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Road Slope and Vehicle Mass Estimation for Light Commercial Vehicle using linear Kalman filter and RLS with forgetting factor integrated approach

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Abstract— **This paper explains application of Kalman filter theory and recursive least squares algorithm with forgetting factor on real time estimation problem of light commercial vehicle mass and road grade on which motor vehicle moves. After a brief survey on mass and slope estimating in literature, there are proposed algorithms theoretical approaches and implementations on a real-time ECU. The test data are obtained from urban, extra-urban and highway experiments with prototypal vehicles.**

Keywords— Parameter estimation, Estimation Observers, Kalman filters, Recursive estimation, State estimation, Least square method, Filtering algorithms, Autoregressive processes.

I. INTRODUCTION

In a continuous effort to improve the control of longitudinal vehicle motion in very different terms it is essential to have to access on-line estimates of parameters as vehicle mass and road slope. Then, vehicle parameters variation plays an even larger role in automated control of vehicles (cars but especially light/heavy commercial vehicles, trucks, tractor & trailers, buses and so on), i.e. light commercial vehicles generally exhibit larger variations in parameters such as vehicle mass (up to 100% differences between loaded and unloaded configurations). Furthermore all main proposed fuel saving evaluation techniques, as in FIAT eco:Drive® system, are dependent on the knowledge of how the road ahead will behave, e.g. modest road grades may prove to be quite a challenge for vehicles with low power-to-weight ratio such as commercial vehicles, and how vehicle mass changes real-time and influences CO_2 emissions. These facts highlight the need for estimation of road slope and vehicle mass on a motor vehicle, in particular on Light Commercial Vehicle.

In general there are important practical needs that a real-time estimator should meet to be viable, especially for economy-priced vehicles. It should, for instance, be:

- Simple enough to run in real time despite onboard processing limitations; designed algorithms have to be implemented at the first in post-processing then in real-time environment with low memory allocation and computational complexity;
- Accurate enough to estimate desired vehicle state/parameter in defined conditions;
- Fast enough to detect changes in a vehicle's states/parameters shortly after it is started and driven onto the road. Estimation needs to be available in according with the activation windows and usability range of estimated information;
- Robust enough to operate successfully despite real signals disturbances and model plant uncertainties;

and especially,

- Inexpensive enough to penetrate the economy-priced vehicle market. This often translates into a minimal instrumentation requirement and no additional sensors.

With this clear point of view, this paper investigates the possibility of achieving slope and mass estimates using vehicle informations on propulsion system state, as motor torque and driveline signal states, together with ESP system (Electronic Stability Program) measures as wheel speed measurements and longitudinal accelerometer available on vehicle CAN network.

II. GENERAL MODEL BASED PROBLEM

A. *Brief survey of mass & slope estimations literature*

Our estimation approach is a model based approach, this aim is strictly linked to need to scale studied algorithms on different production vehicle with a modular re-design. Therefore we first formulate the vehicle longitudinal dynamics equation in general form. This general formulation is useful to understand plant non-linearity and/or time variance complexity.

The equation of motion at front wheels when the driveline is fully engaged and all mechanical power from engine is passed to the wheels, has the general form:

$$m_{at} \cdot \ddot{x} = m \cdot g \cdot \sin(\alpha) + F_P - F_R \qquad \text{with } m_{at} = m + \frac{J_{wheels}}{R^2} + \frac{\eta \cdot J_{motor}}{R^2 \cdot \tau_t^2 \cdot \tau_d^2} \tag{1}$$

