A Model of Search in Credit and Labour Markets with Heterogenous Workers*

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Abstract

How do frictions in credit markets affect firms' choices over which workers to hire? To study this question, I build a search and matching model of credit and labour markets with heterogenous labour. Firms first search for a bank to cover the costs of posting a vacancy. Firms that secure financing then search for workers of varying skill in the labour market. Upon meeting a worker the firm faces a trade-off: hire that worker in the present period and produce output, or wait for a potentially higher skilled worker to come along. Firms' optimal behaviour is determined by tightness in the labour market, itself determined by frictions in both credit and labour markets. Greater credit market frictions drive labour market tightness down, leading firms to seek higher skilled workers.

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1 Introduction

There is mounting evidence that credit market frictions can have effects on real economic outcomes. Bernanke (1983) suggests that increases in banks' cost of intermediation can reduce lending, and subsequent economic activity. This theory has found support in recent empirical evidence from the 2008/2009 financial crisis showing that lending disruptions had a negative impact on firm level employment (Chodorow-Reich, 2014; Huber, 2018). Further, evidence is emerging that these employment losses were concentrated amongst low-educated and low-earning workers (Berton et al., 2018; Hviid and Schroeder, 2021; Moser et al., 2020).

At the same time, it has been widely noted that the financial crisis coincided with a shift in the fundamental, negative relationship between the unemployment rate and the job vacancy rate — the Beveridge curve.[1] An outward shift of the Beveridge curve saw the job vacancy rate increase while the unemployment rate remained high. A growing literature has emerged offering potential explanations for this apparent deterioration of the job matching process. Mismatch in the types of skills demanded and supplied (Şahin et al., 2014), more generous unemployment benefits and lower search effort amongst the unemployed (Kroft et al., 2016), changes in the composition of the labour force (Shimer, 2012), or a combination of these factors (Hobijn and Şahin, 2013) have all been proposed.

Another potential explanation is that employer skill requirements increased during the crisis. Referred to as 'upskilling', this explanation captures the idea that when labour is relatively abundant, employers try and capitalize on the increased likelihood of meeting and hiring a more skilled worker. This explanation has found some empirical support (Hershbein and Kahn, 2018; Modestino et al., 2019), yet the factors that drive upskilling are poorly understood.[2] Firms may seek more skilled workers during crises for a number of reasons. First, the opportunity cost of redirecting search on the labour market may decrease (Hall, 2005). Crises may also bring about Schumpeterian cleansing (Schumpeter, 1942), driving

[1]Shifts in the Beveridge curve often occur during downturns in the business cycle; see Blanchard and P. Diamond (1989), P. A. Diamond and Şahin (2015), and Elsby and Michaels (2013).

[2]Both studies find evidence of increased employer skill requirements in regions that were hardest hit by the 2008/2009 financial crisis. Their explanations, however, differ: Modestino et al. (2019) argue that firms opportunistically increase skill requirements when labour markets are slack while Hershbein and Kahn (2018) argue that routine biased technological change occurred more intensely in regions that were more impacted by the financial crisis. This lead firms in those regions to adopt technologies that are complementary to high-skilled labour and substitutable with low-skilled labour, increasing their demand for skilled labour.

older, less productive firms out of business — firms that typically seek and hire less skilled workers. Crises may also lower the opportunity costs of investing in new technology, leading firms to replace less skilled workers and seek more skilled workers (Hershbein and Kahn, 2018; Jaimovich and Siu, 2020). To date, there has been little theoretical work studying the potential role credit market frictions might play in increasing employer skill requirements. This paper aims to fill this gap. A better understanding of the effects of credit market frictions on firms' demand for skilled labour could help policy makers in determining how to shape policy in preserving labour market matching efficiency.

This paper extends the basic model in Petrosky-Nadeau and Wasmer (2013) and Wasmer and Weil (2004) to study how credit market frictions may affect firms' choices over which workers to hire. In the model, firms require bank financing to cover the costs of posting a vacancy. Firms that successfully meet a bank in the credit market then turn to searching for workers in the labour market. Upon encountering a worker the firm faces a decision: hire the worker and move on to accruing profits from producing output, or defer and wait for a potentially higher skilled worker to come along.

