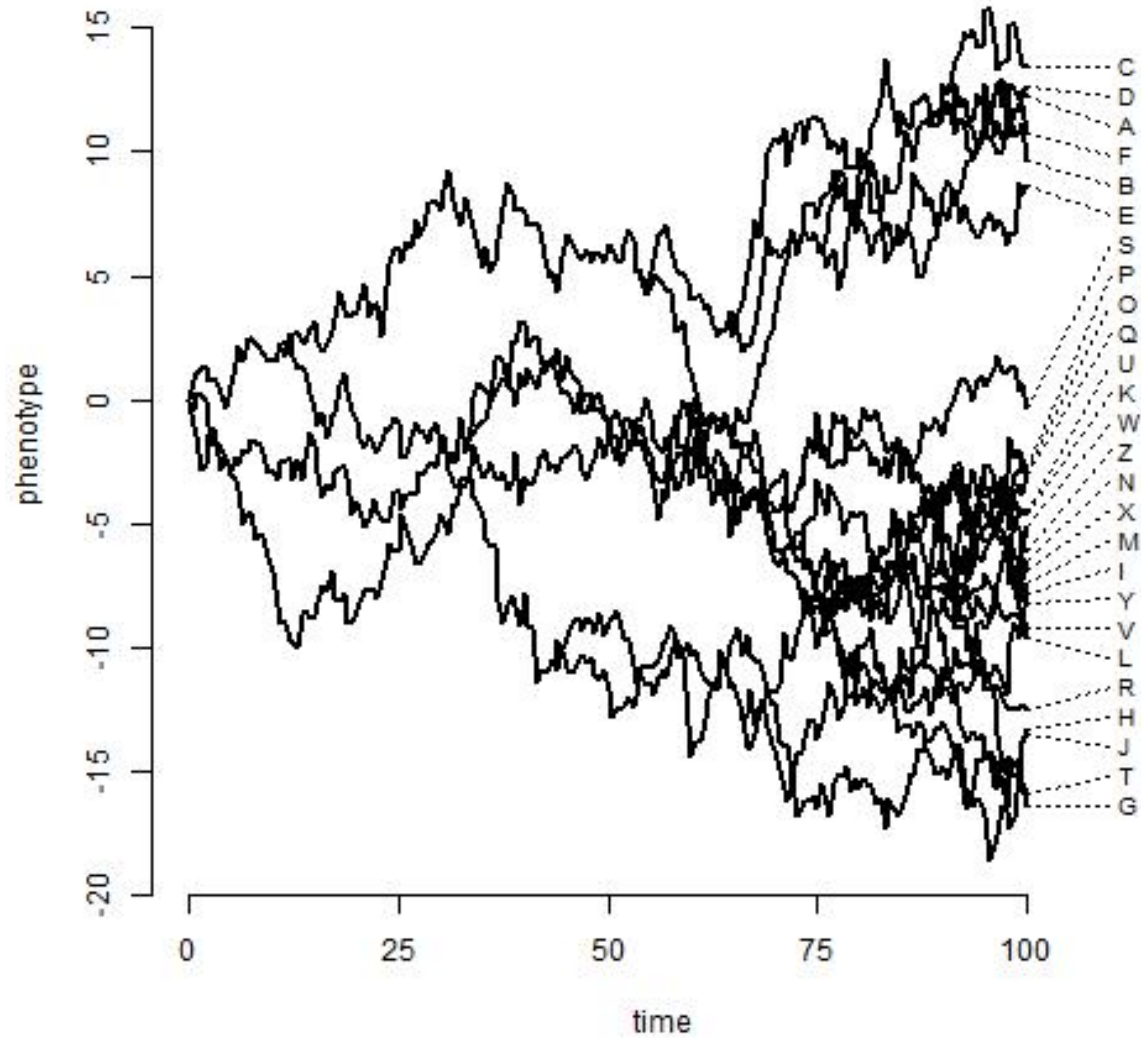


# Modeling adaptive evolution



**Give me your best  
estimate of the  
evolutionary rate  
(back of the napkin  
calculation)**

# Genetic Drift + Natural Selection

Genetic drift only:

$$\Delta z = G/Ne * dW$$

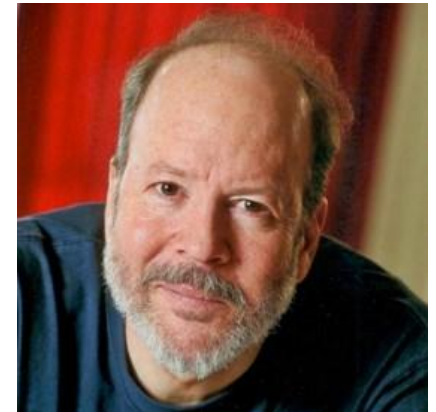
Genetic drift + Natural selection to a Gaussian adaptive peak:

$$\Delta z = G/(\omega + P) * (\theta - z) + G/Ne * dW$$

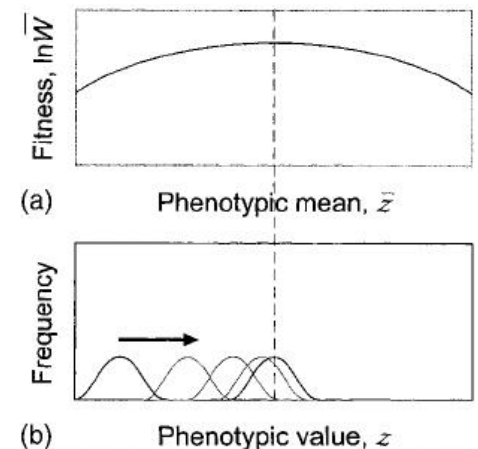
Genetic variance

Width of the adaptive landscape

Distance from the optimum



Russ Lande



# Ornstein-Uhlenbeck Process

$$\Delta z = \alpha(\theta - z) + \sigma dW$$

$\alpha$  - Rate of adaptation

Units: (1/time)

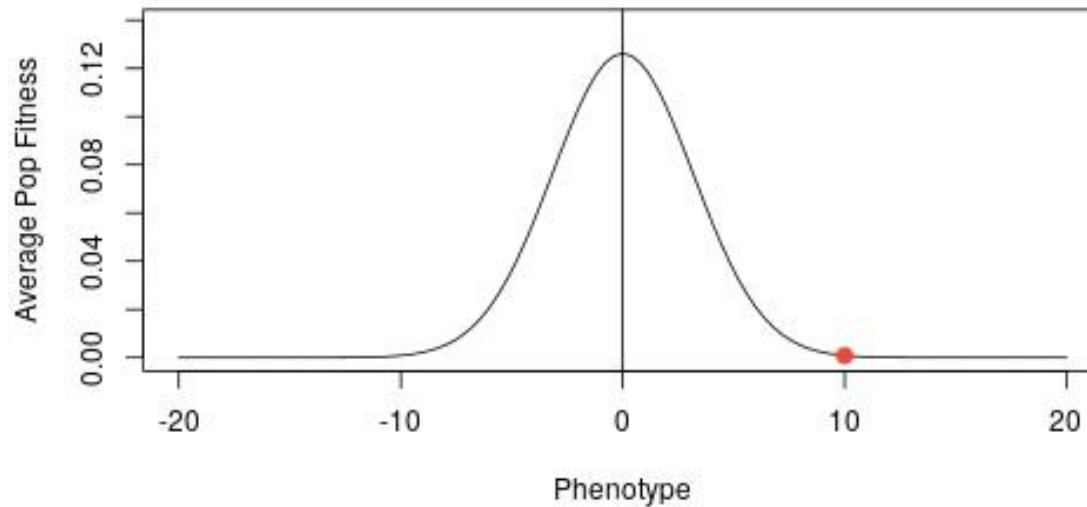
$\sigma$  - Rate of stochastic evolution

