Benchmarking Optimization Methods for Quantum Recurrent Neural Networks

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Project Summary

Quantum Neural Networks (QNNs) remain a promising topic of interest in the field of Machine Learning (ML). With the advent of new simulation tools, it is now possible to run utility-scale simulations of Quantum Circuits (QCs). In particular, the class of QCs known as Variational Quantum Circuits (VQCs) are of great interest to the Quantum Machine Learning (QML) community and offer a quantum analogue to classical Neural Networks (NNs).

However, the types of hybrid classical-quantum models that can be feasibly constructed and trained remain largely unexplored. Moreover, the difficulties associated with training these models—both on classical computers and Quantum Processing Units (QPUs)—warrant further investigation, especially given the high cost of running on quantum hardware.

In light of these challenges, we propose a benchmark of two popular VQC training methods: **Simultaneous Perturbation Stochastic Approximation (SPSA)** and the **Parameter-Shift** rule, within an end-to-end hybrid trainable **Quantum Recurrent Neural Network (QRNN)** model. Using data from the chaotic Kuramoto–Sivashinsky system, we will develop a time-series forecasting model and benchmark the different training methods. This study will help identify best practices for training QRNNs as QPUs become more accessible and performant.

Problem Description

Although many works have explored VQC-based architectures, few have investigated the training behavior of hybrid quantum-classical recurrent networks. The QRNN architecture integrates a recurrent quantum component that presents additional training complexity. The optimization behavior of SPSA and parameter-shift under such conditions remains poorly understood.

Contribution

We will:

- Implement the QRNN model as a fully trainable PyTorch layer.
- Integrate both SPSA and parameter-shift optimization algorithms.
- Benchmark the performance of each approach on chaotic time-series forecasting using CPU and GPU.
- Quantitatively compare efficiency, cost, and loss convergence.

Introduction

Recent progress in QML has demonstrated the feasibility of small-scale quantum learning models. Among these, *Variational Quantum Circuits* (VQCs) are particularly appealing as they allow hybrid training paradigms leveraging both quantum and classical computation. However, training such circuits efficiently remains an open challenge.

The proposed research aims to provide an empirical benchmark comparing SPSA and parameter-shift methods in the context of a Quantum Recurrent Neural Network (QRNN). The Kuramoto–Sivashinsky system will serve as a testbed for assessing forecasting performance.

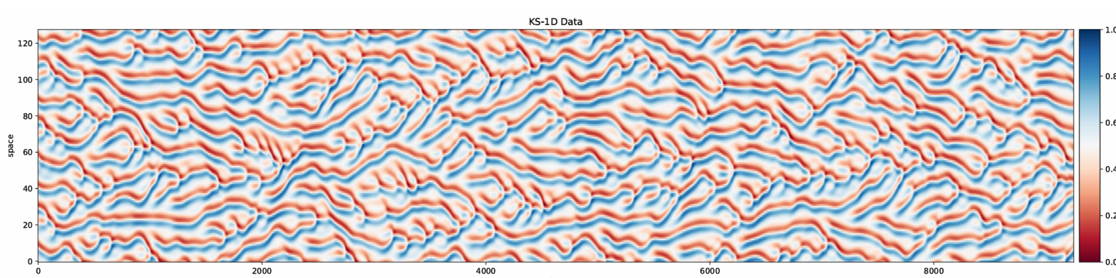

Figure 1: Example of the Kuramoto–Sivashinsky chaotic system used for QRNN benchmarking.

Related Works

Previous work by the authors demonstrated a QRNN model capable of chaotic state prediction on real quantum hardware [1]. The SPSA algorithm [2] and the parameter-shift rule [3] have both been widely applied in variational optimization. However, no prior comparison exists between these methods in a recurrent hybrid setting. Finally, the inherent difficulty of training RNNs in the classical domain [4] provides motivation to assess whether similar issues persist in their quantum analogues.

Hypothesis

The SPSA algorithm is generally assumed to be optimal for VQC optimization since it requires only $\mathcal{O}(n)$ circuit evaluations to compute a gradient, where n is the number of optimization steps. In contrast, the parameter-shift method requires $\mathcal{O}(pn)$ evaluations, where p is the number of parameters in the circuit. As p increases, the parameter-shift runtime grows while SPSA remains constant.

This efficiency is useful given the high cost of quantum computation. However, due to the challenges of training recurrent architectures, it remains uncertain whether SPSA's approximate gradients maintain effectiveness in a hybrid QRNN. We hypothesize that parameter-shift may provide superior optimization stability and convergence in this setting, despite its higher computational cost.

Evaluation

We will conduct experiments using the two aforementioned SPSA and parameter-shift methods in training a QRNN to predict the Kuramoto-Sivashinsky system. Metrics such as the total training-time required for each algorithm, training convergence, and validation accuracy will be used to determine which of the methods is best. In addition, QCs with differing number qubits and parameters will be tested to see if findings still hold with different configurations. We will also include results showing the differences between training on CPU and GPU.

Goals

1. Build a PyTorch-compatible QRNN layer to enable hybrid classical–quantum model training.

2. Implement evaluate SPSA and parameter shift for the network.

3. Accurately predict the dynamics of the KS System.

4. Identify the optimal training process for QRNNs.

References

References

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