Lab 02

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Task 1

For the first task I had to implement LSQ estimator which for an input X and y finds the estimated parameters. I did implement it in *csado7_hw2_task1*. In further usecases I implement the same method in a different form in Task 2.

Task 2

- *1/2. Generate and Plot the generated data points:*

I have generated N points(200) with sigma(of epsilon) being 1. I have created the x datapoints as: $(1, x_1, x_2) = (1, i\times\text{randn}(), i\times\text{randn}())$ where i is a value from the interval [1, N] and randn() returns a random scalar drawn from the standard normal distribution. The plot can be seen on *Figure 1*.

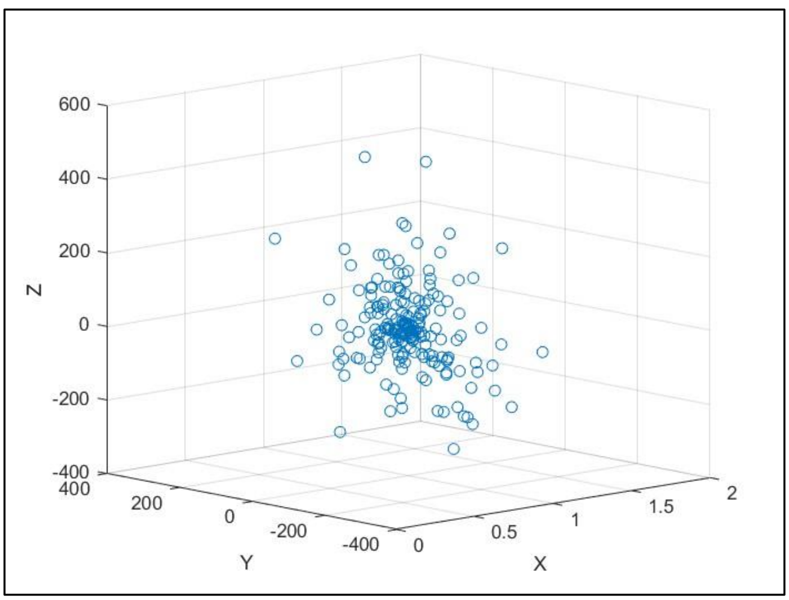

Figure 1 Data Points

- *3. Use least square estimation to estimate the parameters:*

With the LSQ Estimator I did get something similar to the original parameters, which I chose to be: p = (1 2 4). One of the estimations was: p_hat= (0.9776   2.0046   3.9999).

- *4. Calculate the loss function:*

I did calculate the loss function with the equation written in the lab notes. I have stored the value of it in *'L'*. In the first case it was around 223.489.

- 5. Show how the loss function depends on the amplitude of the noise:

I have created a vector having 11 elements, them being: sigma = [1,2, 5,8, 10,15,20, 40,50,80,100]. These values are different sigma values for the noise epsilon. Then I have calculated the LSQ Estimation and the Loss Function for all of these values then I have plotted how these losses change regarding these sigmas. This plot can be seen on *Figure 2*.

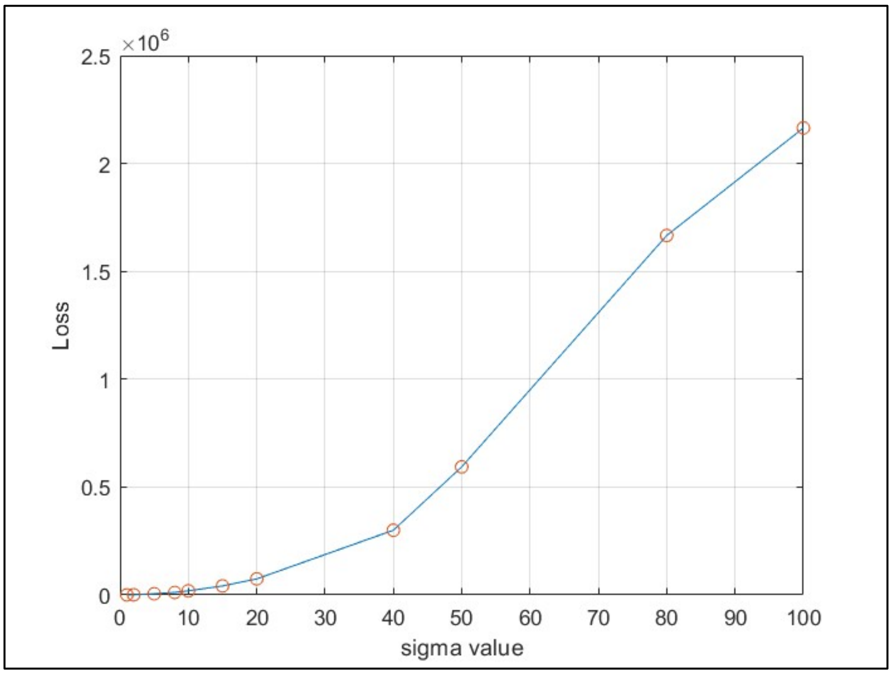

Figure 2 Losses for the corresponding sigma value.

