



Areal statistics

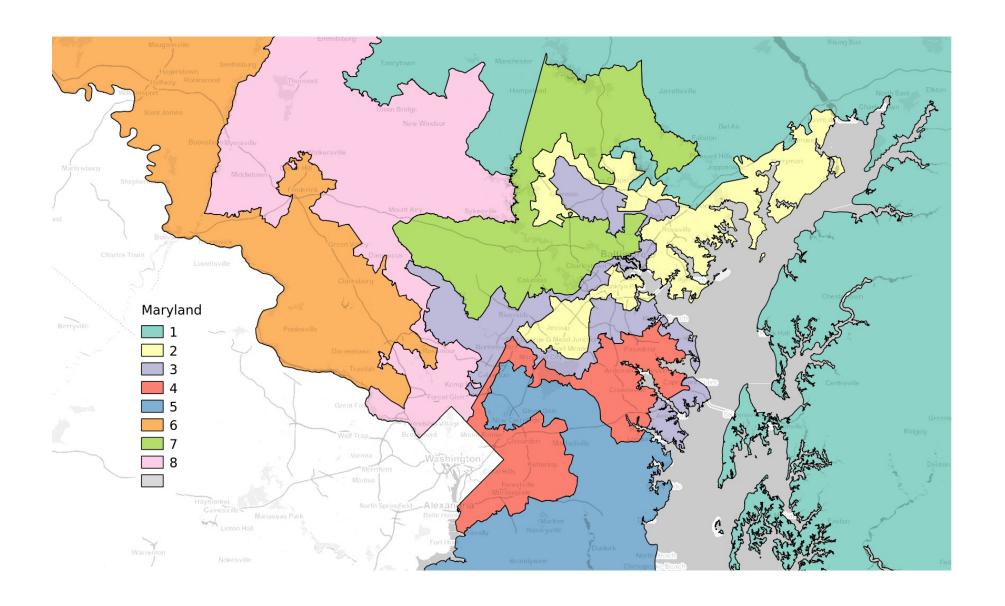
Barry Rowlingson Research Fellow

Borders





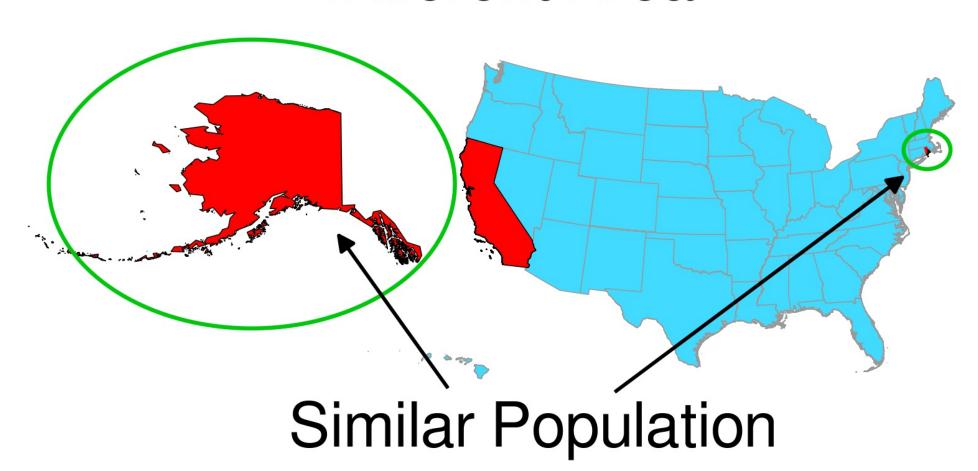
US Congressional Borders





Similar Population

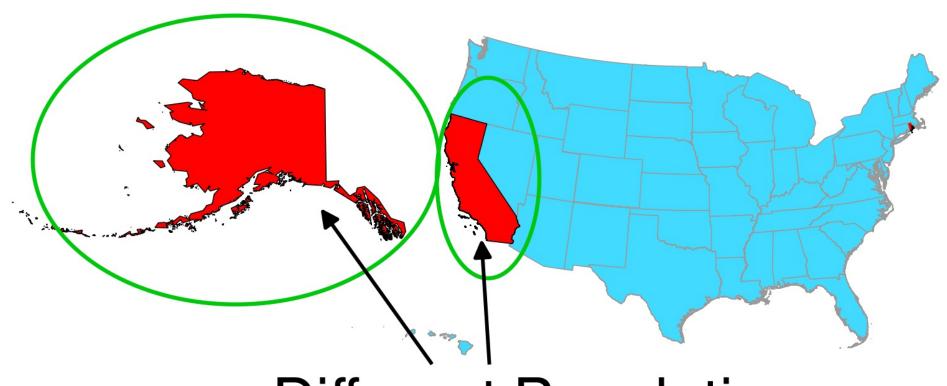
Different Area





Similar Area

