QUANTUM CRYPTOGRAPHY
Master of Logic, University of Amsterdam, June 2022
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Practice problem set 0

You do not have to hand in these exercises, they are for practicing only.

Problem 1: Inner product

For two d-dimensional complex vectors $|a\rangle$ and $|b\rangle$, the inner product $\langle a|b\rangle$ is defined as $\langle a|b\rangle := \langle a||b\rangle = \sum_{i=1}^{d} a_i^* b_i$. Show that $|\langle a|b\rangle|^2 = \langle a|b\rangle\langle b|a\rangle$.

Problem 2: Valid quantum state

Verify that for all $\theta, \varphi \in \mathbb{R}$, $\cos(\theta)|0\rangle + \sin(\theta)e^{i\varphi}|1\rangle$ is a valid qubit state.

Problem 3: Changing basis

Express $|1\rangle$ in the Hadamard basis. That is, find coefficients α and β such that $|1\rangle = \alpha|+\rangle + \beta|-\rangle$.

Problem 4: Measurement

Consider the state $|\psi\rangle = \frac{1}{\sqrt{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$. What are the probabilities p_0, p_1 when we measure $|\psi\rangle$ in the standard basis? What are the probabilities p_+, p_- when we measure $|\psi\rangle$ in the Hadamard basis?

Problem 5: Pauli operations

(a) Write down the Pauli matrices X, Y, and Z.

(b) Verify that they are unitary.

(c) What rotations of the Bloch sphere do the Paulis represent?

Problem 6: Density operators

Which of the following matrices can be interpreted as density operators?

(a)
$$\begin{pmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{3}{4} \end{pmatrix}$$

(b) $\frac{1}{3}|u\rangle\langle u| + \frac{2}{3}|v\rangle\langle v| + \frac{\sqrt{2}}{3}|u\rangle\langle v| + \frac{\sqrt{2}}{3}|v\rangle\langle u|$, where $|u\rangle$ and $|v\rangle$ form an orthonormal basis.

(c)
$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & 0 \end{pmatrix}$$

(d)
$$\begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} \end{pmatrix}$$