Monic arrows in Ω-set .. An arrow $f : A \to B$ in Ω-set is monic iff it satisfies

$$f(a_0, b) \wedge f(a_1, b) \leq [\![a_0 = a_1]\!]_A \tag{1}$$

Proof. Let $g : C \to A$ in Ω-set.

We first prove that (1) implies

$$g(c, a_0) = \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \tag{2}$$

for any c, a_0. To do this we show eacch of

$$g(c, a_0) \leq \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \tag{3}$$

and

$$g(c, a_0) \geq \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \tag{4}$$

For (3), first observe

$$g(c, a_0) \leq [\![a_0 = a_0]\!] \leq \bigsqcup_{b \in B} f(a_0, b) \tag{5}$$

we can therefore chose $b \in B$ with

$$g(c, a_0) \leq [\![a_0 = a_0]\!] \leq f(a_0, b) \tag{6}$$

(6) implies $g(c, a_0) \leq g(c, a_0) \wedge f(a_0, b)$ which in turn implies

$$g(c, a_0) \leq (\bigsqcup_{a \in A} g(c, a) \wedge f(a, b)) \wedge f(a_0, b) \tag{7}$$

from which (3) follows trivially.

For (4), let $b \in B$. We have:

$$(\bigsqcup_{a \in A} g(c, a) \wedge f(a, b)) \wedge f(a_0, b) = \bigsqcup_{a \in A} g(c, a) \wedge f(a, b) \wedge f(a_0, b) \tag{8}$$

$$\leq \bigsqcup_{a \in A} g(c, a) \wedge [\![a = a_0]\!]_A \tag{9}$$

$$\leq g(c, a_0) \tag{10}$$

where the transition from (8) to (9) uses (1), and the transition from (9) to (10) uses (v) p 277. (Is equality true?)

From here it is easy. Suppose $g \circ f(c, b) = h \circ f(c, b)$ for all c and b. Then for any a, $\bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a, b) = \bigsqcup_{b \in B} h \circ f(c, b) \wedge f(a, b)$. Hence $g(c, a) = f(c, a)$ by (2). $\square$