



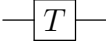

Quantum Computing Cheat Sheet

1 States

$$\begin{aligned}
 |0\rangle &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} & |1\rangle &= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
 |+\rangle &= \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} & |-\rangle &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \\
 |\psi\rangle &= \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, & |\alpha|^2 + |\beta|^2 &= 1
 \end{aligned}$$

2 Unitary Operators

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$ x\rangle \mapsto x\rangle$	
$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$ 0\rangle \mapsto 1\rangle$ $ 1\rangle \mapsto 0\rangle$	$ +\rangle \mapsto +\rangle$ $ -\rangle \mapsto - -\rangle$
$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$ 0\rangle \mapsto i 1\rangle$ $ 1\rangle \mapsto -i 0\rangle$	$ +\rangle \mapsto -i -\rangle$ $ -\rangle \mapsto i +\rangle$
$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$ 0\rangle \mapsto 0\rangle$ $ 1\rangle \mapsto - 1\rangle$	$ +\rangle \mapsto -\rangle$ $ -\rangle \mapsto +\rangle$
$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$ 0\rangle \mapsto +\rangle$ $ 1\rangle \mapsto -\rangle$	$ +\rangle \mapsto 0\rangle$ $ -\rangle \mapsto 1\rangle$

$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	$\begin{aligned} 0\rangle &\mapsto 0\rangle \\ 1\rangle &\mapsto i 1\rangle \end{aligned}$	
$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	$\begin{aligned} 0\rangle &\mapsto 0\rangle \\ 1\rangle &\mapsto e^{i\pi/4} 1\rangle \end{aligned}$	
$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{array}{ll} 00\rangle \mapsto 00\rangle & ++\rangle \mapsto ++\rangle \\ 01\rangle \mapsto 01\rangle & +-\rangle \mapsto --\rangle \\ 10\rangle \mapsto 11\rangle & -+\rangle \mapsto -+\rangle \\ 11\rangle \mapsto 10\rangle & --\rangle \mapsto +-\rangle \end{array}$	

3 Operator identities

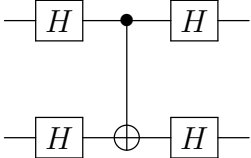
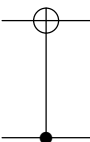
$$X^2 = Y^2 = Z^2 = H^2 = I$$

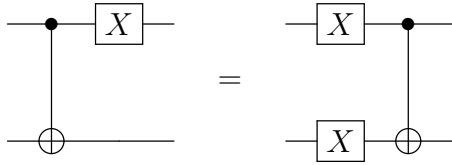
$$T^2 = S \quad S^2 = Z$$

$$XY = iZ \quad YX = -iZ \quad XZ = -iY \quad ZX = iY \quad YZ = -iX \quad ZY = iX$$

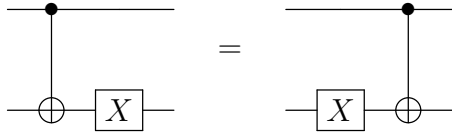
$$\begin{aligned} HX &= ZH & HZ &= XH \\ SX &= XZS & SZ &= ZS \end{aligned}$$

$$\begin{aligned} HXH &= Z & HYH &= -Y & HZH &= X \\ SXS &= Y & SYS &= X & SZS &= Z \end{aligned}$$

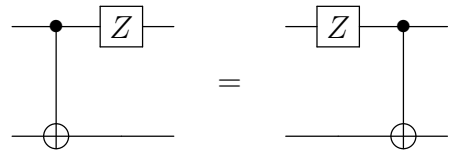
	$=$		$(H \otimes H)\text{CNOT}_{0,1}(H \otimes H) = \text{CNOT}_{1,0}$
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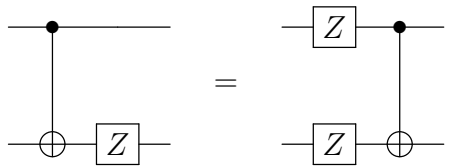
$$\text{CNOT}_{0,1}(X \otimes I) = (X \otimes X)\text{CNOT}_{0,1}$$



$$\text{CNOT}_{0,1}(I \otimes X) = (I \otimes X)\text{CNOT}_{0,1}$$



$$\text{CNOT}_{0,1}(Z \otimes I) = (Z \otimes I)\text{CNOT}_{0,1}$$



$$\text{CNOT}_{0,1}(I \otimes Z) = (Z \otimes Z)\text{CNOT}_{0,1}$$