In the model, firms' optimal behaviour is determined by tightness in the labour market (the ratio of vacancies to the unemployed), itself determined by frictions in both credit and labour markets. Greater credit market frictions drive labour market tightness down, leading firms to seek higher skilled workers. Higher skilled workers are more attractive to firms as their productivity outpaces the wages they negotiate. These model predictions are broadly consistent with the empirical evidence of upskilling in Hershbein and Kahn (2018) and Modestino et al. (2019), and suggest that congestion in credit and labour markets may be an important mechanism driving the observed increase in employer skill requirements.[3] In a quantitative exploration, I parameterize the model to reflect increased credit market frictions and find that these do indeed lead to higher employer skill requirements.

The model developed in this paper builds off the canonical search framework of frictional markets pioneered by Gronau (1971), McCall (1970), and Mortensen (1970). The economy

[3] Congestion is one avenue by which frictions in credit markets may have effects on real economic activity and is featured in Modestino et al. (2016) and Petrosky-Nadeau and Wasmer (2013). Collateral constraints and other limits on the ability to borrow offer additional mechanisms (Boeri et al., 2018; Gavazza et al., 2018; Kiyotaki and Moore, 1997).

in the model consists of banks, firms, and workers that interact in two frictional markets: the credit market and the labour market. In the credit market, firms search for a bank to cover the costs of positing a vacancy in the labour market. Firms face an effort cost of searching for a bank, and banks face a screening cost of searching for a firm. Once they meet, firms and banks engage in Nash bargaining over a per-period financing repayment to be made conditional on the firm meeting a worker. In equilibrium, the credit market frictions together with firms' and banks' relative bargaining power determine the tightness of the credit market, or the ratio of firms to banks searching for one another. Credit market tightness in turn determines the rate at which firms and banks meet each other.

Firms that match with a bank then move on to the labour market to search for a worker. Workers vary in their level of skill distributed according to a known distribution. The rate at which a firm meets any worker is determined by the relative number of each in the labour market — the tightness of the labour market. Given the necessity of banks in financing firms' vacancy costs, tightness in the labour market is determined by frictions and tightness in the credit market. Formally, I show that the presence of search and screening costs in the credit market reduces the number of firms in the labour market, driving equilibrium labour market tightness down. With fewer firms relative to workers in the labour market, the rate at which a firm meets any worker increases. This feature of the model fits well with data showing the fall in new firm entry during the Great Recession as discussed in Gavazza et al. (2018).

Firms that meet workers are then faced with a choice: hire the encountered worker and move on to earnings profits in the goods market, or defer and wait for a higher skilled worker to come along. Firms that do hire a worker engage in Nash bargaining over the total surplus created by the match to determine the worker's wage. Formally, I show that net profits to the firm — output less the wage and repayment to the bank — are strictly increasing in worker skill, providing the firm an incentive to hire more skilled workers. This result reflects the fact that Nash bargaining does not award the worker with their full marginal productivity when they do not have complete bargaining power. The firm's optimal choice of which workers to hire is described by the skill level of the least skilled worker it would optimally choose to hire — a parameter I label the 'reservation skill'. Firms that produce output in the goods market do so facing a constant risk of exogenous separation. Separation sends banks and firms back to the credit market and workers back to the labour market.

Characterizing the reservation skill in equilibrium, I show it to be decreasing in labour market tightness. Taken together the model paints the following picture: credit market frictions increase the hurdle to firm entry in the labour market, decreasing labour market tightness and increasing the rate at which firms meet workers. With a greater chance of meeting a random worker, firms also face a greater chance of meeting a higher skilled worker. The increased likelihood of meeting a higher skilled worker increases the reservation skill, leading firms to seek and hire more skilled workers. This mechanism by which labour market congestion drives employer skill requirements finds support in the literature. Devereux (2002) shows that the education level of new hires in the US tends to be higher when unemployment is high while Quintini (2011) documents that the likelihood of over-qualification rises with the unemployment rate in a sample of OECD countries.

The upshot of the connection between credit market frictions, labour market tightness and employer skill requirements is that it matters which side of the labour market is targeted by policy interventions aimed at increasing matching efficiency as noted by Gavazza et al. (2018). The framework presented in this paper suggests that policies aimed at increasing the search effort of the unemployed may push employer skill requirements higher by decreasing labour market tightness, further exacerbating skill mismatch in the labour market. Policies designed to decrease frictions in the credit market, on the other hand, may improve matching efficiency via a tightening of the labour market. The results of this model also suggest that policies aimed at investing in the skill development of the least skilled workers may be a viable option in combating unemployment during times when credit market frictions are high.