Units: trait units/time

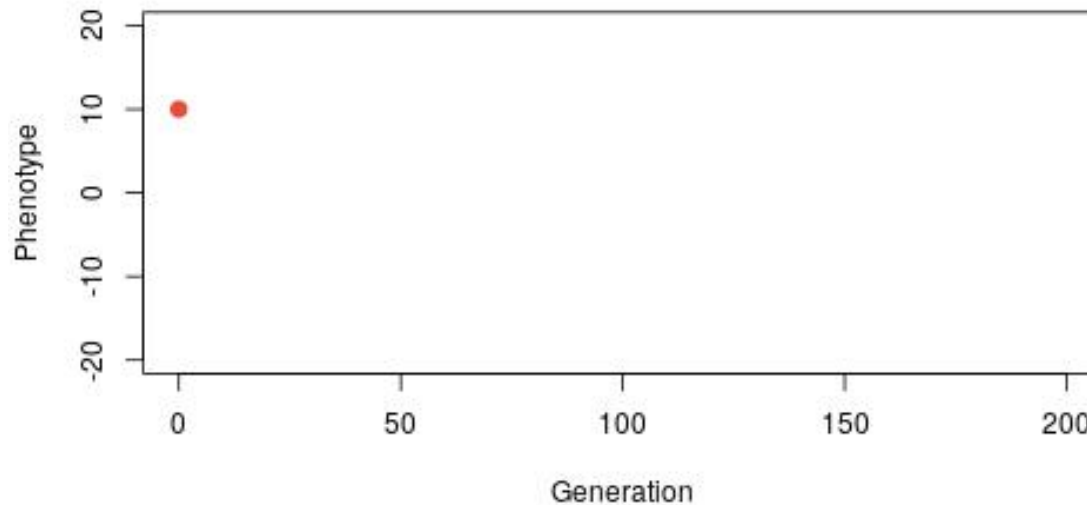
$\theta$  - Phenotypic optimum



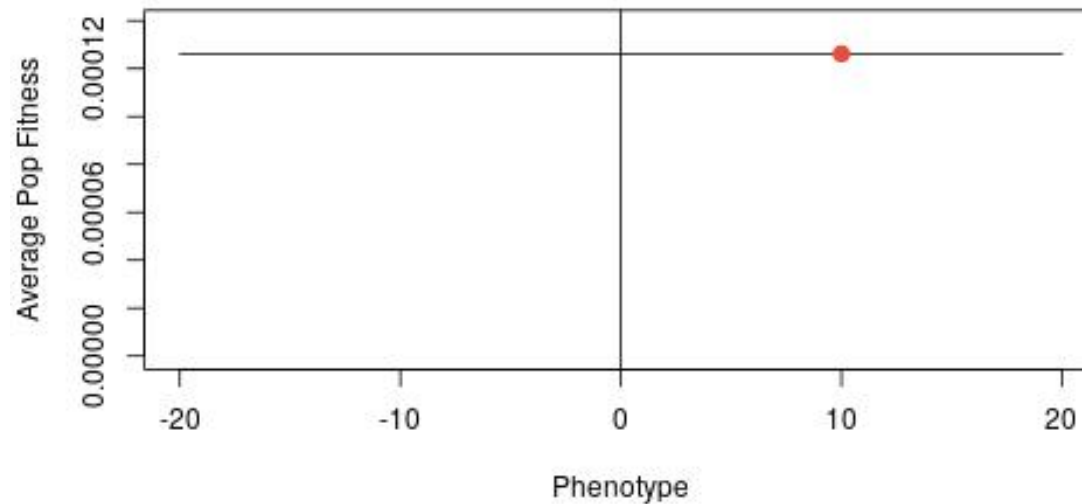
# Ornstein-Uhlenbeck process



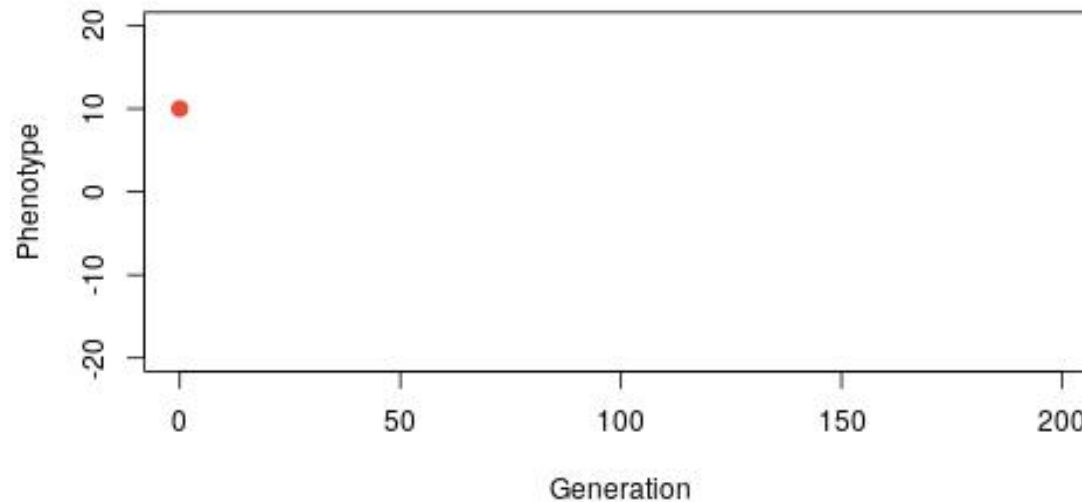
$\alpha = 0.1$   
 $\sigma^2 = 1$



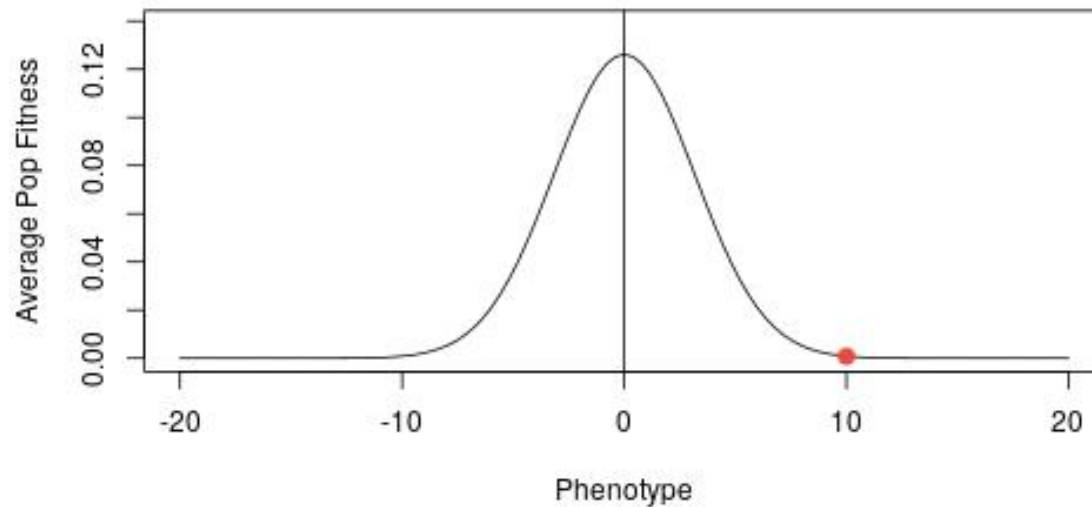