- 6. Show how the loss function depends on the sample size:

I have created a vector having 11 elements, them being: N = [10, 50, 100, 200, 500, 800,1200,2000,5000,7500, 10000]. These values are different sample sizes. Then I have calculated the LSQ Estimation and the Loss Function for all of these values then I have plotted how these losses change regarding different sample sizes. This plot can be seen on *Figure 3*.

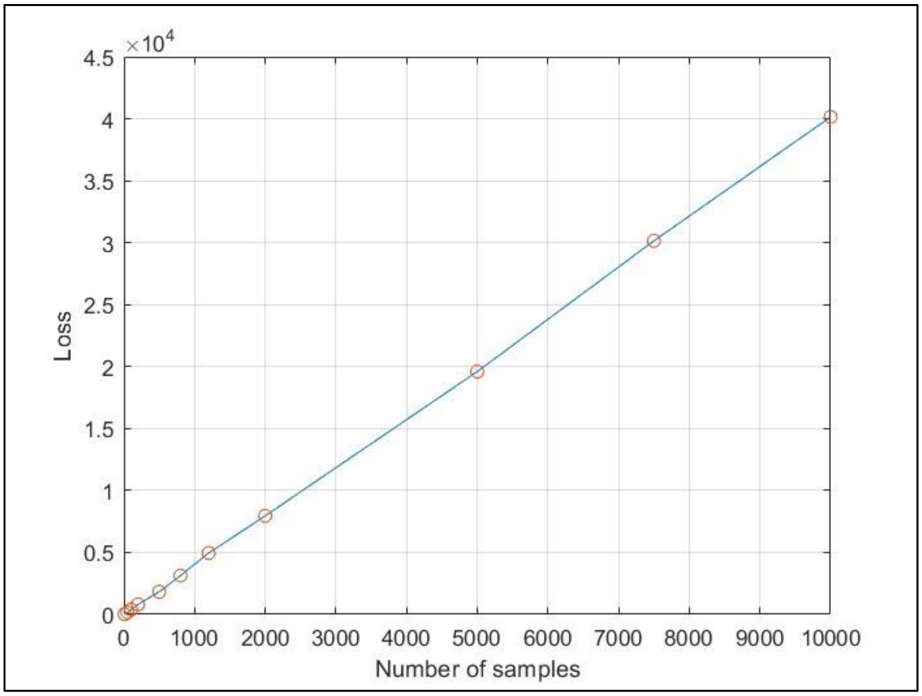

Figure 3 Losses for the corresponding sample sizes.

- <u>7. Create a 3D plot from the computations 5-6:</u>

I have made the same calculations as done before with sigma and the sample size changing simultaneously. This plot can be seen on *Figure 4*.

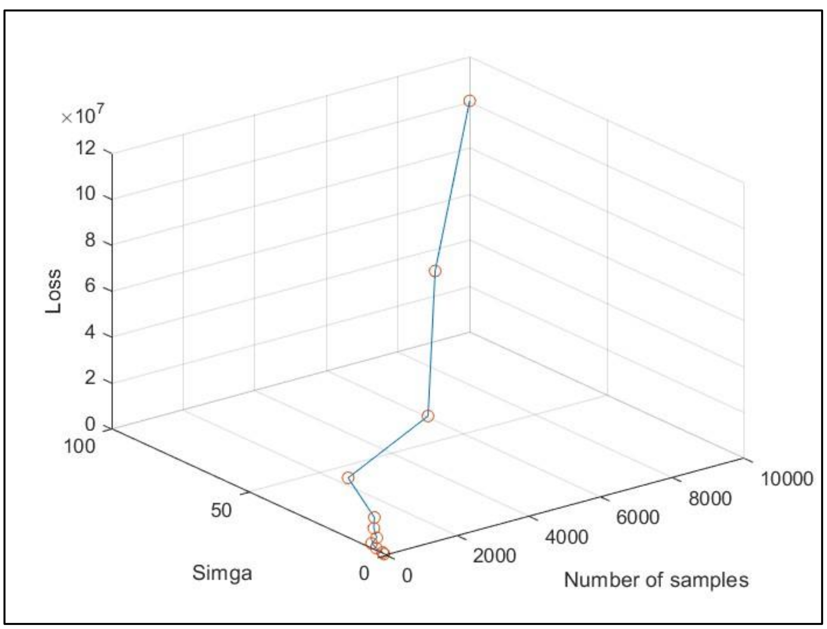

Figure 4 Losses for increasing Sample Size and Sigma.

Note, that having a larger sample should not have that high correspondace for noise in all the cases. This loss function is raw result of the distance of th e grand truth and the estimations.

- *<u>What is the expected value of θ_hatLS? How does the covariance matrix of θ_hatLS depend on Σϵ?</u>*

The expected value for the estimated parameters is to be close to the parameters(so p ~= p_hat in my case). For the convariance, as it is defined as the the product of their deviations from their individual expected values: If the noise covariance gets bigger, the individual expected values got smaller, so thereby the estimation is more precise(as proven in previous task for smaller sigmas) and the covariance matrix of θ_hat would increase as well.