Axiomatic Potentialism

Chris Scambler

All Souls College,
Oxford University

chris.scambler@all-souls.ox.ac.uk

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Overview

1. Background

2. Warm Up: Height Potentialism

3. Height and Width Potentialism

Table of Contents

Potentialism

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- Still deeper roots in mathematics in general.

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- Here we will be focussed on axiomatic potentialism, and on relations between potentialist axiom systems and their first order counterparts.

Table of Contents

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Connection to Standard Set Theory

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Title of the publication

Journal Name 12(3), 45 − 678.

Thanks!