# Ornstein-Uhlenbeck process



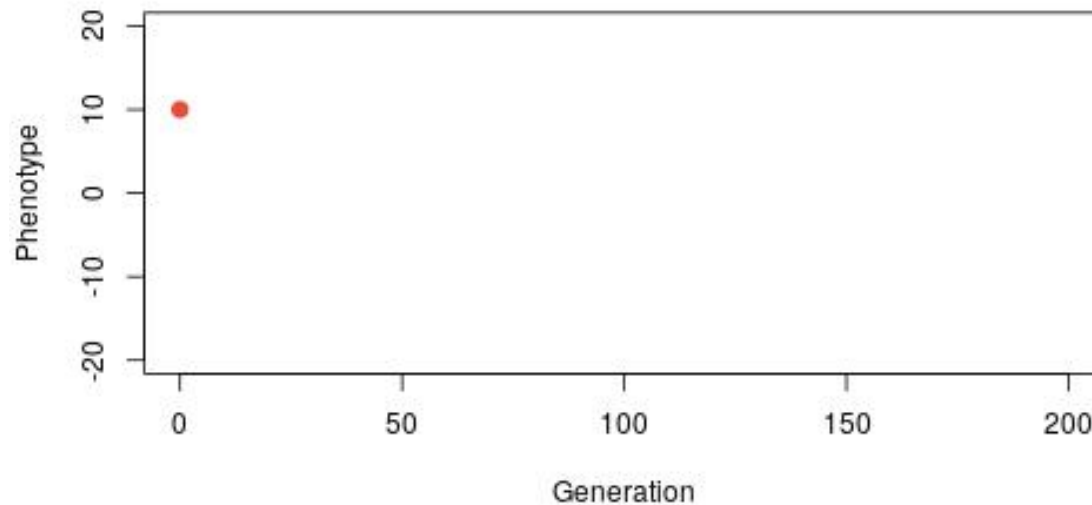
$\alpha = 0.000001$   
 $\sigma^2 = 1$



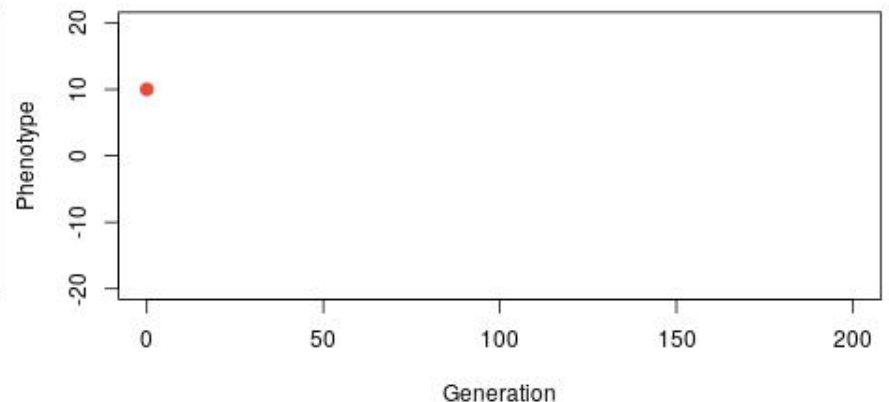
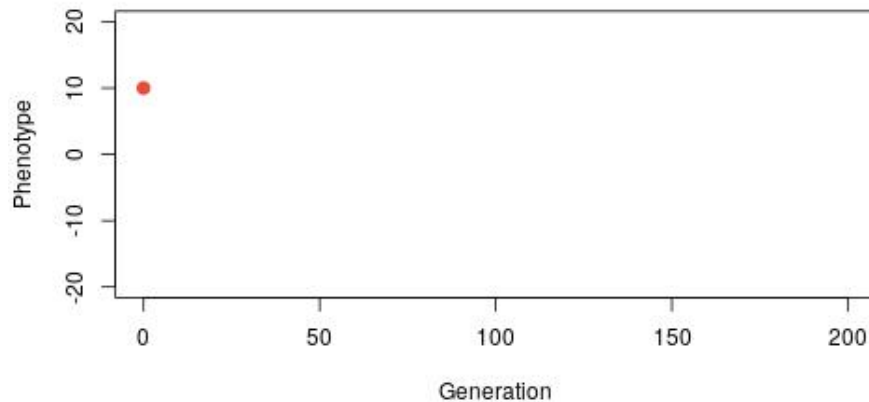
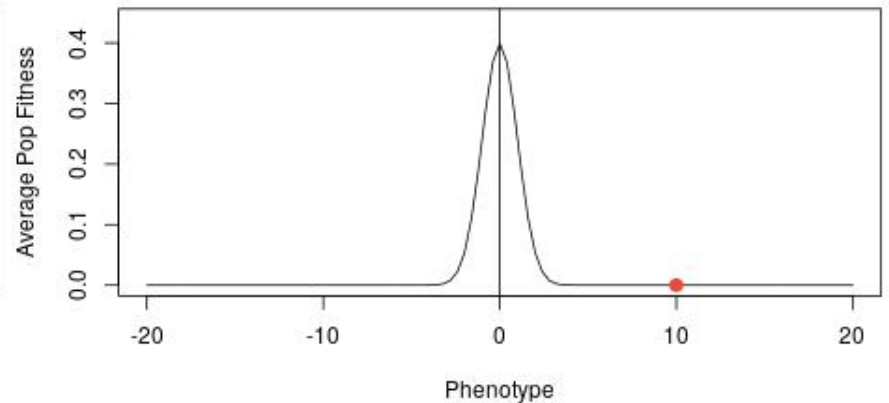
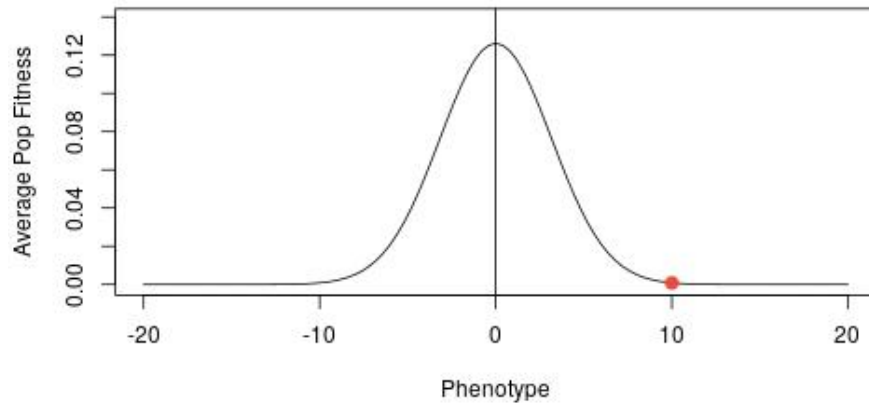
# Ornstein-Uhlenbeck process