Where m is total mass of vehicle, F_p is propulsion force and F_R is resistance force, g is gravitational acceleration, α is slope angle, R wheel radius and m_{at} is equivalent mass at propulsion wheels, so total mass with in addition inertial effects of wheels (J_{wheels}) and motor (J_{motor}) through gearbox and differential. Our unknowns are vehicle mass (m) and road slope ($sin(\alpha)$), where vehicle mass, in practice a model parameter, has very slow dynamics while road slope is a 'true' real-time physical quantity with a medium slow dynamics. Seen differential equation (1) is a classic non-linear, time-variant equation. The simultaneous estimation problem requires an Extended Kalman Filter (EKF) design, but our engineering goal is to realize an integrated estimator which is simplest, enough accurate, quite robust and inexpensive under computational complexity and hardware point of view. The literature presents many algorithms for online estimation of mass and slope. Bae et al., for instance, propose a recursive least squares estimator that utilizes longitudinal force, acceleration, and GPS-based road grade measurements to determine vehicle mass and aerodynamic drag (Bae et al., 2001 [2]). Lingman and Schmidtbauer investigate the possibility by Kalman filtering to estimate slope and mass using available information on propulsion and brake system characteristics, a vehicle speed measurement and the possible improvement through the addition of a longitudinal accelerometer (Lingman et al., 2001, 2002 [10,11]). Grieser proposes again a recursive least squares estimator in which aerodynamic drag forces are simulated online and subtracted from force measurements, rather than estimated (Grieser, 2005 [8]). Vahidi et al. propose a similar estimator that does not require road grade measurements and estimates vehicle mass, drag, and road grade simultaneously using minimal instruments. The algorithm accommodates the time-varying nature of aerodynamic drag and road grade through multi-rate forgetting (Vahidi et al., 2003, 2005 [16,17,18]). Winstead and Kolmanovsky propose an extended Kalman filter that estimates both longitudinal vehicle states and parameters (including mass) for adaptive cruise control (Winstead et al., 2005 [19]). Finally, Fradin propose a combined estimation in two steps: road slope estimation based on vehicle speed and longitudinal acceleration, and then on this base and drive train signals vehicle mass inference (Fradin, 2008 [6]). But historically have been proposed also estimation algorithms for the only mass/slope estimation (Turco et al., 1997 [15]); for example for vehicle mass, algorithms linked to sharp longitudinal accelerations and decelerations which excite vehicle's mass significantly, thereby making this mass easier to estimate. With this in mind, Breen proposes using sharp controlled accelerations and decelerations as part of an event-seeking mass estimation method (Breen, 1996 [4]). Similarly, Klatt proposes to estimate vehicle mass specifically during the sharp accelerations and decelerations introduced by gear shifting (Klatt, 1985 [9]). Reiner et al. propose a similar mass estimator that explicitly compensates for wheel inertia (Reiner et al., 1990 [14]). Further interesting extensions of same approach are proposed by Genise ([7]), Zhu et al. ([20,21]), Bellinger et al., ([3]) and Leminoux et al. ([12]). Otherwise, there were also algorithms for estimation of road slope independently from vehicle mass based on longitudinal acceleration model (Turco et al., 1997 [15]) and (Altenkirch M., 2003 [1]) or based on Kalman filtering applied to same longitudinal model (Lingman et al., 2001, 2002 [10,11]) and further interesting extensions of same approach by (Corigliano et al., 2007 [5]) inferring also vehicle speed sign.

III. PROPOSED MASS AND SLOPE ESTIMATION ALGORITHMS

In coherence with explained simplicity and sustainability of estimations the possibility to obtain our estimation with a single extended states estimator, with its relative problems about convergence, different represented dynamics and non-linearity, is put aside. In short we find suitable to simplify and reduce to the most basic form the problem with a top-down approach. In practice we try to estimate separately road slope and vehicle mass by using different model references and real vehicle measurements, in order to obtain two independent estimations, and then we try to integrate estimations in order to improve estimation performance in a model based framework. With this theoretical base, selected algorithm for implementation of slope observer is a linear Kalman filter based on vehicle longitudinal acceleration model and for mass estimation has been chosen Recursive Least Square with forgetting factor applied to equation (1).

