Kernel Methods

Dual

$$f(x) = \underline{\langle x, w \rangle}$$

$\boxed{x^2}$	x	x^0	y
9	-3	1	9
4	-2	1	4
1	-1	1	1

0	0	1	0
1	1	1	1
4	2	1	4
9	3	1	9

$$f(x) = \langle x, w \rangle$$

$$= \langle \varphi(x), w \rangle$$

$x = (x_0, x_1)$

$y = (y_0, y_1)$

$\langle x, y \rangle = x_0 y_0 + x_1 y_1$

$$(\langle x, y \rangle + 1) = (x_0 y_0 + x_1 y_1 + 1)$$

$$= x_0 y_0 (x_0 y_0 + x_1 y_1 + 1)$$
$$+ x_1 y_1 (x_0 y_0 + x_1 y_1 + 1)$$
$$+ 1 (x_0 y_0 + x_1 y_1 + 1)$$

$$\varphi(x_0, x_1) = (2x_0, 2x_1, 1, 2x_0 x_1, x_0^2, x_1^2)$$

$$\langle \varphi(\vec{x}), \varphi(\vec{y}) \rangle =$$

$$\rightarrow \boxed{(\langle \vec{x}, \vec{y} \rangle + 1)^2}$$

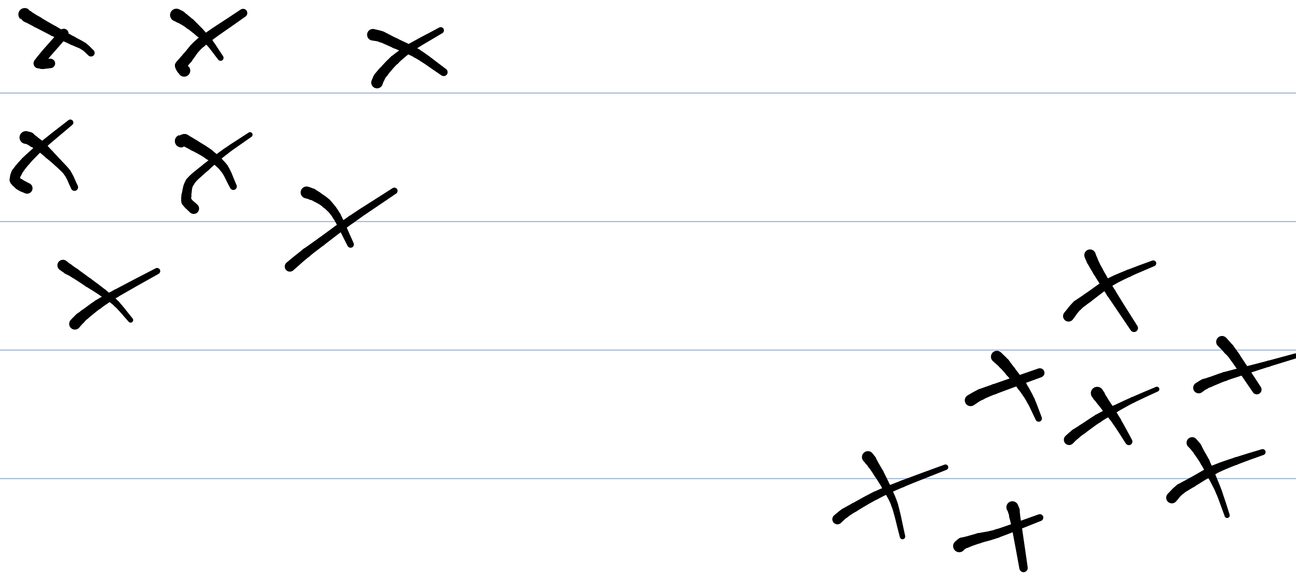
$$\langle x, y \rangle = \sum_i x_i y_i$$

$$\langle x, y \rangle = \langle y, x \rangle$$

$$\langle x, \underline{y + z} \rangle = \langle x, y \rangle + \langle x, z \rangle$$

$$\langle x, k \cdot y \rangle = k \langle x, y \rangle$$

$$\langle x, y \rangle = \cos \measuredangle (x, y) \cdot |x| |y|$$



$$\|x-y\|^2 =$$

$$\langle x-y, x-y\rangle =$$

$$\langle x, x-y\rangle - \langle y, x-y\rangle =$$

$$\langle x,x\rangle - \langle x,y\rangle - \langle y,x\rangle$$

$$+\langle y, y\rangle = \langle x,x\rangle - 2\langle x,y\rangle$$

$+\langle y, y\rangle$

$$c_i = \frac{1}{|A_i|}\sum A_{ij} =$$

j-th $\underline{x}$ vector in A_i

$$\|x - c_i\|^2 =$$

$$\langle x - c_i, x - c_i\rangle =$$

$$\langle x, x\rangle - \underline{2\langle x, c_i\rangle}$$

$$+ \langle c_i, c_i\rangle$$

$$\langle x, c_i \rangle = \langle x, \sum_i \frac{1}{|A_{\cdot}|} A_{ij} \rangle$$