Similar Area



Different Population

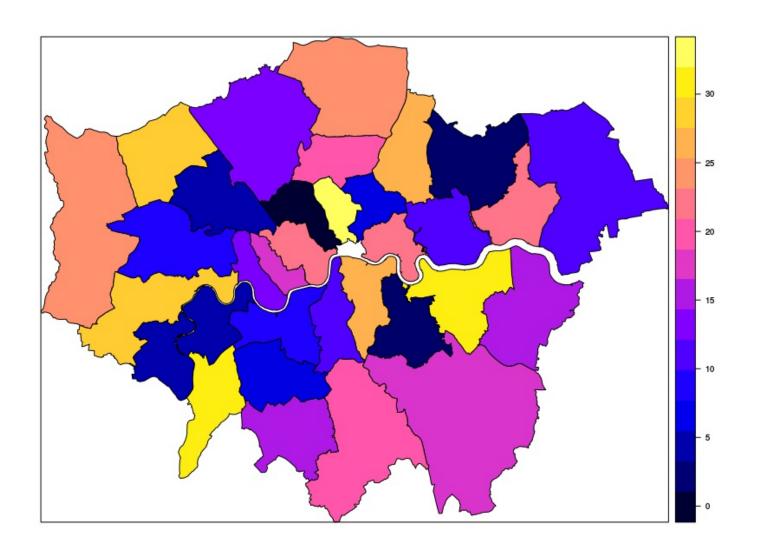
Cartogram



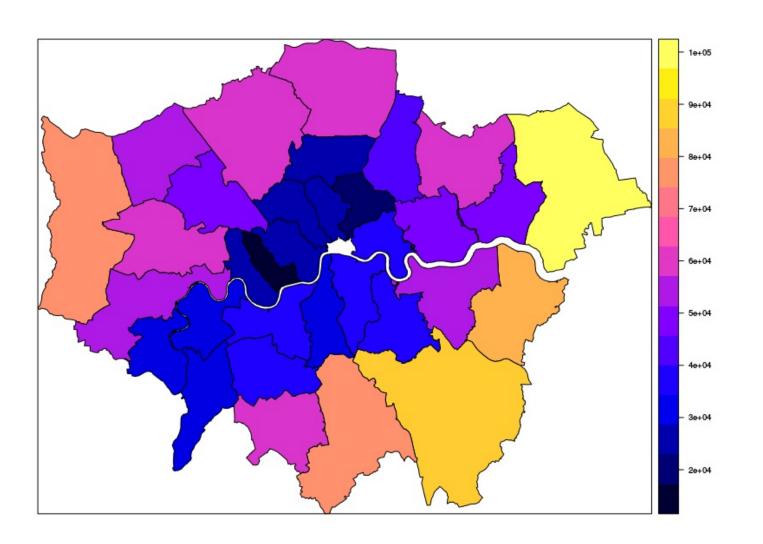
Population Cartogram



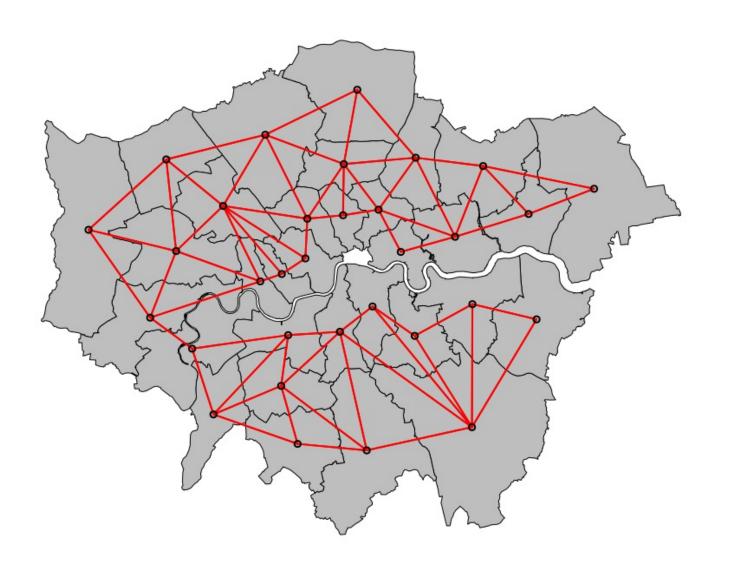
Spatially Random



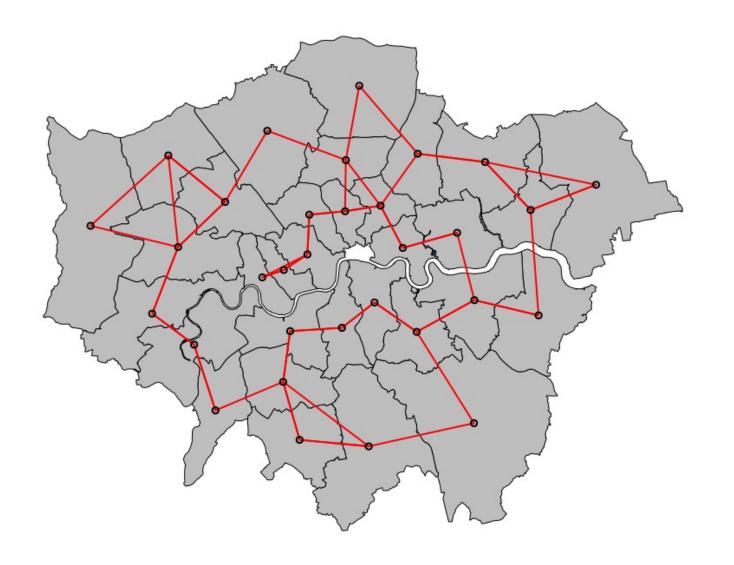
Spatial Structure