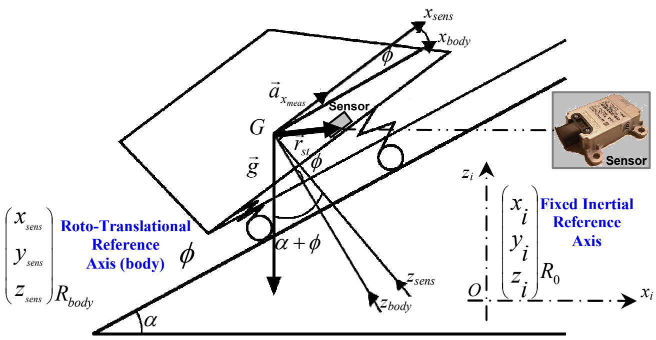

Fig. 1. Longitudinal acceleration measure

A. Slope observer Kalman filtering

To define our model reference observer is fundamental to describe plant with a detailed model useful to quantify entity of different terms in play. In particular we try to express in canonical form longitudinal acceleration measured by sensor in order to understand disturbances and system states for estimation.

According to rigid body dynamics, we can describe sensor position and direction in fixed inertial reference axis always parallel to road R_0 (see fig. 1) as:

$$\begin{pmatrix} x_{sens} \\ y_{sens} \\ z_{sens} \end{pmatrix}^{R_0} = \vec{r}_{sens} = \vec{r}_{OG} + R^{O}_{body} \cdot \vec{r}_{st} \quad (2)$$

Where R_{body} is a reference axis fixed with vehicle which rotates on vehicle CoG of pitch angle . Deriving equation (2):

$$\vec{v}_{sens} = \vec{v}_{OG} + \dot{R}^{O}_{body} \cdot \vec{r}_{st} + R^{O}_{body} \cdot \dot{\vec{r}}_{st} \quad (3)$$

and again,

$$\vec{a}_{sens} = \vec{a}_{OG} + \ddot{R}^{O}_{body} \cdot \vec{r}_{st} + 2 \cdot \dot{R}^{O}_{body} \cdot \dot{\vec{r}}_{st} + R^{O}_{body} \cdot \ddot{\vec{r}}_{st} + \vec{g} \quad (4)$$

$$\begin{pmatrix} \ddot{x}_{sens} \\ \ddot{y}_{sens} \\ \ddot{z}_{sens} \end{pmatrix}^{R_0} = \vec{a}_{sens}^{R_0} = \vec{a}_{OG} + R^{O}_{body} \cdot \ddot{\vec{r}}_{st} + \dot{\omega} \times \left(R^{O}_{body} \cdot \vec{r}_{st} \right) + \dots \quad (5)$$

$$\dots + \omega \times \left(\omega \times \left(R^{O}_{body} \cdot \vec{r}_{st} \right) \right) + 2 \cdot \omega \times \left(R^{O}_{body} \cdot \dot{\vec{r}}_{st} \right) + \vec{g}$$

We obtain three axis accelerations of sensor body in R_0. Then in R_{body}:

$$\vec{a}_{sens}^{R_{body}} = R_o^{body} \cdot \vec{a}_{sens}^{R_o}$$

only about longitudinal axis:

$$a_{x_{sens}}^{R_{body}} = \ddot{x}_G \cdot \cos\phi - \ddot{z}_G \cdot \sin\phi + \ddot{\phi} \cdot z_{sens} + \dots \quad (6)$$

$$\dots - \dot{\phi}^2 \cdot x_{sens} + g \cdot \sin\phi$$

Then, if we consider the road slope angle α:

$$a_{x_{sens}}^{R_{body}} = \ddot{x}_G \cdot \cos\phi - \ddot{z}_G \cdot \sin\phi + \ddot{\phi} \cdot z_{sens} + \dots \quad (7)$$

$$\dots - \dot{\phi}^2 \cdot x_{sens} + g \cdot \sin(\phi + \alpha)$$

This equation is a non-linear, but time-invariant and can be approximate with a general already known linear, time-invariant relation:

