Quantum Computing Cheat Sheet

1 States

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} \quad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}$$

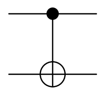
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix}\alpha\\\beta\end{bmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1$$

2 Unitary Operators

$I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$	$\begin{array}{ll} \lvert 0\rangle \mapsto \lvert 0\rangle & \lvert +\rangle \mapsto \lvert +\rangle \\ \lvert 1\rangle \mapsto \lvert 1\rangle & \lvert -\rangle \mapsto \lvert -\rangle \end{array}$	———
$X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$	$\begin{array}{ll} \lvert 0\rangle \mapsto \lvert 1\rangle & \lvert +\rangle \mapsto \lvert +\rangle \\ \lvert 1\rangle \mapsto \lvert 0\rangle & \lvert -\rangle \mapsto -\lvert -\rangle \end{array}$	$-\boxed{X}-$
$Y = \begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$	$\begin{array}{ll} \lvert 0\rangle \mapsto i\lvert 1\rangle & \lvert +\rangle \mapsto -i\lvert -\rangle \\ \lvert 1\rangle \mapsto -i\lvert 0\rangle & \lvert -\rangle \mapsto i\lvert +\rangle \end{array}$	$-\boxed{Y}-$
$Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$	$\begin{array}{ll} \lvert 0\rangle \mapsto \lvert 0\rangle & \lvert +\rangle \mapsto \lvert -\rangle \\ \lvert 1\rangle \mapsto -\lvert 1\rangle & \lvert -\rangle \mapsto \lvert +\rangle \end{array}$	$-\boxed{Z}-$
$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$	$\begin{array}{ll} \lvert 0\rangle \mapsto \lvert +\rangle & \lvert +\rangle \mapsto \lvert 0\rangle \\ \lvert 1\rangle \mapsto \lvert -\rangle & \lvert -\rangle \mapsto \lvert 1\rangle \end{array}$	$-\boxed{H}-$
$R_\theta = \begin{bmatrix}1 & 0\\0 & e^{i\theta}\end{bmatrix}$	$\begin{array}{l} \lvert 0\rangle \mapsto \lvert 0\rangle \\ \lvert 1\rangle \mapsto e^{i\theta}\lvert 1\rangle \end{array}$	$-\boxed{R_\theta}-$
$S = \begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$	$\begin{array}{l} \lvert 0\rangle \mapsto \lvert 0\rangle \\ \lvert 1\rangle \mapsto i\lvert 1\rangle \end{array}$	$-\boxed{S}-$
$T = \begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$	$\begin{array}{l} \lvert 0\rangle \mapsto \lvert 0\rangle \\ \lvert 1\rangle \mapsto e^{i\pi/4}\lvert 1\rangle \end{array}$	$-\boxed{T}-$

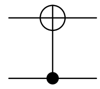
$$\mathrm{CNOT}_{0,1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |01\rangle & |{+}{-}\rangle &\mapsto |{-}{-}\rangle \\ |10\rangle &\mapsto |11\rangle & |{-}{+}\rangle &\mapsto |{-}{+}\rangle \\ |11\rangle &\mapsto |10\rangle & |{-}{-}\rangle &\mapsto |{+}{-}\rangle \end{aligned}$$



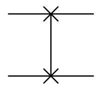
$$\mathrm{CNOT}_{1,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |11\rangle & |{+}{-}\rangle &\mapsto |{+}{-}\rangle \\ |10\rangle &\mapsto |10\rangle & |{-}{+}\rangle &\mapsto |{-}{-}\rangle \\ |11\rangle &\mapsto |01\rangle & |{-}{-}\rangle &\mapsto |{-}{+}\rangle \end{aligned}$$



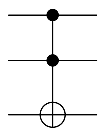
$$\mathrm{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} |00\rangle &\mapsto |00\rangle & \qquad |{+}{+}\rangle &\mapsto |{+}{+}\rangle \\ |01\rangle &\mapsto |10\rangle & |{+}{-}\rangle &\mapsto |{-}{+}\rangle \\ |10\rangle &\mapsto |01\rangle & |{-}{+}\rangle &\mapsto |{+}{-}\rangle \\ |11\rangle &\mapsto |11\rangle & |{-}{-}\rangle &\mapsto |{-}{-}\rangle \end{aligned}$$



$$\mathrm{TOFFOLI} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{aligned} |000\rangle &\mapsto |000\rangle \\ |001\rangle &\mapsto |001\rangle \\ |010\rangle &\mapsto |010\rangle \\ |011\rangle &\mapsto |011\rangle \\ |100\rangle &\mapsto |100\rangle \\ |101\rangle &\mapsto |101\rangle \\ |110\rangle &\mapsto |111\rangle \\ |111\rangle &\mapsto |110\rangle \end{aligned}$$



3 Operator identities

$$X^2 = Y^2 = Z^2 = H^2 = I$$

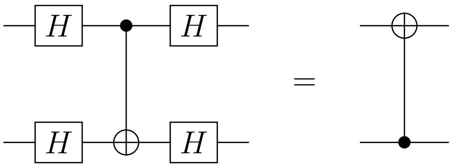
$$T^2 = S \qquad S^2 = Z$$

$$\begin{aligned} XY &= iZ & \qquad YX &= -iZ & \qquad ZX &= iY \\ XZ &= -iY & YZ &= iX & ZY &= -iX \end{aligned}$$

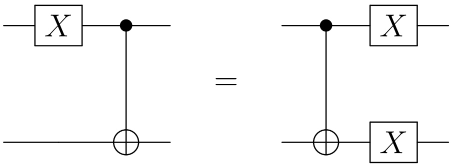
$$\begin{aligned} HX &= ZH & \qquad SX &= XZS \\ HZ &= XH & SZ &= ZS \end{aligned}$$

$$HXH = Z \qquad SXS^\dagger = Y$$

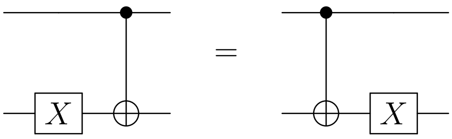
$$HYH = -Y \qquad SYS^\dagger = -X$$
$$HZH = X \qquad SZS^\dagger = Z$$



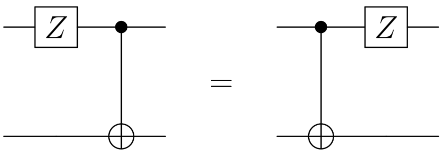
$$(H \otimes H)\mathrm{CNOT}_{0,1}(H \otimes H) = \mathrm{CNOT}_{1,0}$$



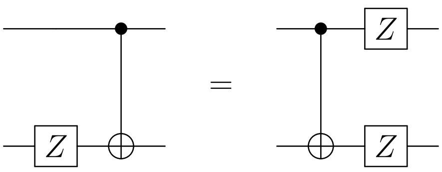
$$(X \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(X \otimes X)$$



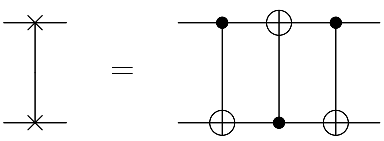
$$(I \otimes X)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(I \otimes X)$$



$$(Z \otimes I)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes I)$$



$$(I \otimes Z)\mathrm{CNOT}_{0,1} = \mathrm{CNOT}_{0,1}(Z \otimes Z)$$



$$\mathrm{SWAP} = \mathrm{CNOT}_{0,1}\mathrm{CNOT}_{1,0}\mathrm{CNOT}_{0,1}$$