Adjacent



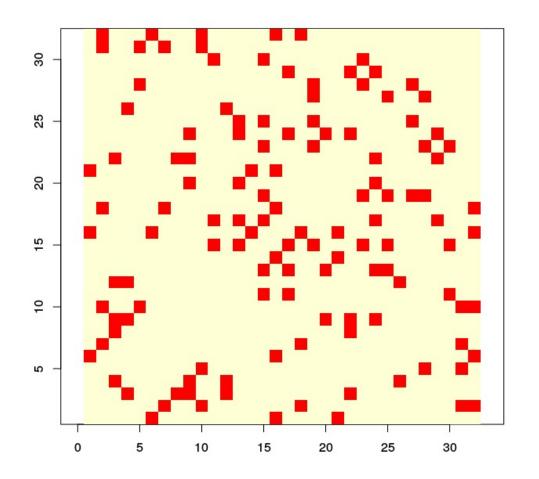
Nearest





Adjacency Representations

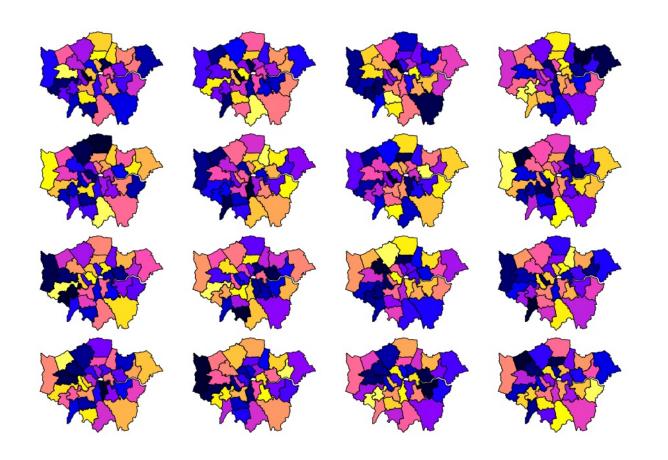
```
[[1]]
[1] 6 16 21
[[2]]
[1] 7 10 18 31 32
[[3]]
       8 9 12 22
[[4]]
[1] 3 9 12 26
[[5]]
[1] 10 28 31
[[6]]
   1 16 32
```



