Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

2 Unitary Operators

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

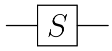
$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad -\boxed{X}-$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad -\boxed{Y}-$$

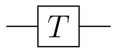
$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad -\boxed{Z}-$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad -\boxed{H}-$$

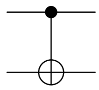
$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$



$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$$



$$\mathrm{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



3 Operator identities

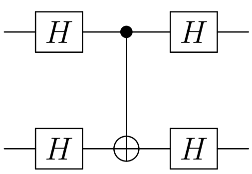
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \quad S^2 = Z$$

$$XY = iZ \quad YX = -iZ \quad XZ = -iY \quad ZX = iY \quad YZ = -iZ \quad ZY = -iX$$

$$HX = ZH \quad HZ = XH$$
$$SX = XZS \quad SZ = ZS$$

$$HXH = Z \quad HYH = -Y \quad HZH$$
$$SXS = Y \quad SYS = X \quad SZS = Z$$



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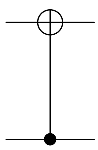
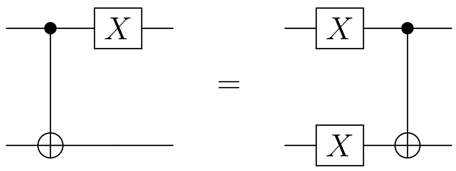
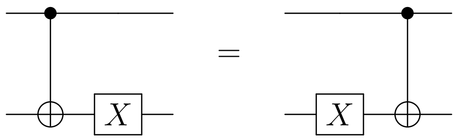


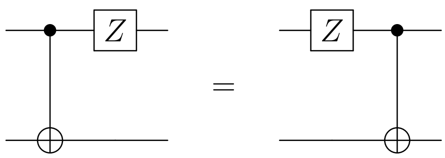
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



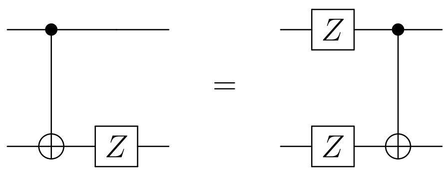
$$\mathrm{CNOT}_{0,1}(X \otimes I) = (X \otimes X)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(I \otimes X) = (I \otimes X)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(Z \otimes I) = (Z \otimes I)\mathrm{CNOT}_{0,1}$$



$$\mathrm{CNOT}_{0,1}(I \otimes Z) = (Z \otimes Z)\mathrm{CNOT}_{0,1}$$