$\alpha = 0.1$   
 $\sigma^2 = 1$



# Ornstein-Uhlenbeck process



$$\alpha = 0.1$$
$$\sigma^2 = 1$$

$$\alpha = 1$$
$$\sigma^2 = 10$$



**How does Hansen 1997  
interpret the “Primary  
Optimum”?**

# What is a Primary Optimum?

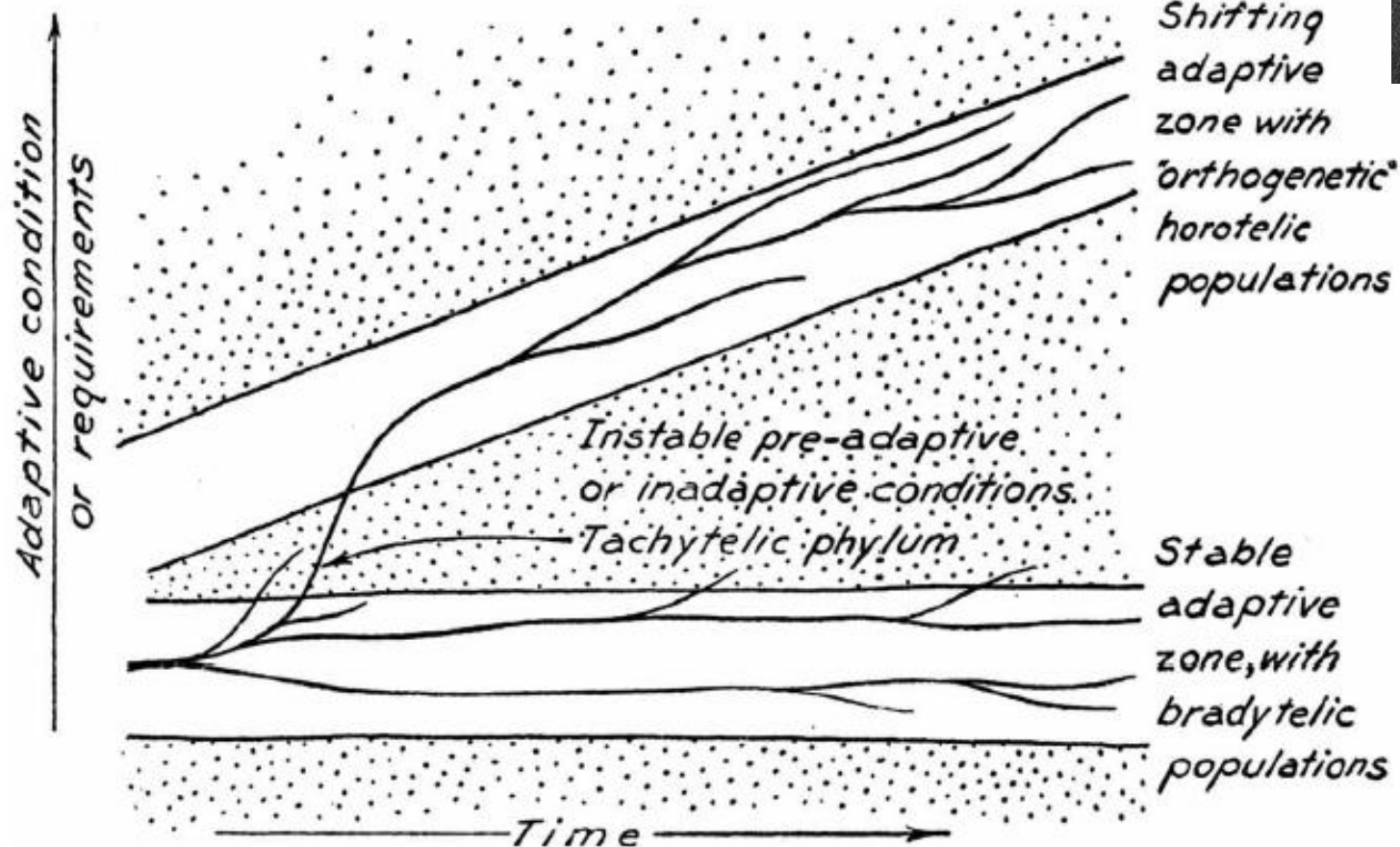
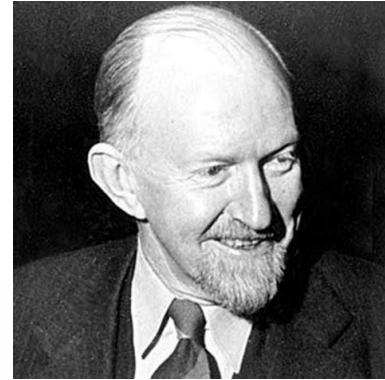
What does alpha measure?

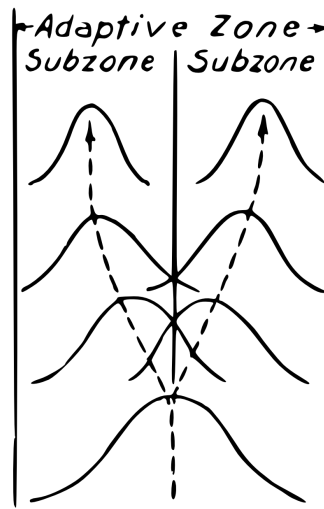
A: Strength of Natural Selection(?)

Problem: What if we measure selection pulling a species *AWAY* from the macroevolutionary optimum in a natural population?

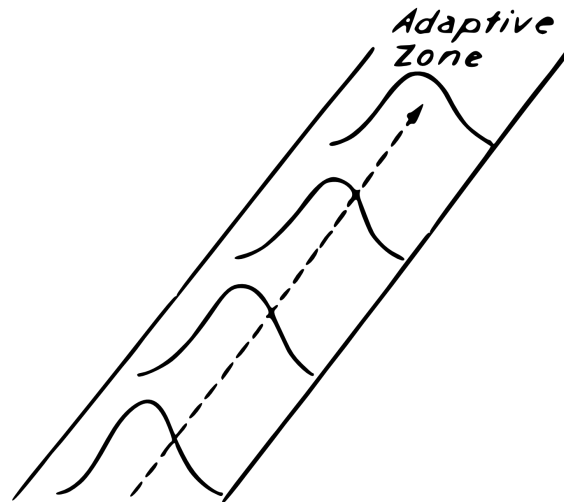
(This is VERY COMMON!)

