Quantum Computing Cheat Sheet

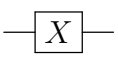
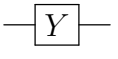
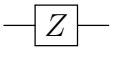
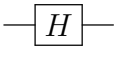
1 States

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

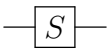
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\\beta\end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

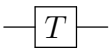
2 Unitary Operators

$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\|x\rangle \mapsto \|x\rangle$		
$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\|0\rangle \mapsto \|1\rangle$ $\|1\rangle \mapsto \|0\rangle$	$\|+\rangle \mapsto \|+\rangle$ $\|-\rangle \mapsto -\|-\rangle$	
$Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$	$\|0\rangle \mapsto i\|1\rangle$ $\|1\rangle \mapsto -i\|0\rangle$	$\|+\rangle \mapsto -i\|-\rangle$ $\|-\rangle \mapsto i\|+\rangle$	
$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	$\|0\rangle \mapsto \|0\rangle$ $\|1\rangle \mapsto -\|1\rangle$	$\|+\rangle \mapsto \|-\rangle$ $\|-\rangle \mapsto \|+\rangle$	
$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$	$\|0\rangle \mapsto \|+\rangle$ $\|1\rangle \mapsto \|-\rangle$	$\|+\rangle \mapsto \|0\rangle$ $\|-\rangle \mapsto \|1\rangle$	

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto i|1\rangle \end{aligned}$$



$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto e^{i\pi/4}|1\rangle \end{aligned}$$



$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{aligned} |00\rangle &\mapsto |00\rangle \quad |{+}{+}\rangle \mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |01\rangle \quad |{+}{-}\rangle \mapsto |{-}{-}\rangle \\ |10\rangle &\mapsto |11\rangle \quad |{-}{+}\rangle \mapsto |{-}{+}\rangle \\ |11\rangle &\mapsto |10\rangle \quad |{-}{-}\rangle \mapsto |{+}{-}\rangle \end{aligned}$$

3 Operator identities

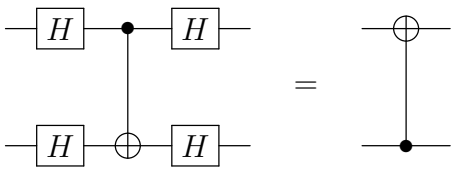
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \qquad S^2 = Z$$

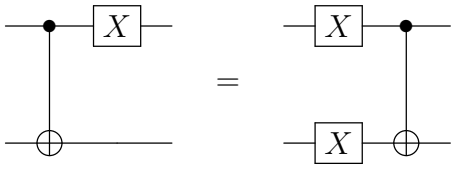
$$\begin{aligned} XY &= iZ \qquad YX = -iZ \qquad ZX = iY \\ XZ &= -iY \qquad YZ = -iZ \qquad ZY = -iX \end{aligned}$$

$$\begin{aligned} HX &= ZH \qquad SX = XZS \\ HZ &= XH \qquad SZ = ZS \end{aligned}$$

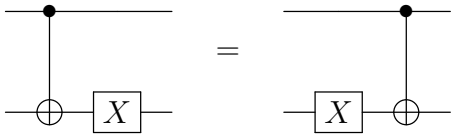
$$\begin{aligned} HXH &= Z \qquad SXS^\dagger = Y \\ HYH &= -Y \qquad SYS^\dagger = X \\ HZH &= X \qquad SZS^\dagger = Z \end{aligned}$$



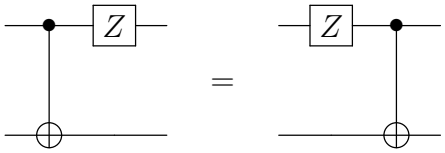
$$(H \otimes H)\text{CNOT}_{0,1}(H \otimes H) = \text{CNOT}_{1,0}$$



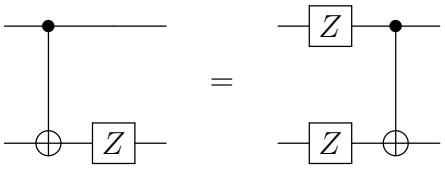
$$\mathrm{CNOT}_{0,1}(X \otimes I) = (X \otimes X)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(I \otimes X) = (I \otimes X)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(Z \otimes I) = (Z \otimes I)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(I \otimes Z) = (Z \otimes Z)\mathrm{CNOT}_{0,1}$$