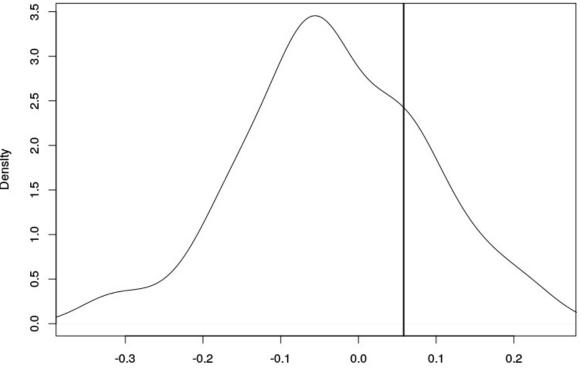
Moran I

$$I = \frac{n}{\sum_{i} \sum_{j} w_{ij}} \frac{\sum_{i} \sum_{j} w_{ij} (z_{i} - \bar{z})(z_{j} - \bar{z})}{\sum_{i} (z_{i} - \bar{z})^{2}}$$

Moran I Test



Density plot of permutation outcomes



Monte-Carlo simulation of Moran I





Let's practice!

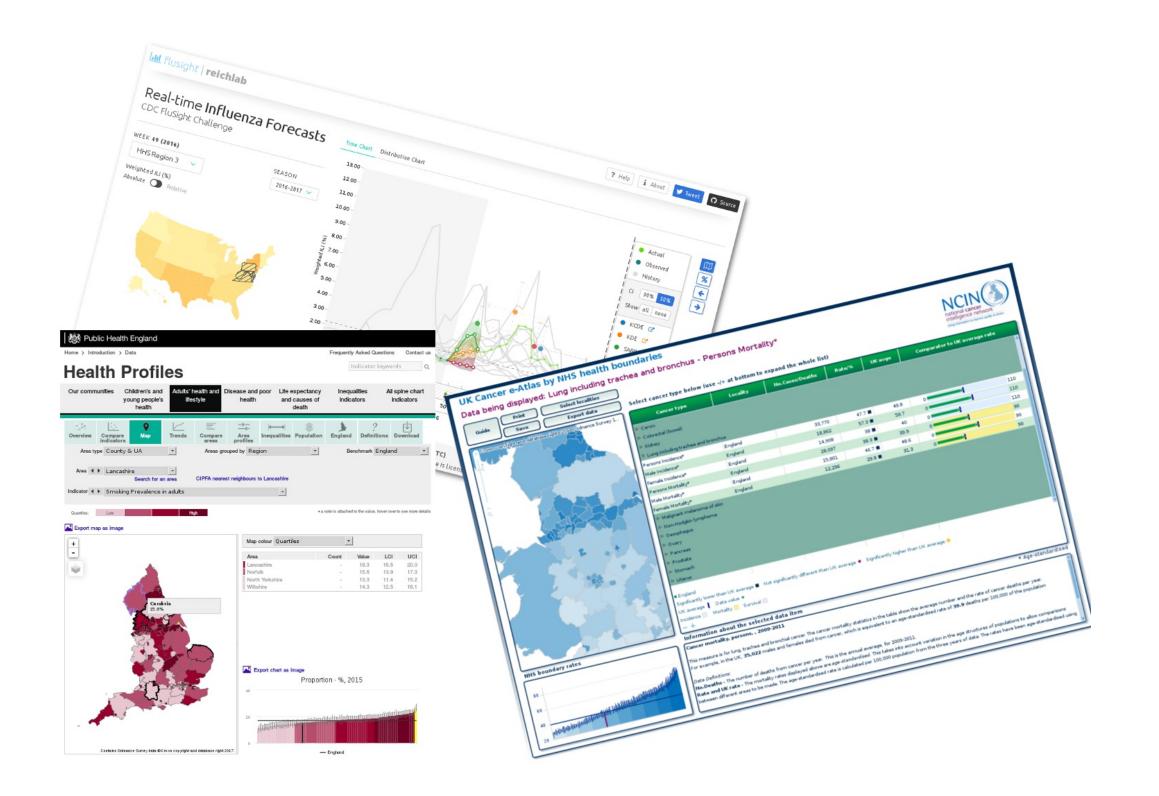




Spatial health data

Barry Rowlingson Spatially Healthy





Incidence Rate

$$R_i = \frac{N_i}{P_i}$$

$${\rm Rate} \ {\bf is} \ \frac{{\rm Number} \ {\rm of} \ {\rm Cases}}{{\rm Population} \ {\rm At} \ {\rm Risk}}$$