$$a_{x_{Meas}} = a_x + a_{disturb} + g \cdot \sin\alpha \quad (8)$$

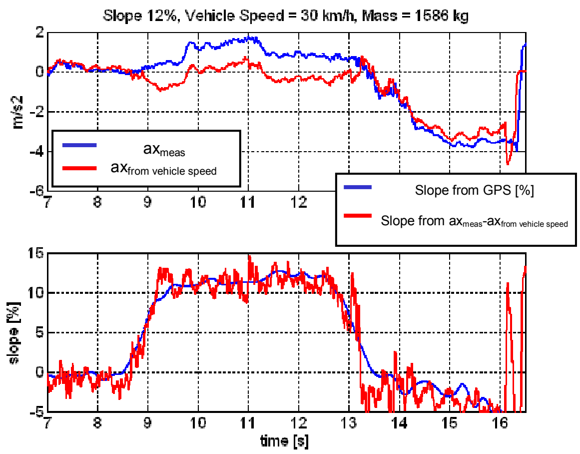

Fig. 2. Longitudinal acceleration measure vs derived from longitudinal vehicle speed

This equation is useful to estimate road slope independently from vehicle mass using the only longitudinal acceleration model and linear Kalman filtering theory. To understand in a superior way the correctness of (8) equation, in Fig. 2 see a compare performed on a 12% well known slope road with one of our prototypal vehicles between GPS measured slope (in blue) and raw slope estimation (in red) obtained from (8) equation as difference between measured longitudinal acceleration and derived vehicle speed. Term $a_{disturb}$ is the part

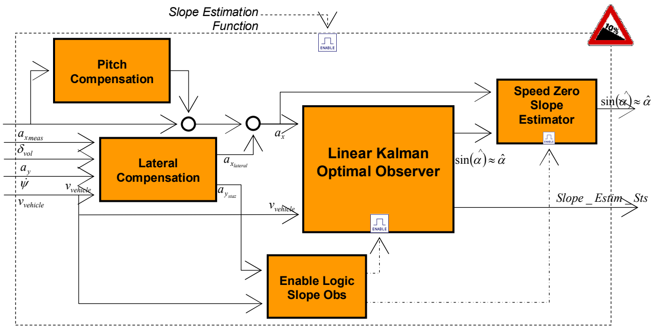

Fig. 3. Road slope estimator functional architecture

of the vehicle's acceleration caused by disturbances not described by the model for the longitudinal dynamics. It should cover all the model errors found in this section. So, given a good description on how the state $a_{disturb}$ is changing, a Kalman filter can be used to estimate system states and to filter this disturbance. The Kalman filter is then designed using:

$$x = [v_{vehicle} \quad \sin\alpha]^T ;\; u = a_{x_{Meas}} ;\; y = v_{vehicle}$$

so that,

$$\dot{v}_{vehicle} = a_x = -g \cdot \sin\alpha + a_{x_{Meas}} - a_{disturb} \Rightarrow \dot{x}_1 = -g \cdot x_2 + u + \upsilon \quad (9)$$

All the process state variables are considered gaussian stochastic ones, so the assumed noise covariance matrix has been defined in coherence with physical characteristics of stochastic variables as seen in (7), and then tuned in order to get the best estimation possible. In this estimation framework, term $a_{disturb}$ may be modeled as a simple process noise υ over slope state.

About slope state we have tried to model according to two different approaches, the first time under the assumption that the state x_2 undergoes slight changes each sampling period (Lingman et al., 2001, 2002, [10,11]), so its first derivative is equal to Gaussian noise,

$$\dot{x}_2 = \omega \quad (10)$$

and a second method according to a first-order Gauss-Markov process (Corigliano et al., 2007, [5]):

$$\dot{x}_2 = -\frac{1}{\tau} \cdot x_2 + \xi \quad (11)$$

Identifying τ parameter according to slowest slope which we have to observe. This last approach is useful especially to filter desired slope dynamics from pitch disturbance, so give the possibility to eliminate pitch disturbance leaving only slope information. While, according to (10) approach, there is need to pre-filter longitudinal acceleration from pitch dynamics also partially. Both approaches are valid and give us interesting estimation results. We prefer (10) approach, so using a partially pitch corrected longitudinal acceleration as input for explained observer.

In addition to described optimal 'dynamic' observer (see 'Linear Kalman Optimal Observer' block in fig. 3) has been developed also a 'Speed Zero Slope Observer' which is activated when dynamic filter is switched off, so in wheel speed measurements blind windows, and smoothly joins dynamic slope estimation to static slope estimation based on the only acceleration measure from inertial sensor avoiding final braking pitch and other noisy effects.

