Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

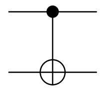
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

2 Unitary Operators

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{array}{ll} \|0\rangle \mapsto \|0\rangle & \|+\rangle \mapsto \|+\rangle \\ \|1\rangle \mapsto \|1\rangle & \|-\rangle \mapsto \|-\rangle \end{array}$	———
$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{array}{ll} \|0\rangle \mapsto \|1\rangle & \|+\rangle \mapsto \|+\rangle \\ \|1\rangle \mapsto \|0\rangle & \|-\rangle \mapsto -\|-\rangle \end{array}$	$-\boxed{X}-$
$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$\begin{array}{ll} \|0\rangle \mapsto i\|1\rangle & \|+\rangle \mapsto -i\|-\rangle \\ \|1\rangle \mapsto -i\|0\rangle & \|-\rangle \mapsto i\|+\rangle \end{array}$	$-\boxed{Y}-$
$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{array}{ll} \|0\rangle \mapsto \|0\rangle & \|+\rangle \mapsto \|-\rangle \\ \|1\rangle \mapsto -\|1\rangle & \|-\rangle \mapsto \|+\rangle \end{array}$	$-\boxed{Z}-$
$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$\begin{array}{ll} \|0\rangle \mapsto \|+\rangle & \|+\rangle \mapsto \|0\rangle \\ \|1\rangle \mapsto \|-\rangle & \|-\rangle \mapsto \|1\rangle \end{array}$	$-\boxed{H}-$
$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$	$\begin{array}{l} \|0\rangle \mapsto \|0\rangle \\ \|1\rangle \mapsto e^{i\theta}\|1\rangle \end{array}$	$-\boxed{R_\theta}-$
$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{array}{l} \|0\rangle \mapsto \|0\rangle \\ \|1\rangle \mapsto i\|1\rangle \end{array}$	$-\boxed{S}-$
$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	$\begin{array}{l} \|0\rangle \mapsto \|0\rangle \\ \|1\rangle \mapsto e^{i\pi/4}\|1\rangle \end{array}$	$-\boxed{T}-$

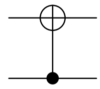
$$\mathrm{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |01\rangle & |{+}{-}\rangle &\mapsto |{-}{-}\rangle \\ |10\rangle &\mapsto |11\rangle & |{-}{+}\rangle &\mapsto |{-}{+}\rangle \\ |11\rangle &\mapsto |10\rangle & |{-}{-}\rangle &\mapsto |{+}{-}\rangle \end{aligned}$$



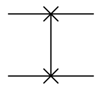
$$\mathrm{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |11\rangle & |{+}{-}\rangle &\mapsto |{+}{-}\rangle \\ |10\rangle &\mapsto |10\rangle & |{-}{+}\rangle &\mapsto |{-}{-}\rangle \\ |11\rangle &\mapsto |01\rangle & |{-}{-}\rangle &\mapsto |{-}{+}\rangle \end{aligned}$$



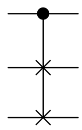
$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |10\rangle & |{+}{-}\rangle &\mapsto |{-}{+}\rangle \\ |10\rangle &\mapsto |01\rangle & |{-}{+}\rangle &\mapsto |{+}{-}\rangle \\ |11\rangle &\mapsto |11\rangle & |{-}{-}\rangle &\mapsto |{-}{-}\rangle \end{aligned}$$



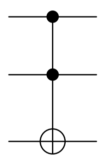
$$\mathrm{CSWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |110\rangle \\ |110\rangle &\mapsto |101\rangle \\ |111\rangle &\mapsto |111\rangle \end{aligned}$$



$$\mathrm{TOFFOLI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |101\rangle \\ |110\rangle &\mapsto |111\rangle \\ |111\rangle &\mapsto |110\rangle \end{aligned}$$



3 Operator identities

$$X^2 = Y^2 = Z^2 = H^2 = I$$

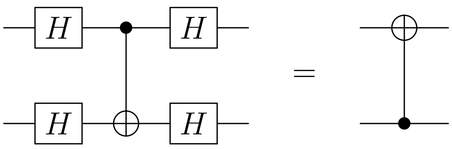
$$T^2 = S \qquad S^2 = Z$$

$$XY = iZ \qquad YX = -iZ \qquad ZX = iY$$

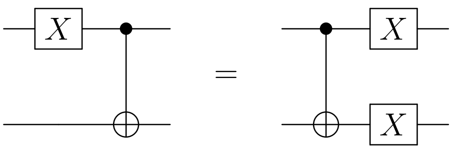
$$XZ = -iY \qquad YZ = iX \qquad ZY = -iX$$

$$\begin{aligned} HX &= ZH & SX &= XZS \\ HZ &= XH & SZ &= ZS \end{aligned}$$

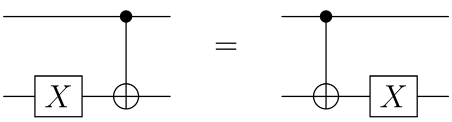
$$\begin{aligned} HXH &= Z & SXS^\dagger &= Y \\ HYH &= -Y & SYS^\dagger &= -X \\ HZH &= X & SZS^\dagger &= Z \end{aligned}$$



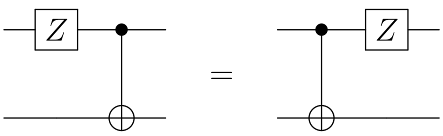
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



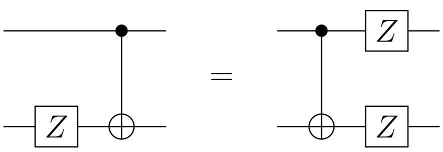
$$(X \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(X \otimes X)$$



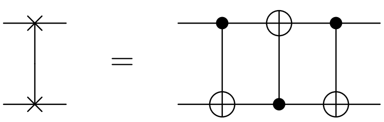
$$(I \otimes X)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(I \otimes X)$$



$$(Z \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes I)$$



$$(I \otimes Z)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Z)$$



$$\mathrm{SWAP} = \mathrm{CNOT}_{0,1}\mathrm{CNOT}_{1,0}\mathrm{CNOT}_{0,1}$$