# Simpson's Adaptive Zones

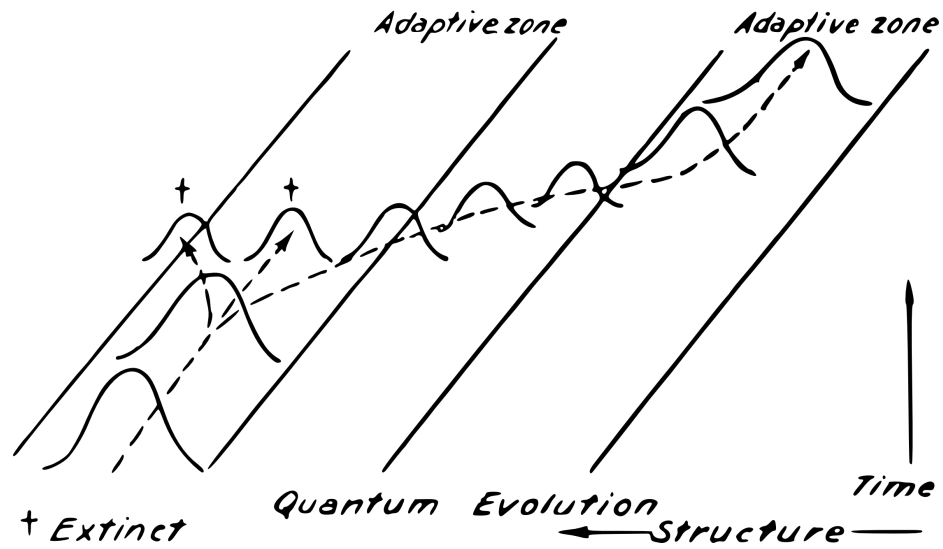




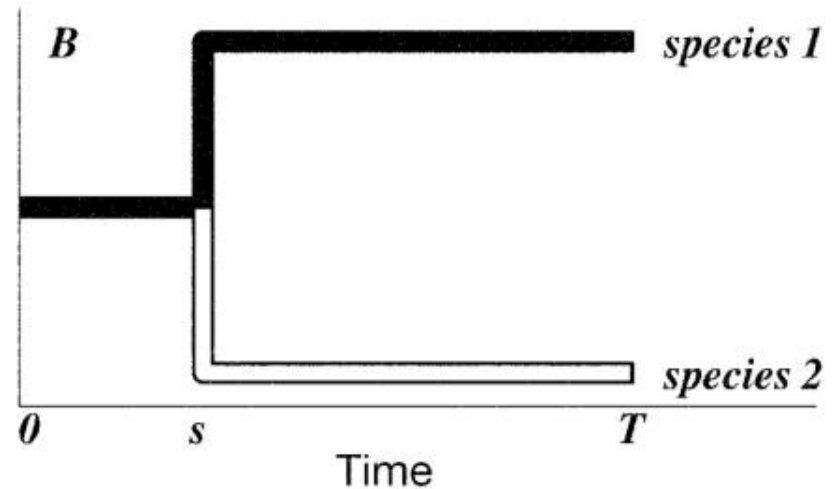
*Speciation*



*Phyletic Evolution*



# Hansen 1997



$$E[X_1(T)] = \theta_0 e^{-\alpha T} + \theta_1 (1 - e^{-\alpha T})$$

$$= W_{10} \theta_0 + W_{11} \theta_1,$$

$$E[X_2(T)] = \theta_0 e^{-\alpha T} + \theta_1 e^{-\alpha(T-s)} (1 - e^{-\alpha s})$$

$$+ \theta_2 [1 - e^{-\alpha(T-s)}]$$

$$= W_{20} \theta_0 + W_{21} \theta_1 + W_{22} \theta_2.$$

# Butler & King 2004

$$V = \frac{\sigma^2}{2\alpha} \begin{bmatrix} 1 - e^{-2\alpha T} & e^{-2\alpha(T-s)}(1 - e^{-2\alpha s}) \\ e^{-2\alpha(T-s)}(1 - e^{-2\alpha s}) & 1 - e^{-2\alpha T} \end{bmatrix}.$$

# Useful parameterization:

**Stationary Variance =  $\sigma^2/(2\alpha)$**

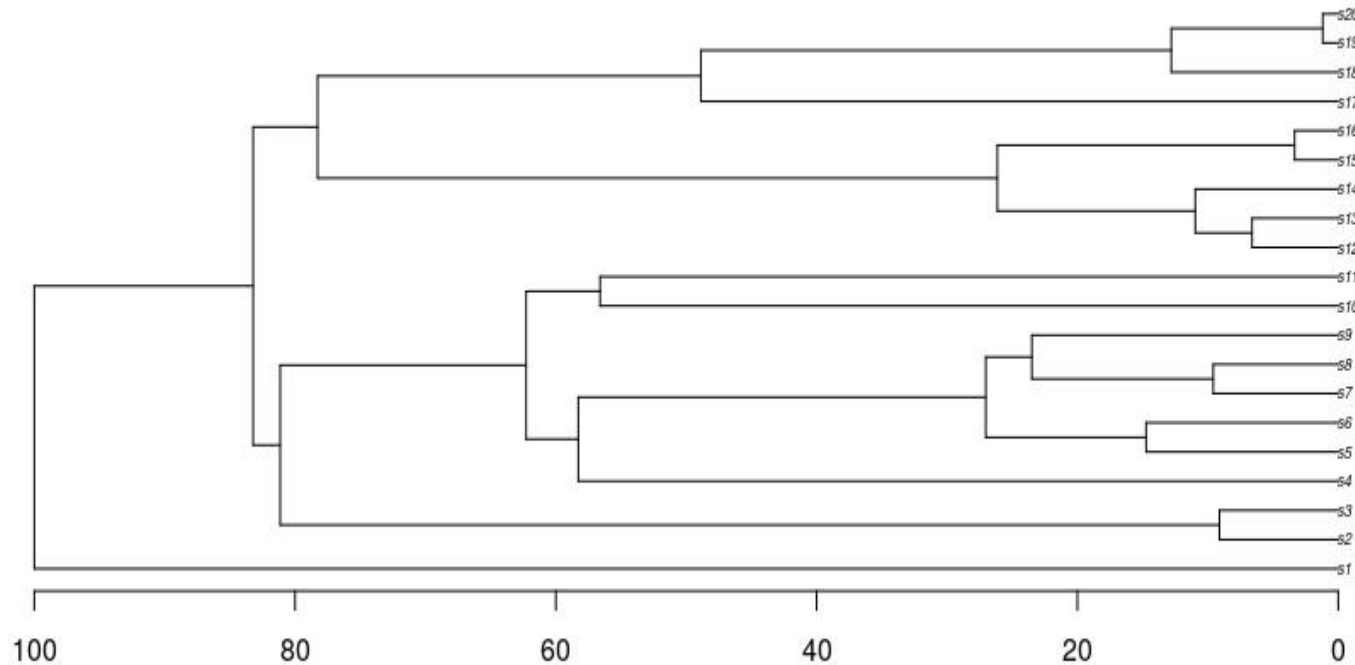
**The amount of variance among a set of lineages with an infinite amount of time evolving at the optimum**

$$\mathbf{V} = \sigma^2 \begin{bmatrix} T & s \\ s & T \end{bmatrix} \quad \mathbf{V} = \frac{\sigma^2}{2\alpha} \begin{bmatrix} 1 - e^{-2\alpha T} & e^{-2\alpha(T-s)}(1 - e^{-2\alpha s}) \\ e^{-2\alpha(T-s)}(1 - e^{-2\alpha s}) & 1 - e^{-2\alpha T} \end{bmatrix}.$$