The model presented in this paper is purposefully kept simple and abstracts from a number of potentially important aspects of credit and labour markets in reality. Directed search, worker skill investments, capital investments, and interactions amongst workers at the same firm are all left out of the model. Further, the model assumes that all firms could potentially hire a worker of any skill level and that workers across the entire skill distribution search for jobs in the same, unsegmented labour market. The model is therefore perhaps best suited for describing the hiring decisions of a firm for a particular job, within occupations.

This paper relates to several literatures. First, it relates to a literature seeking to model the impacts of credit market frictions on the labour market. Many contributions to this literature model market frictions in a search and matching framework. Boeri et al. (2018) consider credit market frictions arising from the limited pledgeability of a firm's assets. The authors show that these frictions lead firms to hoard liquid assets in the form of precautionary savings and at the expense of investing in productive capacity and hiring labour. Jermann and Quadrini (2012) propose a model in which firms must borrow against current capital and require financing to pay wages in advance of production.[4] Negative financial shocks reduce firms' ability to acquire financing and lead them to cut costs by reducing employment. Wasmer and Weil (2004), and later Petrosky-Nadeau (2014) and Petrosky-Nadeau and Wasmer (2013), study the interaction of search frictions in credit and labour markets. They show that screening and search costs in the credit market reduce labour market tightness, amplifying macroeconomic volatility, including swings in employment. To date, however, few papers have examined the effect of credit market frictions on firms' choices over which types of workers to hire. This paper's main contribution, in this regard, is in modelling the relationship between credit market frictions and employer skill requirements.

More broadly, this paper relates to the vast literature on search-theoretic models of the labour market.[5] Building off the early sequential job search models of Gronau (1971), McCall (1970), and Mortensen (1970), the search and matching literature developed by Pissarides (1985) looks to characterize how workers and firms meet in the labour market. The process by worker-firm matches form is captured by the matching function which determines the number of matches from the number of unemployed and the number of vacancies. Standard models of search and matching typically assume that the firm hires every worker it is matched with. Albrecht and Vroman (2002) is one exception which studies firms' hiring decisions over low- and high-skilled workers. The authors use their model to show how employer skill requirements can arise in equilibrium. In contrast, the model presented in this paper considers a continuous distribution of worker skill and shows how frictions outside of the labour market can affect employer skill requirements.

Finally, this paper relates to a largely empirical literature studying the factors affecting firms' demand for skilled labour. In particular, a number of recent papers have exploited detailed data on online job postings. As in the model proposed in this paper, Modestino et al.

[4]The requirement of firms to hold cash to pre-pay wages is also featured in related work by Garín (2015) and Melcangi (2017).

[5]See Rogerson et al. (2005) for a relatively recent survey.

(2016) and Modestino et al. (2019) provide evidence of a causal relationship between labour market tightness and employer skill requirements. Further, the authors provide evidence that this relationship holds even within firms and within narrowly defined jobs. In a similar vein, Hershbein and Kahn (2018) find that the skill requirements of job ads increased more in regions of the US that were harder hit during the 2008/2009 financial crisis. The authors provide evidence that these patterns were driven by upskilling within firms, as well as changes in the composition of employers from older to younger firms. Younger firms demand, on average, more skilled labour raising aggregate demand for skilled labour. Further, the authors find that increases in employer skill requirements are persistent, and argue that their findings are a result of accelerated adoption of skill biased technology. This paper's main contribution to this literature lies in providing a framework showing how credit market frictions can drive upskilling.

The rest of the paper is structured as follows: Section 2 sets up the model and details important outcomes in equilibrium. Section 3 parameterizes and solves for both a baseline specification of the model as well as a specification with increased credit market frictions. Section 4 concludes.