Standardized Morbidity Ratio

SMR in region
$$i = \frac{R_i}{\text{Overall Rate}}$$

Overall Rate

$$\mathbf{R} = \frac{\sum N_i}{\sum P_i}$$

Overall Rate is
$$\frac{\text{Total Cases}}{\text{Total Population}}$$

Expected Cases

$$E_i = \mathbf{R} \times P_i$$

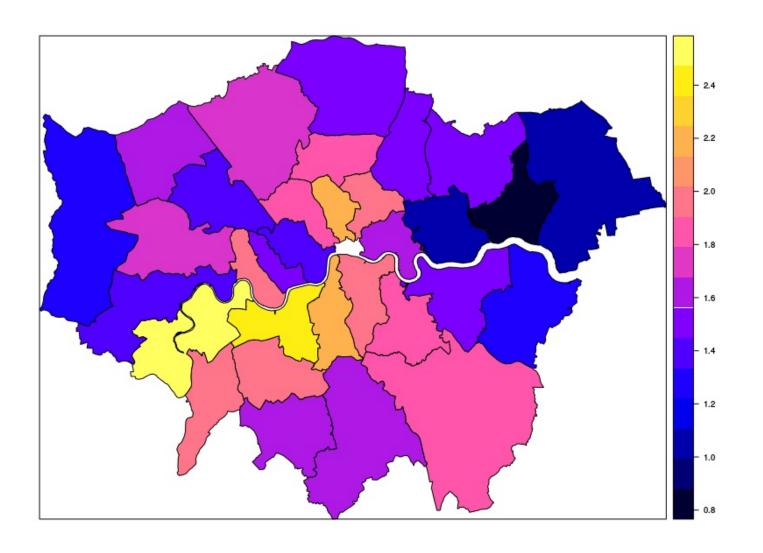
Expected Cases is Overall Rate × Population

Standardized Morbidity Ratio

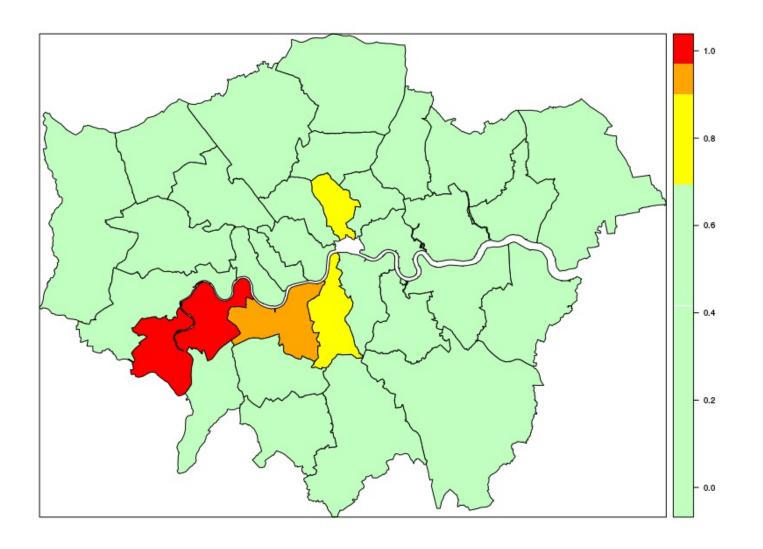
$$SMR_i = \frac{N_i}{E_i}$$

Std Morbidity Rate is
$$\frac{\text{No. of Cases}}{\text{Expected}}$$

SMR Map



Exceedence Probability Map







Let's practice!





