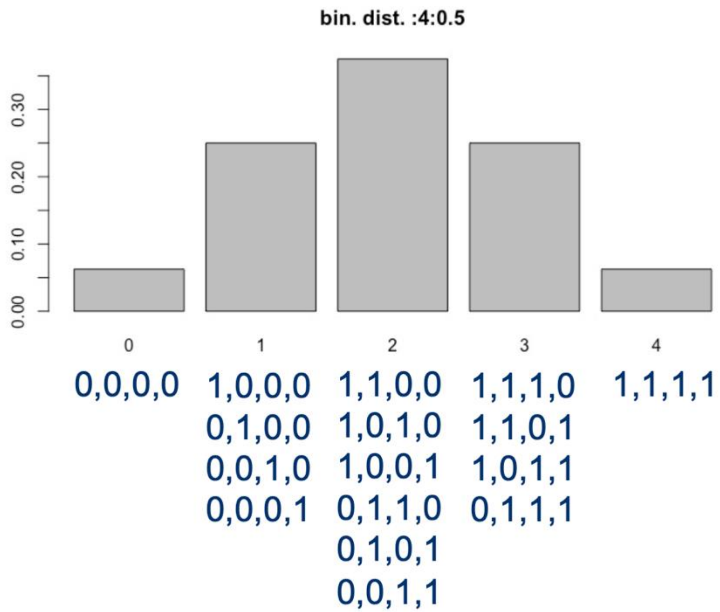

Probability Distributions: common

- **Poisson distribution: counts of events in a fixed time or space**

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

On average, 3.6 people arrive at a booking counter every 10 minutes on weekends. What is the probability that exactly 7 people arrive in 10 minutes?

Probability = 0.0424 (≈ 4.2%)

$$P(7; 3.6) = \frac{e^{-3.6} \cdot 3.6^7}{7!}$$

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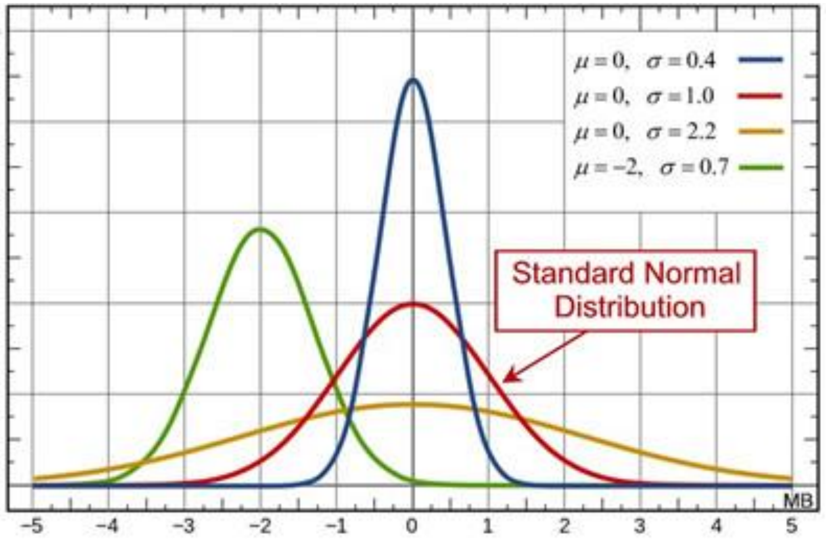
Probability Distributions: common

- **Normal distribution**: continuous “bell curve” e.g. eight, reaction time, blood pressure