B. Mass Recursive Least Square estimation with forgetting factor

For mass estimation part has been developed an estimation algorithm based on longitudinal motion in a condition that a vehicle accelerates with clutch engaged and without turning or turning with a limited lateral acceleration, as described in (1), with these additional hypotheses:

- Nominal known drag and rolling resistances;
- No limitation about available surface adhesion;

The mechanical equilibrium equation at front wheels is:

$$\eta \cdot C_{traction} \cdot \tau_{diff} \cdot \tau_{gear} = \left(M \cdot R + \frac{\eta \cdot J_{mot} \cdot \tau^2_{diff} \cdot \tau^2_{gear}}{R} + \frac{J_{wheels}}{R} \right) \cdot \ddot{x} \quad (12)$$

where τ_{diff} and τ_{gear} are respectively differential and gearbox ratio. $C_{traction}$ is traction torque, so applied motor torque at wheels minus frictions torque from powertrain estimations. In discrete version with sampling time T:

$$\int_{(k-1)T}^{kT} C_{traction} dt = \ldots \quad (13)$$
$$\left(\frac{M \cdot R}{\eta \cdot \tau_{diff} \cdot \tau_{gear}} + \frac{J_{mot} \cdot \tau_{diff} \cdot \tau_{gear}}{R} + \frac{J_{wheels}}{R \cdot \eta \cdot \tau_{diff} \cdot \tau_{gear}} \right) \cdot \ldots$$
$$\left(\dot{x}(kT) - \dot{x}((k-1)T) \right)$$

Approximating the first term of equation with 'Bilinear Transformation Method':

$$\int_{(k-1)T}^{kT} C_{traction} dt = \left[(1-\alpha) \cdot C_m((k-1)T) + \alpha \cdot C_m(kT) \right] \cdot T \quad with\ 0 \le \alpha \le 1$$

with α=1 (Backward Euler Discretization):

$$C_{traction}(k) = \left(\frac{M \cdot R}{\eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T} + \frac{J_{mot} \cdot \tau_{diff} \cdot \tau_{gear}}{R \cdot T} + \frac{J_{wheels}}{R \cdot \eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T} \right) \ldots \quad (14)$$
$$\cdot \left(\dot{x}(k) - \dot{x}((k-1)) \right)$$

In order to be robust to slope disturbance we can pass to consider in place of vehicle speed difference, measured longitudinal acceleration multiplied to sampling time:

$$C_{traction}(k) = \left(\frac{M \cdot R}{\eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T} + \frac{J_{mot} \cdot \tau_{diff} \cdot \tau_{gear}}{R \cdot T} + \frac{J_{wheels}}{R \cdot \eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T} \right) \ldots \quad (15)$$
$$\cdot (\ddot{x}(k) \cdot T)$$

This longitudinal equation (15) can be rewritten in following regression form:

$$y(k) = \varphi^T(k) \cdot \theta(k) + \xi(k) \quad (16)$$

with:

$$y(k) = C_{traction}(k)$$

$$\varphi^T(k) = \ddot{x}(k) \cdot T$$

$$\theta(k) = \frac{M \cdot R}{\eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T} + \frac{J_{mot} \cdot \tau_{diff} \cdot \tau_{gear}}{R \cdot T} + \frac{J_{wheels}}{R \cdot \eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T}$$

From $\theta(k)$, vehicle mass M:

$$M = \frac{\theta(k) \cdot \eta \cdot \tau_{diff} \cdot \tau_{gear} \cdot T}{R} - \frac{\eta \cdot J_{mot} \cdot \tau_{diff}^2 \cdot \tau_{gear}^2}{R^2} - \frac{J_{wheels}}{R^2} \quad (17)$$