**Phylogenetic half-life =  $\ln 2/(2\alpha)$**

**The expected amount of time for a lineage to get halfway to the optimum**

# Interpreting half-life



$\ln 2 / \alpha$

BM-like

White noise

OU

# Phylogenetic signal - What does it mean?

Pagel's Lambda

Blomberg's K

Phylogenetic halflife



# AC/DC model

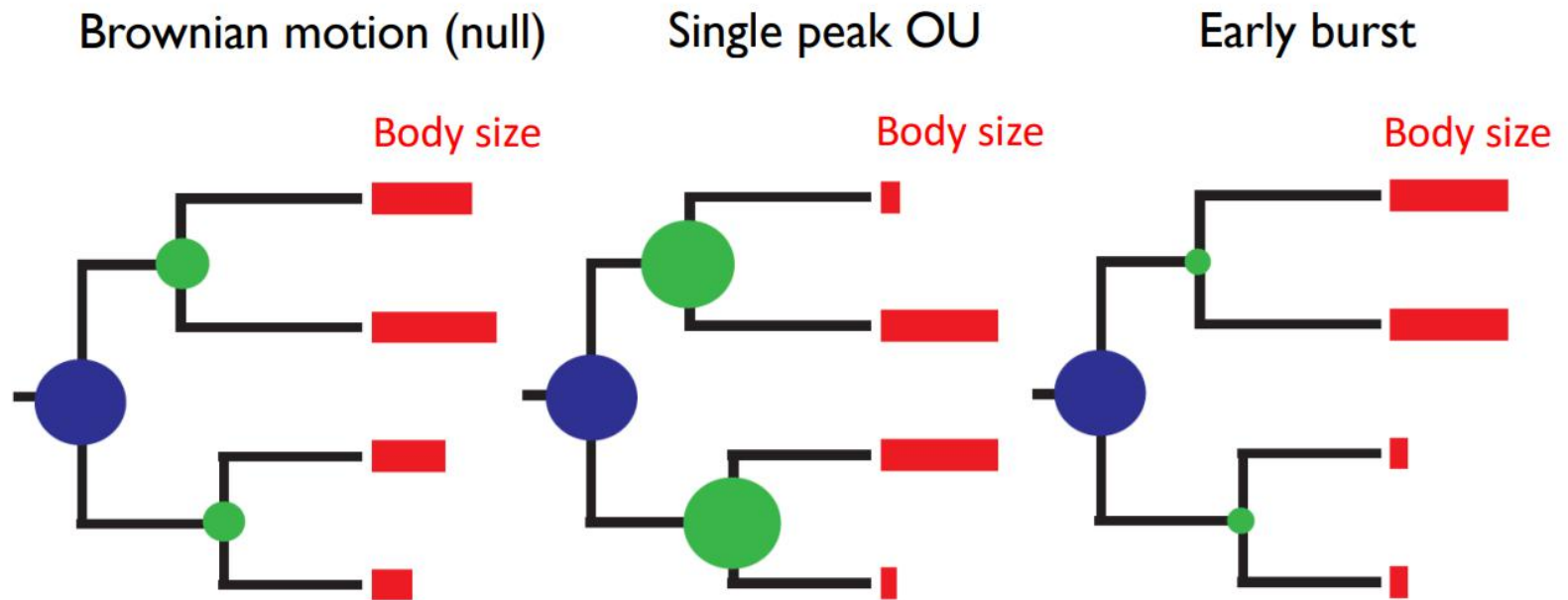
BM, but with a declining rate parameter:

$$\sigma^2(t) = \sigma_0^2 e^{bt}$$

If  $b$  is negative, declining rates (i.e. Early burst)

If  $b$  is positive, increasing rates (late bursts, not identifiable from OU in ultrametric trees)

# Early Burst -> Brownian Motion -> Ornstein-Uhlenbeck



# Early Burst -> Brownian Motion -> Ornstein-Uhlenbeck

## EARLY BURSTS OF BODY SIZE AND SHAPE EVOLUTION ARE RARE IN COMPARATIVE DATA

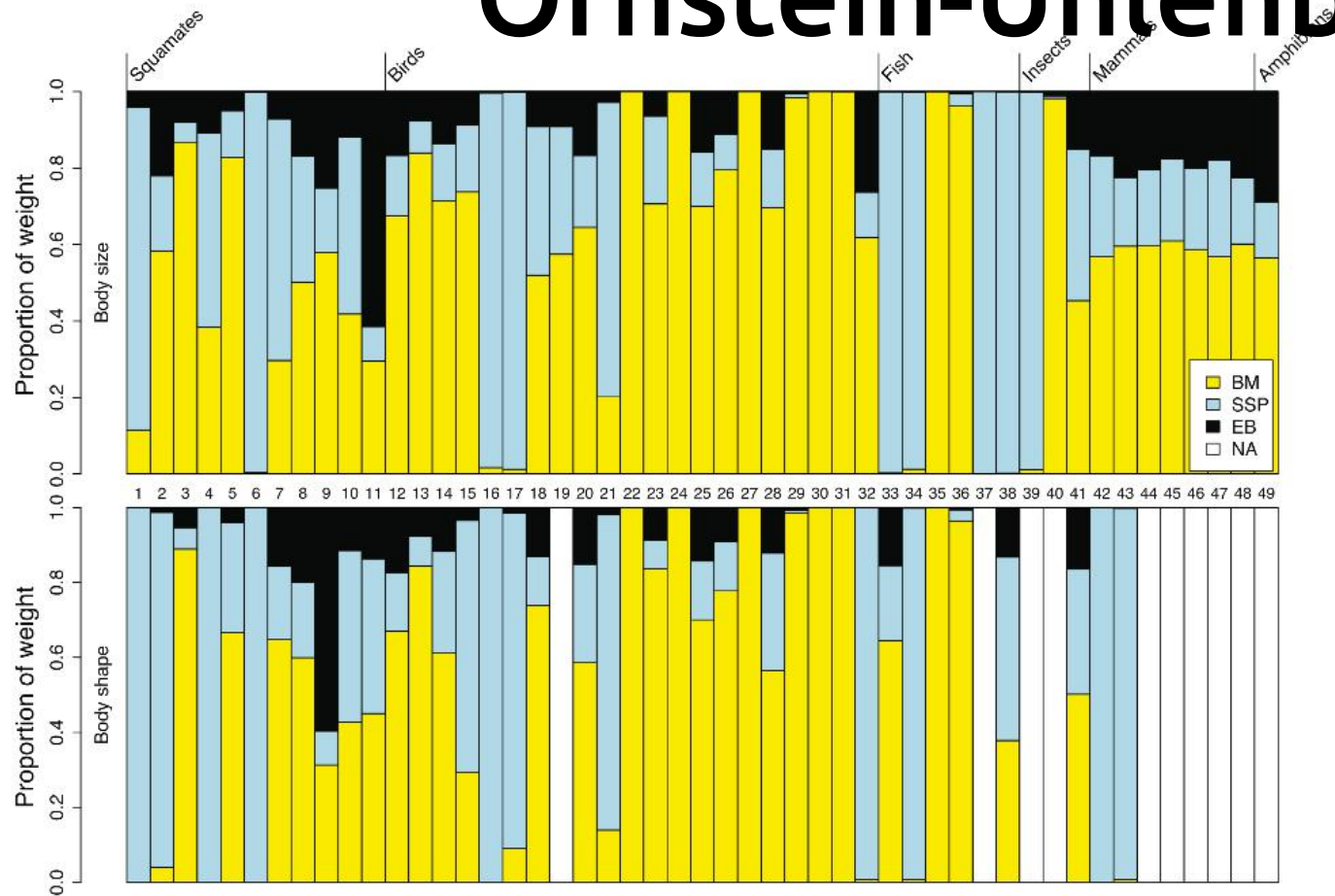
Luke J. Harmon,<sup>1,2,3</sup> Jonathan B. Losos,<sup>4</sup> T. Jonathan Davies,<sup>5</sup> Rosemary G. Gillespie,<sup>6</sup> John L. Gittleman,<sup>7</sup> W. Bryan Jennings,<sup>8</sup> Kenneth H. Kozak,<sup>9</sup> Mark A. McPeck,<sup>10</sup> Franck Moreno-Roark,<sup>11</sup> Thomas J. Near,<sup>12</sup> Andy Purvis,<sup>13</sup> Robert E. Ricklefs,<sup>14</sup> Dolph Schluter,<sup>2</sup> James A. Schulte II,<sup>11</sup> Ole Seehausen,<sup>15,16</sup> Brian L. Sidlauskas,<sup>17,18</sup> Omar Torres-Carvajal,<sup>19</sup> Jason T. Weir,<sup>2</sup> and Arne Ø. Mooers<sup>20</sup>