$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

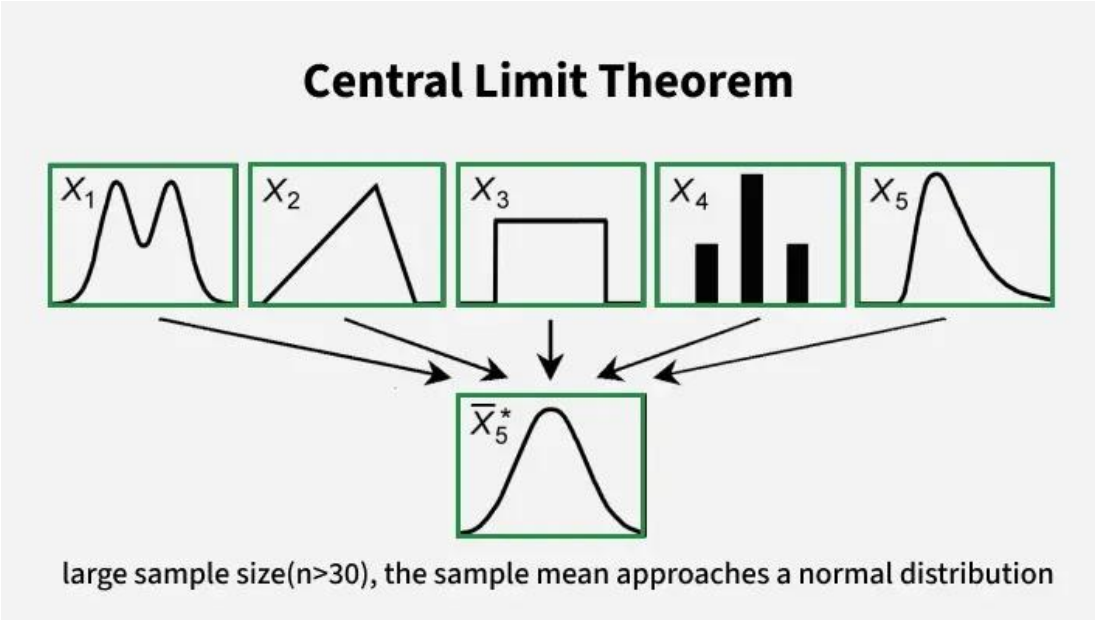
- Probability defined by only two parameters (mean and standard deviation)
- Symmetry
- Mean, median and mode are identical
- Approximately 95% of the area of a normal distribution is within two standard deviations of the mean

Central Limit Theorem (CLT)

•**Regardless of population distribution, the sample mean tends to be normal as sample size ↑**

•**The distribution of the sample mean:** $\bar{X} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$

•**Why it matters:** Justifies t-tests, ANOVA, regression, Explains why averaging trials reduces noise



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Quick Check Questions – Session 1

- Why do we need statistics in neuroscience?
- You collect reaction times (in ms) from 5 students: **220, 230, 400, 240, 250**.
  - What is the **mean**?
  - What is the **median**?
  - Which measure (mean or median) is more robust to the outlier (400)?
- A neuron fires on each trial with probability **p = 0.2**. What is the expected number of spikes in 10 trials?
- Suppose you flip a coin 5 times and calculate the proportion of heads. If you repeat this experiment many times, what will the distribution of these proportions look like as the number of repeats grows?

Hypothesis Testing & P-values

- **Hypothesis testing** asks: is the observed effect real or just chance?

- Two competing hypotheses:
    - **Null hypothesis (H_0):** no effect / no difference
    - **Alternative hypothesis (H_1):** there is an effect / difference

- **Test statistic** → measures signal vs. noise

- **P-value** = probability of observing this data (or more extreme) **if H_0 is true**

- If $p < \alpha$ (usually 0.05) → reject H_0

Hypothesis Testing & P-values

***Tasty Beer* produce 0.5l beer bottles**:

• There is a little bit of variation in the amount of beer in each bottle
• The consumer production agency thinks they are cheating their customers, so they take a random sample of bottles to look at:

Sample: 0.499 0.488 0.478 0.508 0.490 0.504 0.488 0.502 0.508 0.506

• How can they test whether *Tasty Beer* is really filling too little beer into their bottles?

Hypothesis Testing & P-values

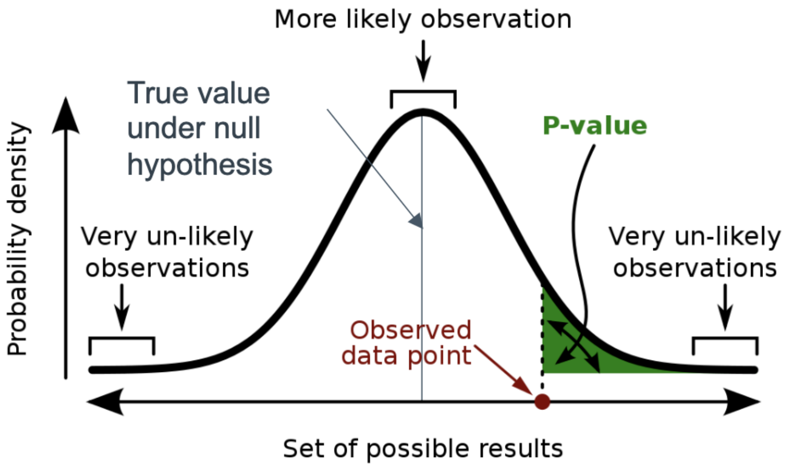
- H0 = *Tasty Beer* fills at least 0.5l into each bottle on average
- H1 = *Tasty Beer* fills less than 0.5l into each bottle on average

How likely are we to observe the sampled values if we assume that H0 is rue (*Tasty Beer* fills at least 0.5l on average into each bottle)?