2 A Model of Search in Credit and Labour Markets

2.1 Environment

The environment follows the basic setup in Petrosky-Nadeau and Wasmer (2013), yet allows for firms to encounter workers of varying skill in the labour market. Time is discrete and discounted at rate $r \in [0, 1]$ over an infinite horizon. There are three basic types of agents: banks, firms, and workers. There is a unit mass of homogenous banks, and a unit mass of homogenous firms. Workers differ in their level of skill distributed according to the known distribution, $F(s)$. Firms and banks search for one another in the credit market while firms and workers search for one another in the labour market. A successful worker-firm match then produces output in the goods market.

The initial stage occurs on the credit market. Firms require credit to finance setup costs and to recruit labour. Firms expend an effort cost e per period searching for a bank to provide this initial financing. Firms are successful in their search with probability p_t. On the other side of the market, banks incur a per-period screening cost of a and meet with a firm with probability $\tilde{p}_t$.

Firms that match with a bank then move on to the labour market to search for workers. From this point on, a bank's fate is tied to the firm it is matched with. For simplicity, and following Petrosky-Nadeau and Wasmer (2013), I assume that search on the labour market occurs within the same period as search in the credit market. There is a per-period cost associated with recruiting workers, γ, that is financed by the bank. Firms and banks that match bargain over a lump-sum repayment, ψ, to be made to the bank in each period the firm is active in the goods market. The repayment is determined via Nash bargaining over the total surplus to the firm and the bank in the labour market — prior to the firm searching for a worker. I characterise the repayment in Section 2.4. With probability q a firm meets a worker in the labour market; that worker has an independently and identically distributed skill level s drawn from $F(s)$. The firm then faces a choice: hire the given worker and move on to producing output in the goods market or return to the labour market to search for a different worker.

In the goods market, workers of skill level s produce output $y(s)$ which is increasing in s with decreasing marginal returns ($y'(s) > 0$ and $y''(s) < 0$). Following Petrosky-Nadeau and Wasmer (2013), the wage paid to a worker of skill level s, $w(s)$, is determined via Nash bargaining over the total surplus created from employment, that is, the surplus to the bank, the firm, and the worker. The firm and the bank collectively bargain together with the worker, where α captures the bargaining power of the worker relative to the firm-bank block.[6] In Section 2.5.1 I solve the Nash bargaining problem for $w(s)$; for now, let us note that profits net of labour and financing costs are given by

$$\pi(s) = y(s) - w(s) - \psi$$

where $\pi(s)$ is invertible. On the other side of the labour market, workers are either employed at the wage rate $w(s)$ or enjoy the value of nonemployment (leisure, unemployment benefits,

[6]The choice to model the wage setting process as the bargaining outcome between the worker and the firm-bank block is twofold. First, it is shown in Wasmer and Weil (2004) that if wage bargaining were to occur strictly between the worker and the firm, the firm would have an incentive to increase its debt burden to reduce the surplus and thus the workers wage: the type of strategic surplus shrinking behaviour modelled in Baldwin (1983) and Dasgupta and Sengupta (1993). Second, the wage that results from bargaining between the worker and the firm-bank block allows a more ready comparison in volatility to the classic labour search model with exogenous wages.

etc.), b.

In Section 2.5.2 I will show that the environment described thus far implies that net profits to the firm are strictly increasing in worker skill. That is, a firm would prefer to employ a worker of a higher skill, all things equal. This means that there is a skill level, s_R, at which the firm is indifferent between hiring a worker of that skill, or deferring in the hope of encountering a more skilled worker. Following convention in the search and matching literature, I call this skill level the 'reservation skill' and assume that a firm hires the worker when indifferent. In Section 2.6 I characterize the reservation skill in detail; for now, I simply note that the probability a firm meets a worker it wants to hire is $(1 - F(s_R))$, and the probability that a firm-bank match will move on to the goods market is then $q(1 - F(s_R))$.