<sup>1</sup>*Department of Biological Sciences, University of Idaho, Moscow, Idaho 83844*

<sup>2</sup>*Biodiversity Centre, University of British Columbia, Vancouver, BC V6T1Z4, Canada*

<sup>3</sup>*E-mail: lukeh@uidaho.edu*

# Early Burst -> Brownian Motion -> Ornstein-Uhlenbeck



**Figure 2.** Akaike weights for three models of phenotypic evolution (BM, Brownian motion; SSP, single stationary peak; EB, early burst; NA, not applicable because shape data were unavailable) for all phylogenetic trees in the dataset. Numbers correspond to the datasets as listed in Table 1. Relative area of the bar filled with any color is proportional to the Akaike weight for that model given the data.

# Model Adequacy and the Macroevolution of Angiosperm Functional Traits

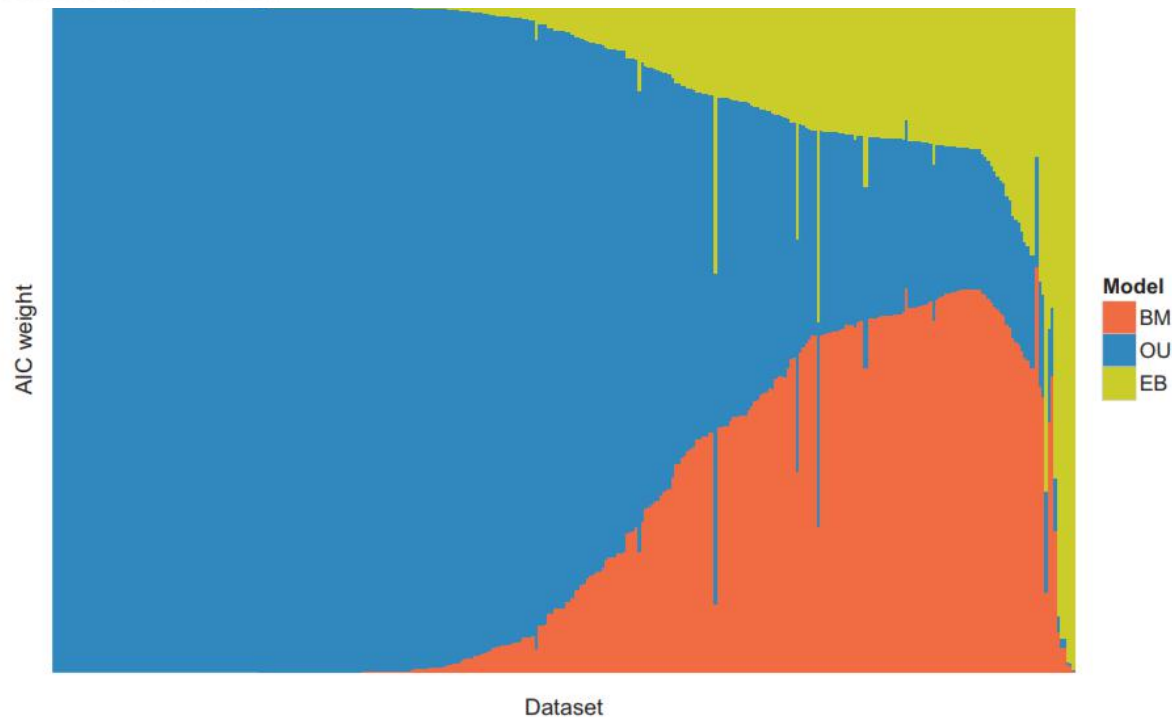
Matthew W. Pennell,<sup>1,\*</sup> Richard G. FitzJohn,<sup>2</sup> William K. Cornwell,<sup>3</sup> and Luke J. Harmon<sup>1</sup>

1. Department of Biological Sciences and Institute for Bioinformatics and Evolutionary Studies, University of Idaho, Moscow, Idaho 83844;