Hypothesis Testing & P-values

What is the probability (P) of observing the sampled data under the null hypothesis? • If P is less than a selected threshold α (often 0.05), the null hypothesis is rejected – Statistical significant

The P value is defined as the probability, under the null hypothesis H0, of obtaining a result equal to or more extreme than what was actually observed – P(result | H0).



A **p-value** (shaded green area) is the probability of an observed (or more extreme) result assuming that the null hypothesis is true.

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Errors in Hypothesis Testing

- **Type I error (false positive):** reject H_0 when it's true

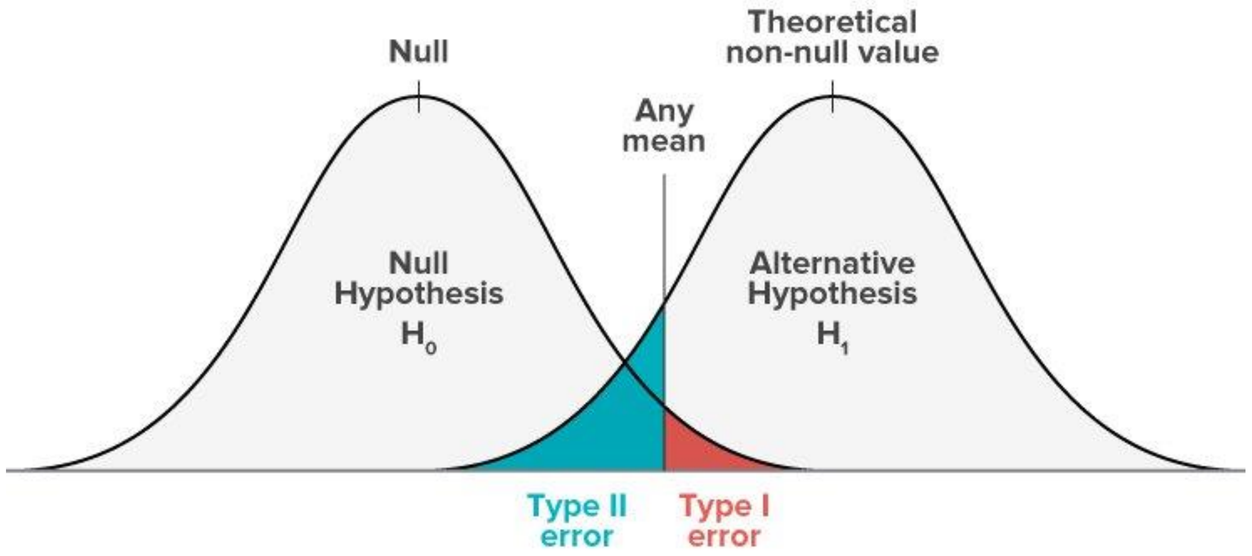
Probability = α (significance level, usually 0.05)

- **Type II error (false negative):** fail to reject H_0 when H_1 is true

Probability = β

- **Power = 1 − β** = chance of correctly detecting a true effect

Balancing errors is key to good study design



Multiple Testing Problem

- Each test has **false positive rate = α** (usually 0.05)
- Doing many tests increases chance of false positives
- Probability of at least one false positive after *n* tests:

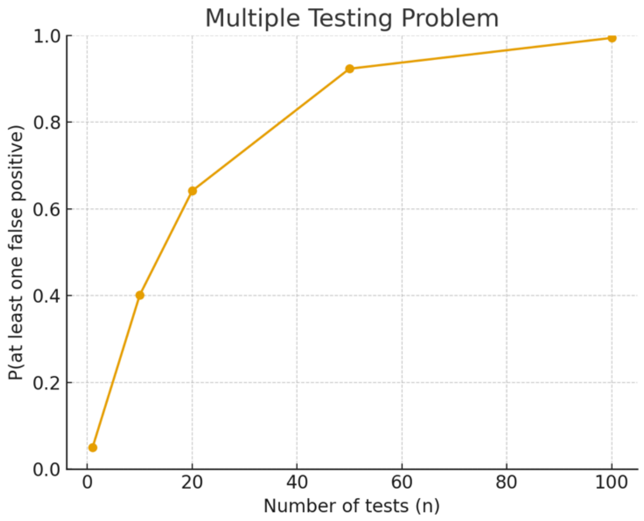
$$P = 1 - (1 - \alpha)^n$$

- Example:

10 tests → 40% chance of ≥1 false positive
20 tests → 64%
50 tests → 92%

$$\alpha_{\text{adj}} = \frac{\alpha}{n}$$

- **Solutions: Bonferroni correction**
- Other methods: FDR, Holm–Bonferroni



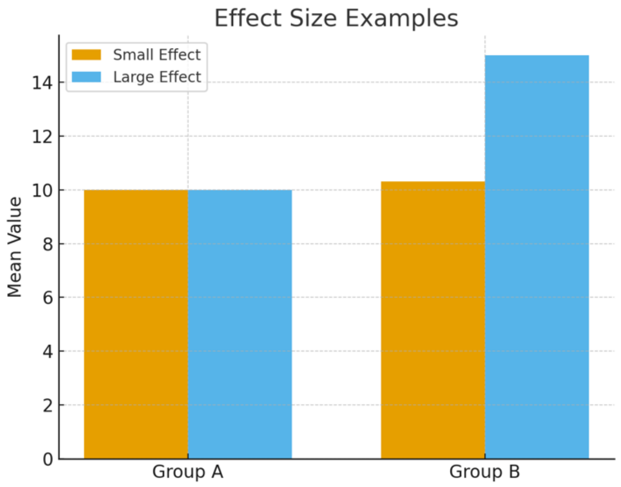
Effect Size

- **P-values** only tell us if an effect is likely not due to chance

- **Effect size** tells us how *big* the difference really is

- Examples:
  Clinical trial: drug reduces symptoms by 1% vs 30%
  Neuroscience: brain volume difference of 1 mm³ vs 100 mm³

- Common measures:
  Cohen's d (mean difference / SD)
  Correlation coefficient (r)



Parametric vs. Non-parametric Tests

- **Parametric tests**
  Assume data follow a known distribution (often normal)
  Examples: t-test, ANOVA, Pearson correlation

- **Non-parametric tests**
  "Distribution-free," no assumption of normality
  Examples: Wilcoxon, Mann-Whitney U, Spearman correlation

- **Why not always non-parametric?**
  Parametric tests usually have **more power**
  Easier to model population & confounders
  More flexible (e.g., regression models)

The T-test

- **Goal:** compare means
- Types:
  - **One-sample t-test**: compare mean to known value
  - **Independent two-sample t-test**: compare means of 2 groups
  - **Paired t-test**: compare before/after in the same group
- Formula

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

- Signal = difference from null mean
- Noise = standard error (spread / √n)

Experimental Design

- **Randomized Controlled Trials (RCTs):**
  Participants randomly assigned to control vs. experimental group
  Goal: reduce bias, isolate treatment effect
- **Advantages:**
  Washes out population bias
  Reduces observer bias
- **Challenges:**
  Needs large enough sample size
  Balancing cost vs. power