Where identified $\theta(k)$ is the parameter vector and $\xi(k)$ denotes lumped disturbance which can degrade the estimation performance of vehicle mass and which must be minimized. In order to overcome this degradation has been used RLS algorithm with forgetting factor (μ) which minimizes prediction error according to quadratic principle, and reduces RLS problem about progressive decay of algorithm reactivity with sampling aging (Parkum et al., 1992, [13]):

$$\begin{aligned}
&\theta(k) = \theta(k-1) + K(k) \cdot \varepsilon(k) \\
&K(k) = Vff(k) \cdot \varphi(k) \\
&\varepsilon(k) = y(k) - \varphi^T(k) \cdot \theta(k-1) \\
&Vff(k) = (1/\mu) \cdot \left(Vff(k-1) - \beta_{k-1}^{-1} \cdot Vff(k-1) \cdot \varphi(k) \cdot \varphi^T(k) \cdot Vff(k-1) \right) \\
&\beta_{k-1} = \mu + \varphi^T(k) \cdot Vff(k-1) \cdot \varphi(k)
\end{aligned} \quad (18)$$

Where $Vff(k)$ is not so different from estimation variance for every sampling period and forgetting factor μ has been managed in a typical mode for estimation of constant parameters:

$$\begin{aligned}
&\mu(k) = \rho \cdot \mu(k-1) + (1-\rho) \\
&\mu(0) = \mu_0 \\
&with\ \rho \in (0;1)\ and\ \mu_0 \in (0;1)
\end{aligned} \quad (19)$$

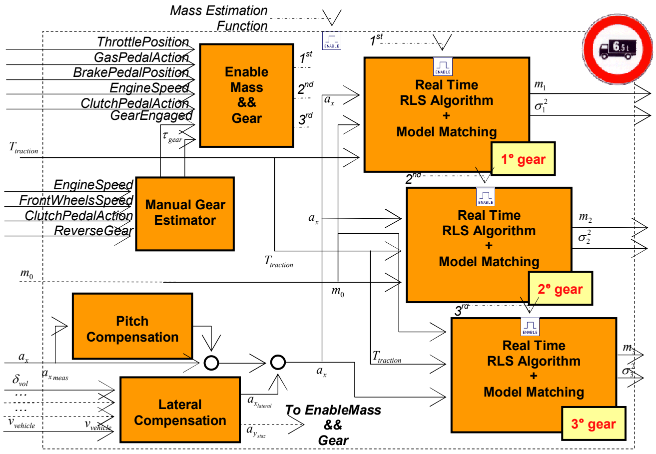

Fig. 4. Vehicle mass estimator functional architecture

Explained algorithm has been computationally replied for lower powertrain gears, i.e. 1st, 2nd and 3rd, and activated where longitudinal acceleration entity and powertrain "torque estimated measure" are significant (see fig. 4). Mixed mass estimation from different gears is obtained by using a weighted mean where performed choice about weights is:

$$\omega_i = \frac{1}{\sigma_i^2}.$$

The weighted mean in this case is:

$$\overline{M} = \frac{\sum_i \left(M_i / \sigma_i^2 \right)}{\sum_i \left(1 / \sigma_i^2 \right)}$$

and the variance of the weighted mean is:

$$\sigma_{\overline{M}}^2 = \frac{1}{\sum_i \left(1 / \sigma_i^2 \right)}$$

The significance of formulation in fig. 5 is that this weighted mean is the maximum likelihood estimator of the mean of the probability distributions under the assumption that they are independent and normally distributed with the same mean.

