Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

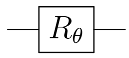
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

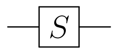
2 Unitary Operators

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\vert x\rangle \mapsto \vert x\rangle$	———
$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\vert 0\rangle \mapsto \vert 1\rangle \quad \vert +\rangle \mapsto \vert +\rangle$ $\vert 1\rangle \mapsto \vert 0\rangle \quad \vert -\rangle \mapsto -\vert -\rangle$	$-\boxed{X}-$
$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	$\vert 0\rangle \mapsto i\vert 1\rangle \quad \vert +\rangle \mapsto -i\vert -\rangle$ $\vert 1\rangle \mapsto -i\vert 0\rangle \quad \vert -\rangle \mapsto i\vert +\rangle$	$-\boxed{Y}-$
$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\vert 0\rangle \mapsto \vert 0\rangle \quad \vert +\rangle \mapsto \vert -\rangle$ $\vert 1\rangle \mapsto -\vert 1\rangle \quad \vert -\rangle \mapsto \vert +\rangle$	$-\boxed{Z}-$
$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	$\vert 0\rangle \mapsto \vert +\rangle \quad \vert +\rangle \mapsto \vert 0\rangle$ $\vert 1\rangle \mapsto \vert -\rangle \quad \vert -\rangle \mapsto \vert 1\rangle$	$-\boxed{H}-$

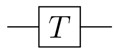
$$R_\theta = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto e^{i\theta}|1\rangle \end{aligned}$$



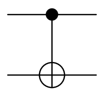
$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto i|1\rangle \end{aligned}$$



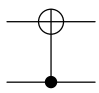
$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \qquad \begin{aligned} |0\rangle &\mapsto |0\rangle \\ |1\rangle &\mapsto e^{i\pi/4}|1\rangle \end{aligned}$$



$$\mathrm{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{aligned} |00\rangle &\mapsto |00\rangle \\ |01\rangle &\mapsto |01\rangle \\ |10\rangle &\mapsto |11\rangle \\ |11\rangle &\mapsto |10\rangle \end{aligned} \qquad \begin{aligned} |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |{+}{-}\rangle &\mapsto |{-}{-}\rangle \\ |{-}{+}\rangle &\mapsto |{-}{+}\rangle \\ |{-}{-}\rangle &\mapsto |{+}{-}\rangle \end{aligned}$$



$$\mathrm{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{aligned} |00\rangle &\mapsto |00\rangle \\ |01\rangle &\mapsto |11\rangle \\ |10\rangle &\mapsto |10\rangle \\ |11\rangle &\mapsto |01\rangle \end{aligned} \qquad \begin{aligned} |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |{+}{-}\rangle &\mapsto |{+}{-}\rangle \\ |{-}{+}\rangle &\mapsto |{-}{-}\rangle \\ |{-}{-}\rangle &\mapsto |{-}{+}\rangle \end{aligned}$$



3 Operator identities

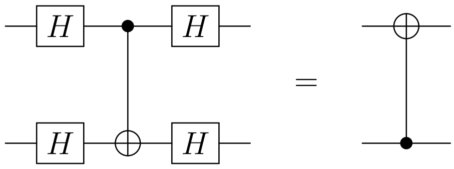
$$X^2 = Y^2 = Z^2 = H^2 = I$$

$$T^2 = S \qquad S^2 = Z$$

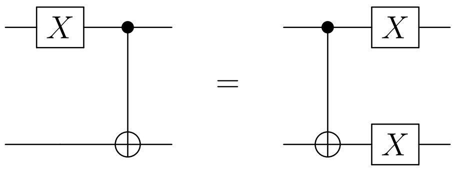
$$\begin{aligned} XY &= iZ & YX &= -iZ & ZX &= iY \\ XZ &= -iY & YZ &= iX & ZY &= -iX \end{aligned}$$

$$\begin{aligned} HX &= ZH & SX &= XZS \\ HZ &= XH & SZ &= ZS \end{aligned}$$

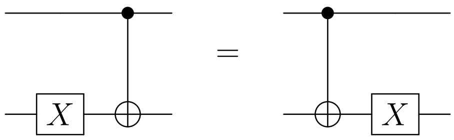
$$
\begin{aligned}
HXH &= Z & \qquad SXS^\dagger &= Y \\
HYH &= -Y & \qquad SYS^\dagger &= -X \\
HZH &= X & \qquad SZS^\dagger &= Z
\end{aligned}
$$



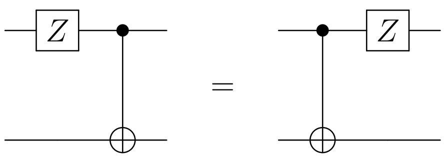
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



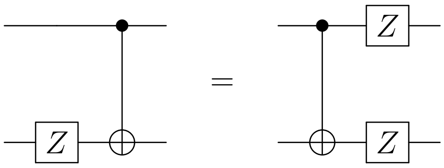
$$(X \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(X \otimes X)$$



$$(I \otimes X)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(I \otimes X)$$



$$(Z \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes I)$$



$$(I \otimes Z)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Z)$$