Power & Sample Size

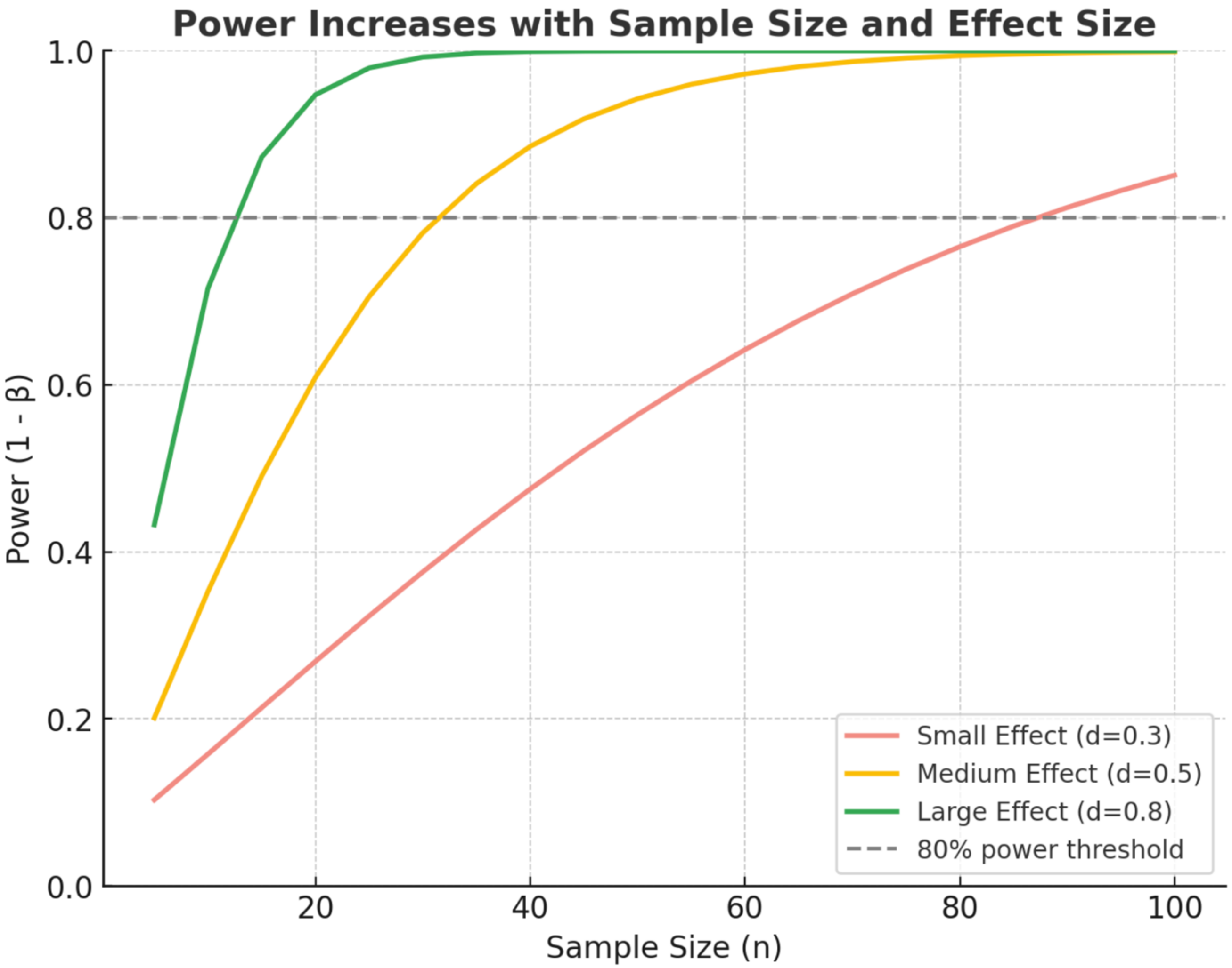
- **Power = 1 − β**
  - Probability of detecting a true effect
- Depends on:
  - **Effect size** (bigger effects easier to detect)
  - **Sample size (n)** (more data → less noise)
  - **Significance level (α)**

- **Formula (sample size estimate):**

$$n = \left(\frac{Z \cdot \sigma}{E}\right)^2$$

- Z = z-score for confidence (e.g., 1.96 for 95%)
- σ = standard deviation
- E = margin of error

Power & Sample Size



Quick Check Questions – Session 2

- You compare patient and control brain volumes.
  What is your **null hypothesis (H_0)**?
  What does a **p-value = 0.01** mean in this context?
- Imagine you run 20 independent statistical tests, each at $\alpha = 0.05$.
  What is the chance of getting at least one false positive?
  Why might effect size be more useful than just a p-value?
- You measure the number of siblings of 30 students. The data are skewed.
  Should you use a **parametric** or **non-parametric** test?
  If you compare pre- and post-training reaction times in the same group, which t-test do you use?
- Why is an underpowered study problematic in neuroscience?

Take-home Messages

- **Visualize your data first** – plots tell you more than tables

- **P-values ≠ truth** – always think about effect size

- **Beware of errors & multiple testing** – false positives are easy to create

- **Good design & power matter** – plan before you collect data