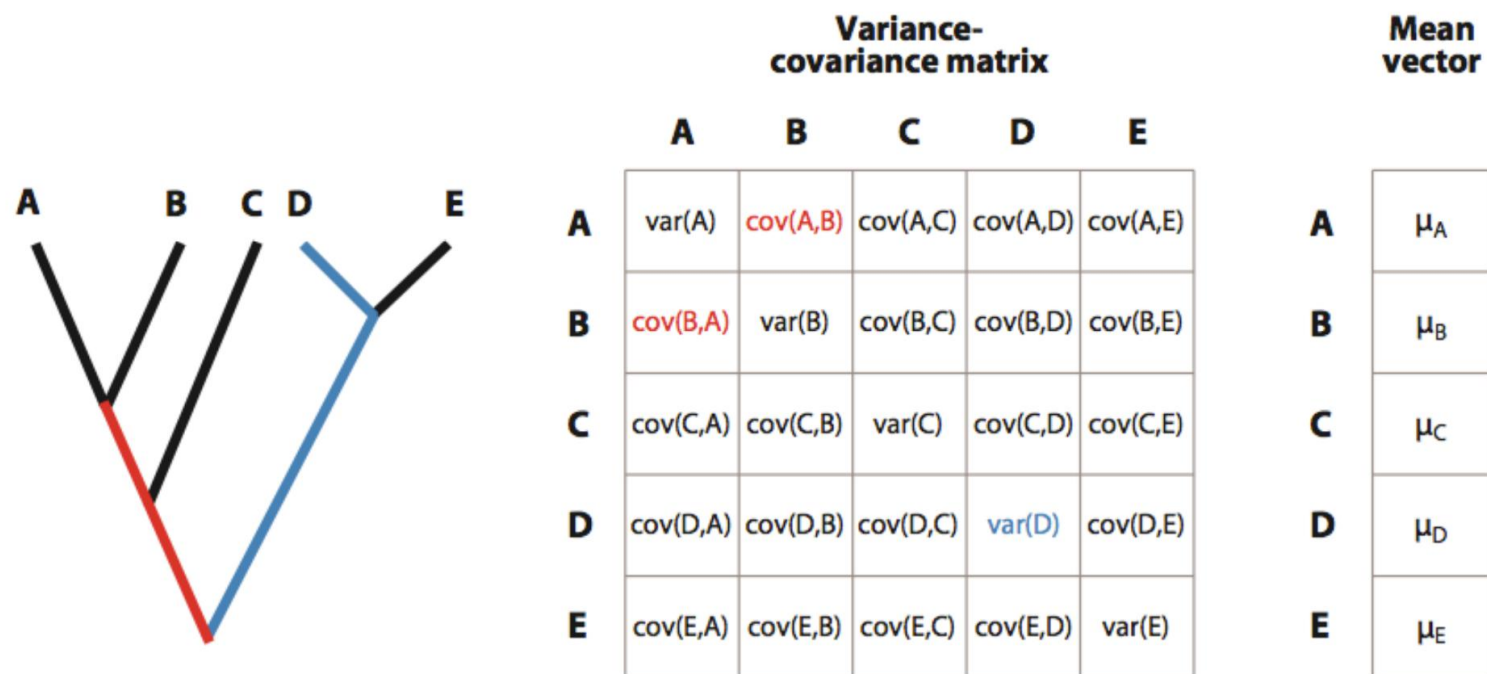
2. Department of Biological Sciences, Macquarie University, Sydney, New South Wales 2109, Australia; 3. School of Biological, Earth, and Environmental Sciences, University of New South Wales, Sydney, New South Wales 2052, Australia

*Submitted April 7, 2014; Accepted March 31, 2015; Electronically published June 12, 2015*

*Online enhancement: supplemental PDF.*



**Figure 3:** The relative support, as measured by Akaike information criterion (AIC) weight, for the three models used in our study (Brownian motion [BM], Ornstein-Uhlenbeck [OU], and early burst [EB]) across all 337 data sets. An OU model is highly supported for a majority of the data sets.



**Figure 4**

Multivariate normal distribution. The figure shows a tree, the tree's variance-covariance matrix, and the vector of means (which, under Brownian motion, would equal the root state). Highlighted are the branches leading to covariance between taxa A and B (*red*) and the branches leading to variance in D (*blue*).

# Why we must account for “measurement” error

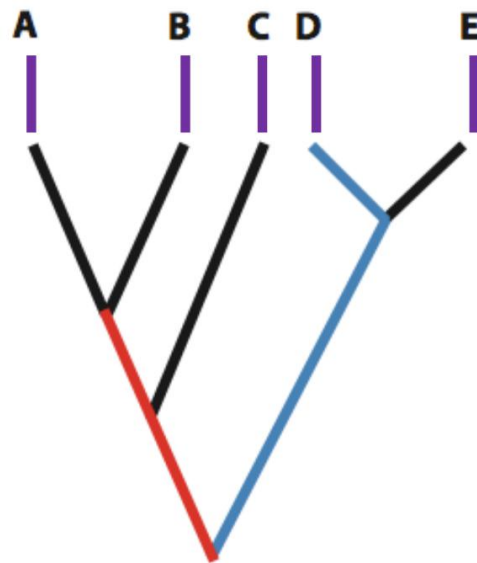


Figure 4

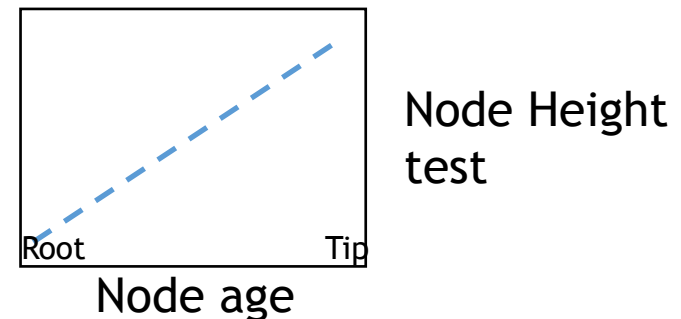
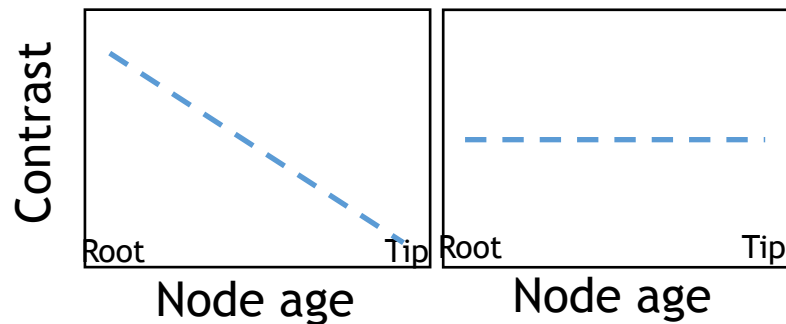
Variance-covariance matrix						Mean vector	
	A	B	C	D	E		
A	var(A) + error	cov(A,B)	cov(A,C)	cov(A,D)	cov(A,E)	A	$\mu_A$
B	cov(B,A)	var(B) + error	cov(B,C)	cov(B,D)	cov(B,E)	B	$\mu_B$
C	cov(C,A)	cov(C,B)	var(C) + error	cov(C,D)	cov(C,E)	C	$\mu_C$
D	cov(D,A)	cov(D,B)	cov(D,C)	var(D) + error	cov(D,E)	D	$\mu_D$
E	cov(E,A)	cov(E,B)	cov(E,C)	cov(E,D)	var(E) + error	E	$\mu_E$

# EB

# BM

# OU

# WN



Measurement error +  
“Biological error”





# PGLS + OU : Not a full OU model

Statistically, much like PGLS + Lambda

Appropriate only for “Allometric-type” relationships  
between predictor and response

OU model in variance, but not in mean  
(instantaneous adaptation)

Phylogenetic half-life with no predictors =  
measure of phylogenetic signal

..... with predictors =  
measure of REMAINING  
phylogenetic signal in residuals

**Next time: Shifting adaptive  
zones**