$$= \frac{1}{|A_{\cdot}|} \sum_j \langle x, \underline{A_{ij}} \rangle$$

$$\langle c_i, c_i \rangle = \langle \sum A, \sum A \rangle$$

$$= \sum_j \sum_k \frac{1}{|A_{i}|} \frac{1}{|A_{\cdot}|} \langle A_{ij}, A_{ik} \rangle$$

$$\boxed{w_d = \left(\overbrace{\cdots\cdots\cdots}^{c} \right)}$$

$$\underline{w_d} = \sum_i \boxed{\underline{\underline{c_i}}} \, x_i$$

$$\Sigma \quad N$$

$$\underline{\langle x_i, w \rangle} =$$

$$\langle x_i, \sum_j c_j x_j \rangle =$$

$$\underline{\sum c_j \langle x_i, x_j \rangle}$$

$$\underline{L(w)} = \sum_i \ell(\underline{\langle w, x_i \rangle}, y_i) + \lambda \|w\|^2$$

$$\nabla L = \sum \boxed{\frac{d\,\ell(p_i, y_i)}{\partial p_i}}\, x_i$$

$$1+2\lambda w$$

$$p_i = \langle w, x_i \rangle$$

$$X\vec{\beta} = y$$

$$\|X\beta - y\|^2 + \lambda \|\beta\| \quad \text{minimum}$$

representer thm

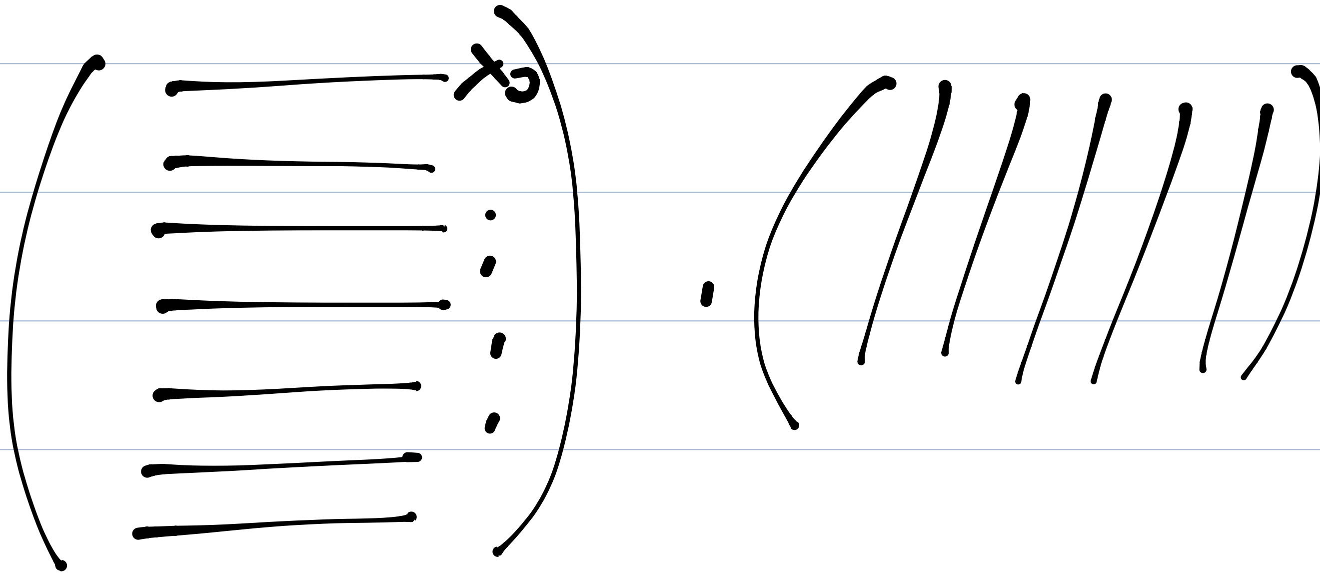
$$\beta = \sum_i c_i x_i \Leftarrow$$

$$\beta = X^T c$$

$$\|XX^T c - y\|^2 + \lambda\|\beta\|^2$$

$$= \|XX^T c - y\|^2 + \lambda c^T X X^T c$$

$$XX^T = K$$



$$= \|Kc - y\|^2 + \lambda c^T K c$$

$$K_{ij} = \langle x_i, x_j \rangle$$