Generalized linear models in space

Barry Rowlingson Generally Linear

Linear Model

$$Y \sim N(X\beta, \sigma^2)$$



Generalized Linear Model

$$Y \sim D(\mu(X\beta))$$



Poisson (Count) GLM

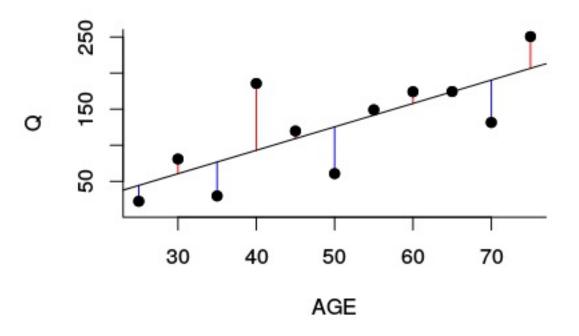
$$Y \sim Poisson(e^{X\beta})$$

Parameter Estimation

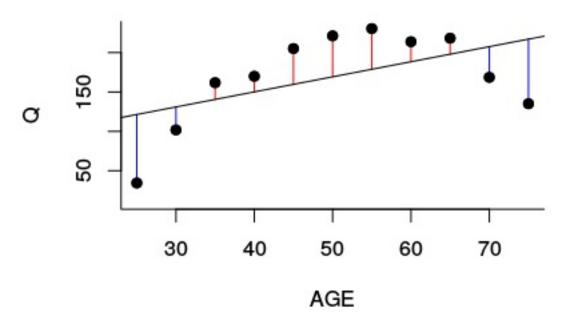
> summary(fit)

	Estimate	Std. Error	t value	$\Pr(> t)$	
(Intercept)	24.28	25.44	0.95	0.3448	
AGE	1.69	0.53	3.16	0.0028	**
SHOESIZE	0.17	1.61	0.11	0.9155	

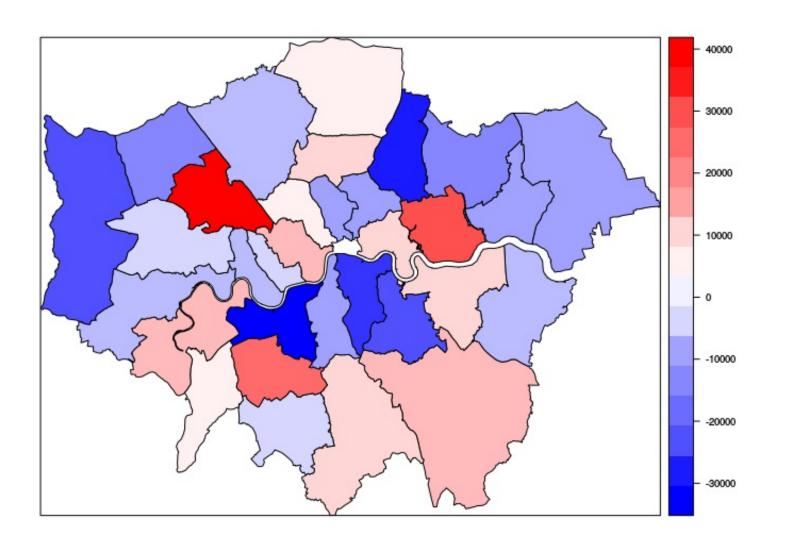
Fit



Misfit

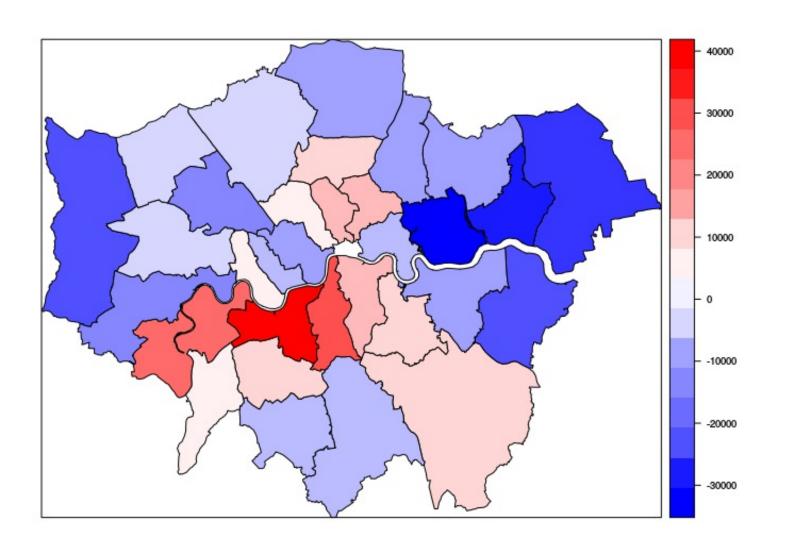


Random Residuals





Spatially Structured Residuals







Let's practice!