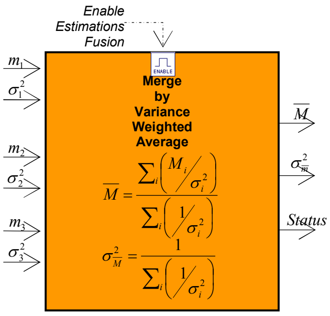

Fig. 5. Vehicle mass estimators fusion

C. *Real time estimations integration*

Estimation functions have been developed to be robust to reciprocal disturbance, in practice slope disturbance hasn't influence on mass estimation by using suitably dynamic pitch-compensated longitudinal acceleration measure instead of vehicle wheels speed, while slope estimation is based on longitudinal acceleration sensor model, so, it guarantees robustness to powertrain uncertainties and mass varying by avoiding vehicle motion equation use. Otherwise mass varying disturbance is not fully rejected, there is a not erasable disturbance on longitudinal acceleration measure, static pitch acceleration offset due to vehicle mass distribution. If you consider a vehicle which runs on a slope with constant speed or still on flat road, there is a static offset on longitudinal acceleration sensor measure (see suspension schematization in fig. 1), this error is due to load distribution on vehicle, and its entity is linked to vehicle suspension stiffness. This last disturbance on slope estimation may be reduced by integrating slope and mass estimations. So, knowing vehicle load attitude diagrams and using mass estimation function is possible to recognize particular conditions of vehicle load distribution and to correct longitudinal acceleration measure in order to limit this not erasable error on slope estimation improving further estimation precision.

IV. EXPERIMENTAL RESULTS

The estimation algorithms have been evaluated on test track with already known slopes and on public road with known mass with urban, extra-urban and highway experiments on a prototypal vehicle. Required signals were sampled from CAN bus available on normal production vehicles and algorithms are tested on real-time ECU. In the below fig. 6 a normal run on FGA Balocco Langhe test track with a compare between real-time estimation and commercial GPS road slope off-line calculation. This off-line calculation is delayed in order to eliminate the time delay from slope estimation filter.

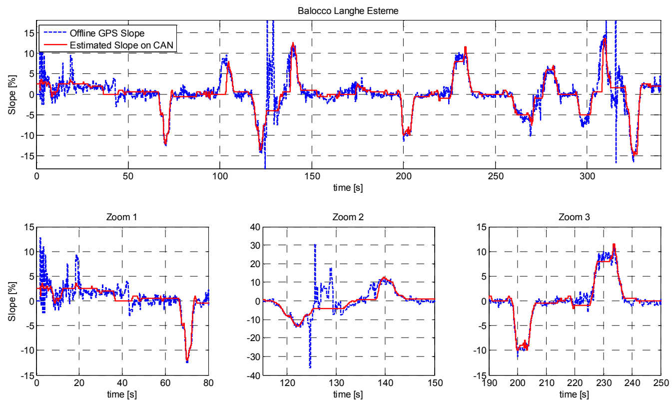

Fig. 6. Balocco Lange Slope Estimation Test

Fig. 7. A screen capture from real time test on Balocco test track

About mass estimation has been performed a series of tests with different scenarios, urban and extra urban, and using different driving styles, normal/sport and normal/eco. In fig. 8, you can see three examples of urban test with different vehicle masses. In details, mass estimation signal time lines in fig. 8 start from a vehicle mass value of empty vehicle. Then 'Estimated mass on CAN' is CAN network published vehicle weight signal, which is sampled from 'Internal Realtime Estimated Mass' signal when estimation value can be defined plausible.

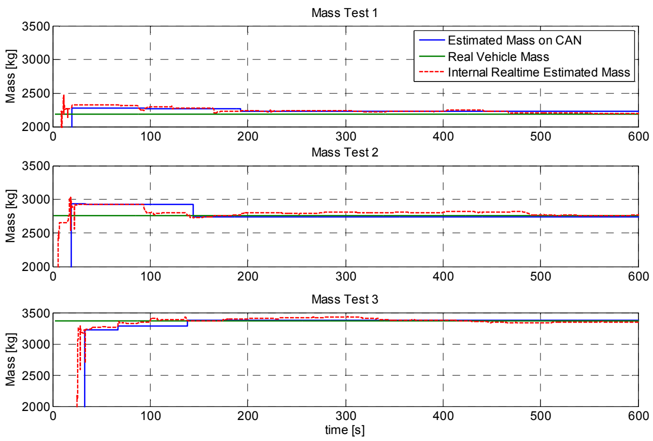

Fig. 8. Three urban scenario mass estimation tests

Described tests as other tests carried out on public roads and test tracks have demonstrated the effectiveness of these two integrated observers as an intelligent algorithm developed exploiting different physical and numerical methods to observe at best vehicle mass and road slope on which motor vehicle moves.