With probability k firms and workers are exogenously separated from one another. In this event, the firm-bank relationship also dissolves and the firm goes back to the credit market to search for a bank to finance its recruitment costs to hire a new worker. Together, the values of a firm in the credit (C), labour (L), and goods (G) markets can be described by the following Bellman equations:

$$J_{C,t} = -e + p_t J_{L,t} + (1 - p_t)\left(\frac{1}{1+r}\right) J_{C,t+1} \tag{1}$$

$$J_{L,t} = \left(\frac{1}{1+r}\right)\left[q\int_0^\infty \max\{J_{L,t+1}, J_{G,t+1}(s)\}dF(s) + (1-q)J_{L,t+1}\right] \tag{2}$$

$$J_{G,t}(s) = \pi(s) + \left(\frac{1}{1+r}\right)[kJ_{C,t+1} + (1-k)J_{G,t+1}(s)] \tag{3}$$

The values of a bank in these markets are captured by the following Bellman equations:

$$B_{C,t} = -a + \tilde{p}_t B_{L,t} + (1 - \tilde{p}_t)\left(\frac{1}{1+r}\right) B_{C,t+1} \tag{4}$$

$$B_{L,t} = -\gamma + \left(\frac{1}{1+r}\right)[q[1 - F(s_R)]B_{G,t+1} + (1-q)B_{L,t+1}] \tag{5}$$

$$B_{G,t} = \psi + \left(\frac{1}{1+r}\right)[kB_{C,t+1} + (1-k)B_{G,t+1}] \tag{6}$$

Workers of all skill face the following value of employment, $W(s)$:

$$W_t(s) = w(s) + \left(\frac{1}{1+r}\right)[kU_{t+1}(s) + (1-k)W_{t+1}(s)] \tag{7}$$

while workers with a skill level at or above the reservation skill level face the following value of unemployment, $U(s)$:

$$U_t(s) = b + \left(\frac{1}{1+r}\right)[fW_{t+1}(s) + (1-f)U_{t+1}(s)] \tag{8}$$

where f is the rate at which a nonemployed worker meets a firm in the labour market.

2.2 Matching

2.2.1 Credit Market

The total number of firm-bank matches in the credit market is given by the matching function $m_C(\mathcal{J}_t, \mathcal{B}_t)$. $\mathcal{J}_t$ and $\mathcal{B}_t$ are the total number of firms and banks in the credit market at time t respectively. $m_C()$ is assumed to exhibit constant returns to scale. Define $\phi_t \equiv \frac{\mathcal{J}_t}{\mathcal{B}_t}$ as the ratio of firms to banks in the credit market, or the tightness of the credit market from the perspective of the firm. The number of firm-bank matches in any period can be expressed as a function of credit market tightness as follows

$$\text{matches} = m_C(\mathcal{J}_t, \mathcal{B}_t) = \frac{m_C(\mathcal{J}_t, \mathcal{B}_t)\mathcal{J}_t}{\mathcal{J}_t} = m_C\left(\frac{\mathcal{B}_t}{\mathcal{J}_t}, \frac{\mathcal{J}_t}{\mathcal{J}_t}\right)\mathcal{J}_t = m_C(\phi_t)\mathcal{J}_t$$

Thus, banks meet firms at rate

$$\frac{m_C(\phi_t)\mathcal{J}_t}{\mathcal{B}_t} = m_C(\phi_t)\phi_t \equiv \tilde{p}(\phi_t) \quad \text{where} \quad \tilde{p}'(\phi_t) > 0$$

and firms meet banks at rate

$$\frac{m_C(\phi_t)\mathcal{J}_t}{\mathcal{J}_t} = m_C(\phi_t) \equiv p(\phi_t) \quad \text{where} \quad p'(\phi_t) < 0$$

As the credit market becomes tighter (relatively more firms to banks), the probability that a firm finds a bank decreases, and the probability that a bank finds a firm increases.

Following Petrosky-Nadeau and Wasmer (2013), I assume that the firm's repayment to the bank, ψ, is negotiated once the bank and the firm meet, and before the firm hires a worker. ψ solves the Nash bargaining problem between the firm and the bank over their collective surplus in the labour market, $J_{L,t} + B_{L,t}$. With $\beta \in (0,1)$ capturing the bargaining power of the bank relative to the firm, the Nash bargaining outcome requires that

$$\beta J_{L,t} = (1-\beta)B_{L,t} \tag{9}$$

2.2.2 Labour Market

The total number of firm-worker matches in the labour market is given by the matching function $m_L(\mathcal{V}_t, \mathcal{U}_t)$. $\mathcal{V}_t$ and $\mathcal{U}_t$ are the total number of firms (vacancies) and unemployed workers in the labour market at time t respectively. $m_L()$ is assumed to exhibit constant returns to scale. Define $\theta_t \equiv \frac{\mathcal{V}_t}{\mathcal{U}_t}$ as the ratio of firms to unemployed workers in the labour market, or the tightness of the labour market from the perspective of the firm. The number of worker-firm matches in any period can be expressed as a function of labour market tightness as follows