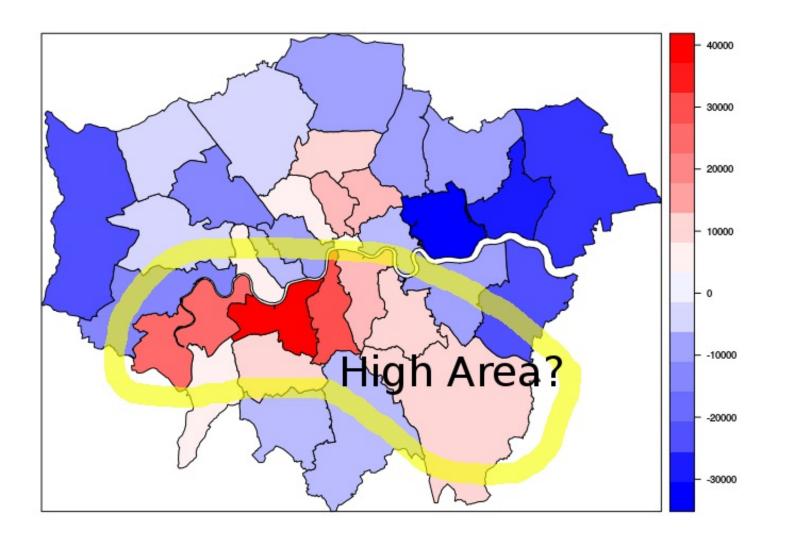




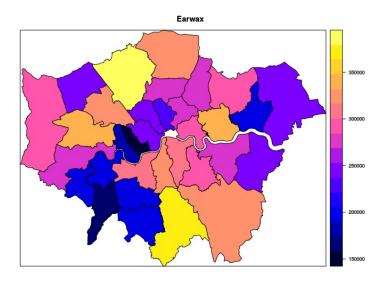
Correlation in spatial GLMs

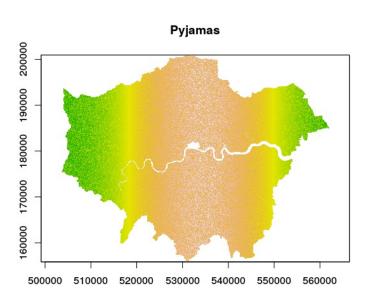
Barry Rowlingson Spatially Correlated

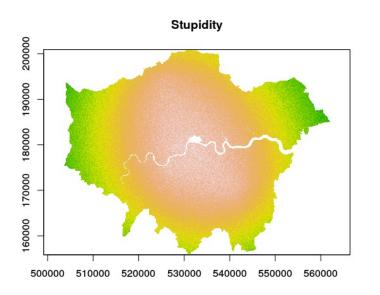
Correlated Residuals

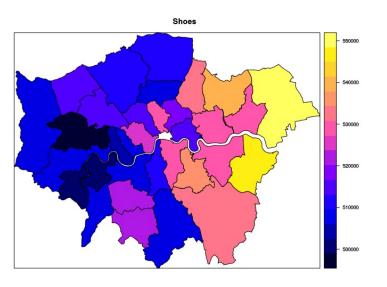


