Lab 04

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Task 1

For this lab we had to estimate the parameters of a given ARX system with the help of loglikelihood and least squares estimation.

- *Simulate the system for N = 1000 samples with the input signal.*

For the given ARX system, I had chose the sigma values and the parameters. The parameters for θ_1 and θ_2 were 0.3 and 0.7 respectively. The sigma values were sqrt(1.2) whe k as smaller than 500 in the process and sqrt(3) when k was at elast 500. After this I have implemented the log_likelihood function with these parameters (sigma values) were hard coded in.

- *Plot the log-likelihood surface as a function of θ_1 and θ_2 and estimate the parameters.*

For the domain I have chosen [0,1] having 'n' elements as both θ_1 and θ_2 are in this range. For n I chose 200. The plot can be seen on *Figure 1*. For estimating the parameters, I have chosen the methods recommended in the seminar, which is to find the minimum of this surface and the indices corresponding to this values would index the correct theta values in their range array. Which values were $\theta_1 = 0.2864$ and $\theta_2 = 0.7186$ for one of the runs.

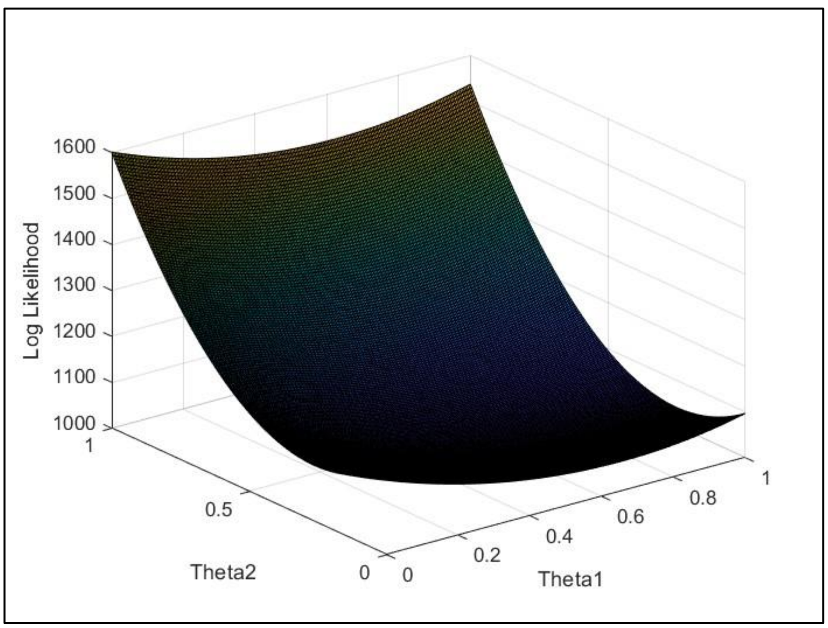

Figure 1 Loglikelihood for different parameter values for the given ARX system.

- *<u>Run 100 simulations of the model and also perform a simple least squares estimation. Compute and compare the covariance matrices.</u>*

For calculating the loglikelihood I have changed the number of theta values in the range [0 1] from 200 to a 100 so it would run faster. I lost precision by doing this decrease, however the main focus of this exercise was on the implementation on itself. I have stored the estimated parameters of the least squares estimation in *est_lse* while the parameters estimated by the help of the loglikelihood were stored in *est_log*. I have plotted the results which can be seen on *Figure 2*. The black lines represent the ground truth.

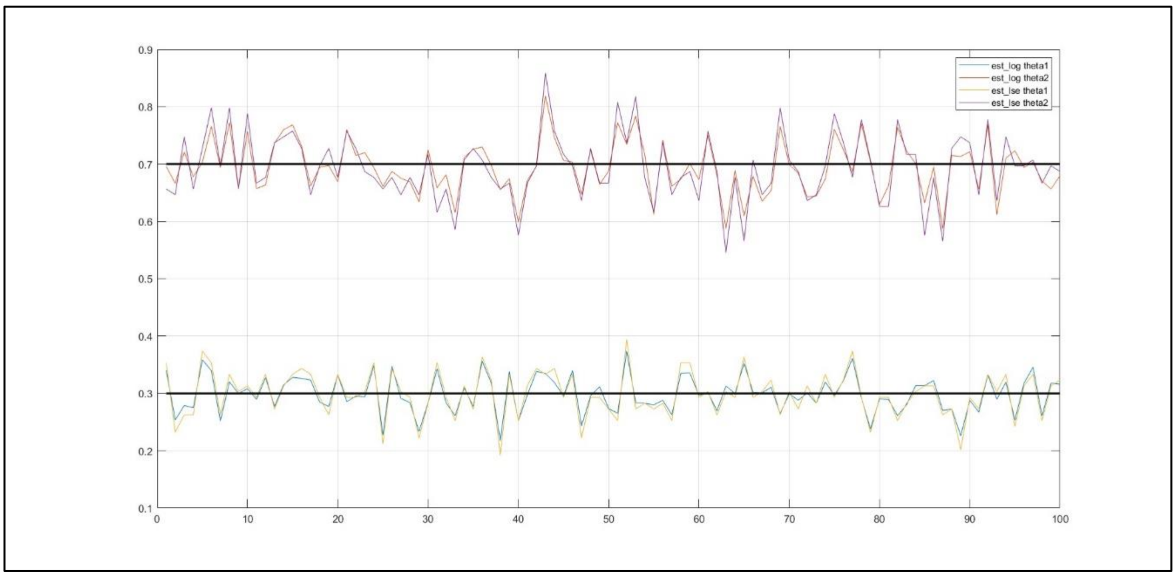

Figure 2 Values of est_log and est_lse, results of the 100 simulations. Black lines represent the ground truth.

The covariance matrices were the following:

1. Covariance matrix of estimation *est_log*:
   $$\begin{bmatrix} 0.0011 & -0.001 \\ -0.000 & 0.0027 \end{bmatrix}$$
2. Covariance matrix of estimation *est_lse*:
   $$\begin{bmatrix} 0.008 & 0.0 \\ 0.0 & 0.0018 \end{bmatrix}$$

In general, LSE estimation performed better, however it could change by increasing the number of theta values (increasing 'n') in the given range. There is no real variance between estimated values.