

Monic arrows in Ω -set .. An arrow $f : A \rightarrow B$ in Ω -set is monic iff it satisfies

$$f(a_0, b) \wedge f(a_1, b) \leq \llbracket a_0 = a_1 \rrbracket_A \quad (1)$$

Proof. Let $g : C \rightarrow A$ in Ω -set.

We first prove that (1) implies

$$g(c, a_0) = \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \quad (2)$$

for any c, a_0 . To do this we show each of

$$g(c, a_0) \leq \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \quad (3)$$

and

$$g(c, a_0) \geq \bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a_0, b) \quad (4)$$

For (3):

$$g(c, a_0) = g(c, a_0) \wedge \llbracket a_0 = a_0 \rrbracket \quad (5)$$

$$= \bigsqcup_{b \in B} g(c, a_0) \wedge f(a_0, b) \quad (6)$$

$$= \bigsqcup_{b \in B} (\bigsqcup_{a \in A} g(c, a) \wedge f(a, b)) \wedge f(a_0, b) \wedge g(c, a_0) \quad (7)$$

$$= \bigsqcup_{b \in B} f \circ g(c, b) \wedge f(a_0, b) \wedge g(c, a_0) \quad (8)$$

which latter is what we want.

For (4), let $b \in B$. We have:

$$(\bigsqcup_{a \in A} g(c, a) \wedge f(a, b)) \wedge f(a_0, b) = \bigsqcup_{a \in A} g(c, a) \wedge f(a, b) \wedge f(a_0, b) \quad (9)$$

$$\leq \bigsqcup_{a \in A} g(c, a) \wedge \llbracket a = a_0 \rrbracket_A \quad (10)$$

$$\leq g(c, a_0) \quad (11)$$

where the transition from (9) to (10) uses (1), and the transition from (10) to (11) uses (v) p 277. (Is equality true?)

From here it is easy. Suppose $g \circ f(c, b) = h \circ f(c, b)$ for all c and b . Then for any a , $\bigsqcup_{b \in B} g \circ f(c, b) \wedge f(a, b) = \bigsqcup_{b \in B} h \circ f(c, b) \wedge f(a, b)$. Hence $g(c, a) = f(c, a)$ by (2). \square