New Covariates







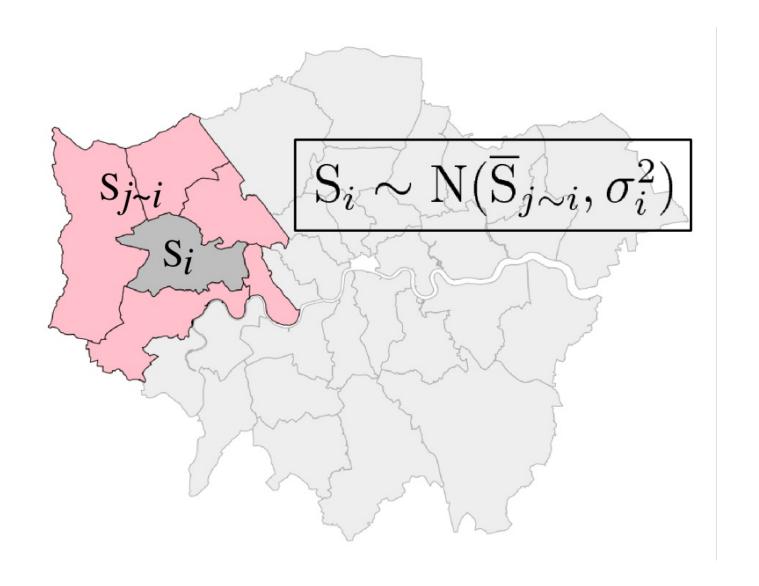




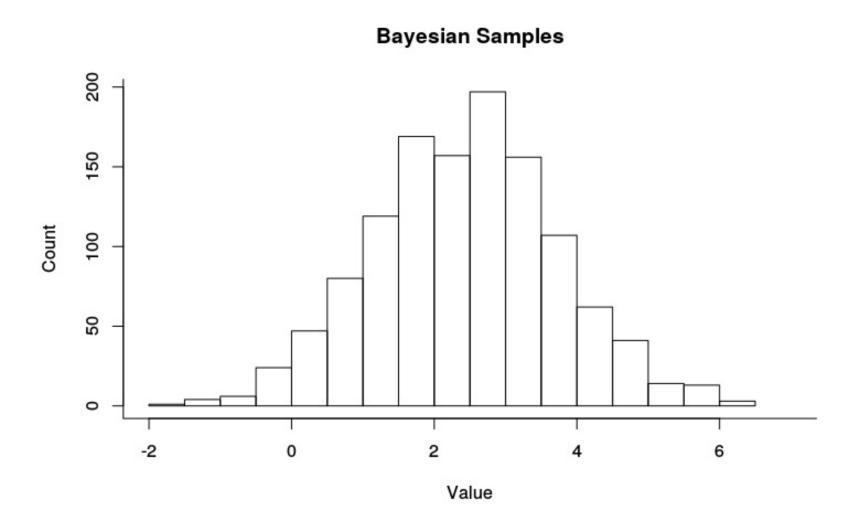
Model for Spatial Correlation

$$Y = X\beta + \mathbf{S}(x, y)$$

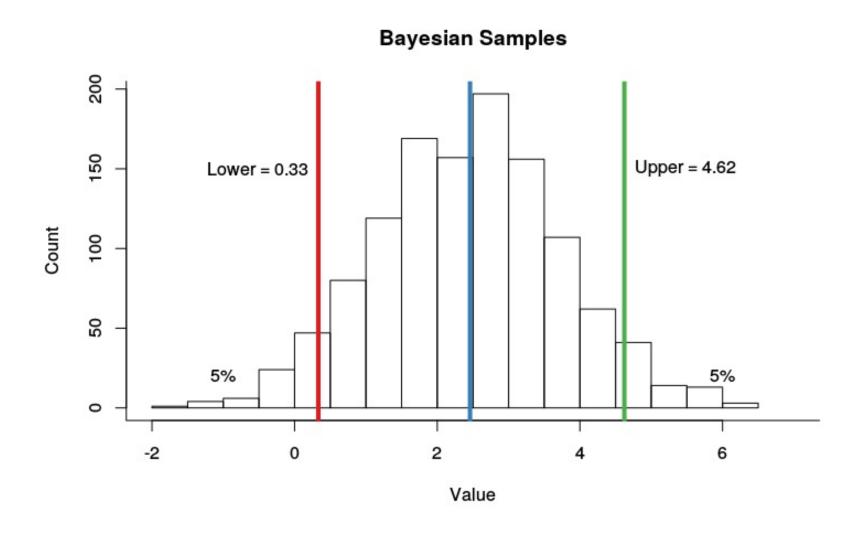
Conditional Autocorrelation



Bayesian Statistics



Bayesian Statistics







Let's practice!