$$\text{matches} = m_L(\mathcal{V}_t, \mathcal{U}_t) = \frac{m_L(\mathcal{V}_t, \mathcal{U}_t)\mathcal{U}_t}{\mathcal{U}_t} = m_L\left(\frac{\mathcal{V}_t}{\mathcal{U}_t}, \frac{\mathcal{U}_t}{\mathcal{U}_t}\right)\mathcal{U}_t = m_L(\theta_t)\mathcal{U}_t$$

Firms therefore meet (but don't necessarily hire) unemployed workers at rate

$$\frac{m_L(\theta_t)\mathcal{U}_t}{\mathcal{V}_t} = \frac{m_L(\theta_t)}{\theta_t} \equiv q(\theta_t) \quad \text{where} \quad q'(\theta_t) < 0$$

and unemployed workers meet (but don't necessarily join) firms at rate

$$\frac{m_L(\theta_t)\mathcal{U}_t}{\mathcal{U}_t} = m_L(\theta_t) \equiv f(\theta_t) \quad \text{where} \quad f'(\theta_t) > 0$$

As the labour market becomes tighter (relatively more firms to workers), the probability that a firm meets with a random unemployed worker decreases, and the probability that an unemployed worker meets a firm increases.

2.3 Equilibrium

I start by considering the asset values for the firm and the bank in the credit market to derive a condition for equilibrium credit market tightness. Assuming free entry for both firms and banks in the credit market ($J_{C,t} = B_{C,t} = 0 \quad \forall t$), we have from (1) and (4)

$$0 = -e + p(\phi_t)J_{L,t} \quad \Rightarrow \quad J_{L,t} = \frac{e}{p(\phi_t)} \tag{10}$$

$$0 = -a + \tilde{p}(\phi_t)B_{L,t} \quad \Rightarrow \quad B_{L,t} = \frac{a}{\tilde{p}(\phi_t)} \tag{11}$$

The Nash bargaining condition in (9) together with (10) and (11) give us

$$\beta\left(\frac{e}{p(\phi_t)}\right) = (1-\beta)\left(\frac{a}{\tilde{p}(\phi_t)}\right)$$
$$\phi_t^* = \frac{(1-\beta)}{\beta}\left(\frac{a}{e}\right) \quad \forall t$$

since $\tilde{p}(\phi_t) \equiv m_C(\phi_t)\phi_t$ and $p(\phi_t) \equiv m_C(\phi_t)$. Credit market tightness is therefore a constant and depends positively on banks' screening costs and negatively on firms' search effort costs. This means that the values of the firm and the bank in the labour market, (10) and (11), are also constants. In Section 2.2.1 I showed that the rates at which firms and banks meet in the credit market were dictated by tightness in the credit market. In equilibrium, therefore, when banks' screening costs go up, relatively fewer banks enter the credit market, credit market tightness goes up, and the probability that a bank meets a firm increases, while the probability a firm meets a bank decreases. Conversely, when firms' effort costs of searching for a bank go up, relatively fewer firms enter the credit market and the probability a bank meets a firm decreases, while the probability a firm meets a bank increases.

Next I consider the asset values for the firm and the bank in the labour and goods markets to characterise equilibrium labour market tightness. I start by simplifying (2). In particular, note that we can express the firm's maximization problem as a function of the reservation skill. Once meeting a worker in the labour market, the probability that the worker is not hired is the probability that the worker has skill level below s_R, or $F(s_R)$. Therefore, we can rewrite (2) as

$$J_{L,t} = \left(\frac{1}{1+r}\right)\left[q(\theta_t)\int_{s_R}^{\infty} J_{G,t+1}(s)dF(s) + (1 - q(\theta_t)[1 - F(s_R)])J_{L,t+1}\right]$$

Together with (10) we then have

$$\frac{e}{p(\phi^*)} = \left(\frac{1}{1+r}\right)\left[q(\theta_t)\int_{s_R}^{\infty} J_{G,t+1}(s)dF(s) + (1 - q(\theta_t)[1 - F(s_R)])\frac{e}{p(\phi^*)}\right]$$

Rearranging terms leaves us with

$$\frac{e}{p(\phi^*)} = \left(\frac{q(\theta_t)}{r + q(\theta_t)[1 - F(s_R)]}\right)\int_{s_R}^{\infty} J_{G,t+1}(s)dF(s) \tag{12}$$

(12) represents an entry condition for the firm, requiring that the expected entry costs in terms of effort be equal to the discounted, expected value of the firm in the goods market.