V. CONCLUSIONS

It has been shown through plain explanations and practical tests that developed 'Slope & Mass' Estimations are mature to use on light commercial vehicles with quite simple, stable, model-based mature algorithms, and without additional sensors outwards normal production ESP sensors and powertrain signals.

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REFERENCES

[1] Altenkirch M., 2003, "Method and device for determining the geometric vehicle inclination of a motor vehicle", W.O. Patent # 200340652

[2] Bae, H. S., Ryu, J., and Gerdes, J. C., 2001, "Road Grade and Vehicle Parameter Estimation for Longitudinal Control Using GPS", Proceedings of the IEEE Intelligent Transportation Systems Conference.

[3] Bellinger, S. M., Kalill, J. F., and Tyler, B. C., 2003, "System and Method for Estimating Vehicle Mass", U.S. Patent # 6,567,734.

[4] Breen, M. T., 1996, "System and Method for Determining Relative Vehicle Mass", U.S. Patent # 5,482,359.

[5] Corigliano E. and Borodani P., "Method and system for recognizing the sign of the velocity and for estimating the road slope", U.S. Patent # 7,269,494

[6] Fradin L., 2008, Method and device for estimating longitudinal load, in particular when applied to automobiles, E.P. Patent # 1935733

[7] Genise, T. A., 1996, "Control method/System Including Determination of an Updated Value Indicative of Gross Combination Weight of Vehicles", U.S. Patent # 5,490,063.

[8] Grieser, J., 2005, "Method for Determining an Estimate of the Mass of a Motor Vehicle", U.S. Patent #6,980,900.

[9] Klatt, A., 1985, "Method and Apparatus to Automatically Determine the Weight or Mass of a Moving Vehicle", U.S. Patent # 4,548,079.

[10] Lingman, A., and Schmidtbauer, B., 2002, "Road Slope and Vehicle Mass Estimation Using Kalman Filtering", Vehicle System Dynamics, 37, pp. 12-23.

[11] Lingman, A., and Schmidtbauer, B., 2001, "Method for estimation of the mass of a vehicle which is driven on a road with varying inclination and method for estimation of road inclination", S.E. Patent # 200102776

[12] Leminoux G., Plevin E., Pothin R., 2007, "Method and device for estimating the total mass of a motor vehicle", U.S. Patent # 2007/0038357

[13] Parkum J.E., Poulsen N.K., and Holst J.. Recursive forgetting algorithms. International Journal of Control, 55, 1:109–128, 1992.

[14] Reiner, K., Rieker, H., and Stoll, J., 1990, "Device for Determining the Mass of a Motor Vehicle", U.S. Patent # 4,941,365.

[15] Turco P. and Vincenzi M., 1997, "Sistemi per la diagnosi Model Reference", C.R.F./FIAT internal report.

[16] Vahidi, A., Druzhinina, M., Stefanopoulou, A., and Peng, H., 2003, "Simultaneous Mass and Time-Varying Grade Estimation for Heavy-Duty Vehicles", Proceedings of the American Control Conference, Denver, CO.

[17] Vahidi, A., Stefanopoulou, A., and Peng, H., 2003, "Experiments for Online Estimation of Heavy Vehicle's Mass and Time-Varying Road Grade", Proceedings of the 2003 ASME International Mechanical Engineering Congress and Exposition.

[18] Vahidi, A., Stefanopoulou, A., and Peng, H., 2005, "Recursive Least Squares with Forgetting for Online Estimation of Vehicle Mass and Road Grade: Theory and Experiments", Vehicle System Dynamics, 41(1), pp. 31-55.

[19] Winstead, V., and Kolmanovsky, I., 2005, "Estimation of Road Grade and Vehicle Mass via Model Predictive Control", Proceedings of the IEEE Conference on Control Applications.

[20] Zhu, G. G., Taylor, D. O., and Bailey, T. L., 2000, "Recursive Vehicle Mass Estimation", U.S. Patent # 6,167,357.

[21] Zhu, G. G., Taylor, D. O., and Bailey, T. L., 2002, "Recursive Vehicle Mass Estimation System", U.S.Patent # 6,438,510.