Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1\\0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0\\1 \end{bmatrix}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} \end{bmatrix} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha\\\beta \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

2 Unitary Operators

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \qquad I|0\rangle = |0\rangle$$

$$I|1\rangle = |1\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \qquad X|0\rangle = |1\rangle$$

$$X|1\rangle = |0\rangle$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \qquad Y|0\rangle = i|1\rangle$$

$$Y|1\rangle = -i|0\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \qquad Z|0\rangle = |0\rangle$$

$$Z|1\rangle = -|1\rangle$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad H|0\rangle = |+\rangle$$

$$H|1\rangle = |-\rangle \qquad -H|-$$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad S|0\rangle = |0\rangle$$

$$S|1\rangle = i|1\rangle \qquad -S|-$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \qquad T|0\rangle = |0\rangle$$

$$T|1\rangle = e^{i\pi/4}|1\rangle \qquad -T|-$$

$$CNOT|00\rangle = |00\rangle$$

$$CNOT|01\rangle = |01\rangle$$

$$CNOT|10\rangle = |11\rangle$$

$$CNOT|11\rangle = |10\rangle$$

3 Operator identities

$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \quad S^2 = Z$$

$$XY = iZ$$
 $YX = -iZ$ $XZ = -iY$ $ZX = iY$ $YZ = -iZ$ $ZY = -iX$

$$HX = ZH$$
 $HZ = XH$
 $SX = XZS$ $SZ = ZS$

$$HXH = Z$$
 $HYH = -Y$ HZH
 $SXS = Y$ $SYS = X$ $SZS = Z$

$$\mathrm{CNOT}_{0,1}(X \otimes I) = (X \otimes X)\mathrm{CNOT}_{0,1}$$

$$\mathrm{CNOT}_{0,1}(I \otimes X) = (I \otimes X)\mathrm{CNOT}_{0,1}$$

$$\mathrm{CNOT}_{0,1}(Z \otimes I) = (Z \otimes I)\mathrm{CNOT}_{0,1}$$

$$\mathrm{CNOT}_{0,1}(I \otimes Z) = (Z \otimes Z)\mathrm{CNOT}_{0,1}$$