We can derive a similar such condition for banks, combining (5) and (11)

$$\frac{a}{\tilde{p}(\phi^*)} = -\gamma + \left(\frac{1}{1+r}\right)\left[q(\theta_t)[1 - F(s_R)]B_{G,t+1} + (1 - q(\theta_t))\frac{a}{\tilde{p}(\phi^*)}\right]$$

Rearranging terms leaves us with

$$\frac{a}{\tilde{p}(\phi^*)} = -\gamma\left(\frac{1+r}{r + q(\theta_t)}\right) + \left(\frac{q(\theta_t)[1 - F(s_R)]}{r + q(\theta_t)}\right)B_{G,t+1} \tag{13}$$

Again, (13) represents an entry condition, this time for the bank, requiring that the expected entry costs in terms of screening be equal to the discounted, expected value of the bank in the goods market, less the vacancy costs covered.

Combining (12) and (13) leaves us with a single market entry condition

$$\begin{aligned}\frac{e}{p(\phi^*)} + \frac{a}{\tilde{p}(\phi^*)} = \left(\frac{q(\theta_t)}{r + q(\theta_t)[1 - F(s_R)]}\right) \int_{s_R}^{\infty} J_{G,t+1}(s) dF(s) + \\ \left(\frac{q(\theta_t)[1 - F(s_R)]}{r + q(\theta_t)}\right) B_{G,t+1} - \gamma \left(\frac{1 + r}{r + q(\theta_t)}\right)\end{aligned} \tag{14}$$

The equilibrium value of labour market tightness, θ_t^*, satisfies this condition. The left hand side of (14) represents total expected frictions in the credit market which, in equilibrium, are set equal to the total discounted, expected surplus to the firm and the bank from producing in the goods market.

As in Wasmer and Weil (2004), we can show that credit market frictions in this setting imply a lower equilibrium labour market tightness. To see this, consider (13). In the absence of credit market frictions, equilibrium labour market tightness, θ_t^{NF}, satisfies

$$-\gamma \left(\frac{1 + r}{r + q(\theta_t^{NF})}\right) + \left(\frac{q(\theta_t^{NF})[1 - F(s_R)]}{r + q(\theta_t^{NF})}\right) B_{G,t+1} = 0 \quad \Rightarrow \quad q(\theta_t^{NF})[1 - F(s_R)] = \gamma(1 + r)$$

In the presence of credit market frictions, equilibrium labour market tightness, θ_t^*, satisfies

$$\begin{aligned}-\gamma \left(\frac{1 + r}{r + q(\theta_t^*)}\right) + \left(\frac{q(\theta_t^*)[1 - F(s_R)]}{r + q(\theta_t^*)}\right) B_{G,t+1} &> 0 \\ q(\theta_t^*)[1 - F(s_R)] &> \gamma(1 + r) = q(\theta_t^{NF})[1 - F(s_R)] \\ q(\theta_t^*) &> q(\theta_t^{NF})\end{aligned}$$

Since $q'(\theta_t) < 0$, we have $\theta_t^{NF} > \theta_t^*$.

2.4 The Repayment

In this section I characterize the financing repayment made from the firm to the bank. A full derivation is presented in Appendix A. A key feature of the repayment is that it is only dependent on the reservation skill level and not the skill level of a hired worker.

Following Petrosky-Nadeau and Wasmer (2013), I assume that the repayment is determined via Nash bargaining between the firm and the bank over their total surplus in the labour market. This implies that the repayment will satisfy the usual Nash bargaining condition in (9). Conceptually, it is already clear that the repayment will not depend on the skill of the hired worker since it is bargained over prior to the firm producing output. Substituting the values of the firm and the bank in the labour market in to (9) we can show that the repayment is giving by the following

$$\psi = \frac{\beta}{[1-F(s_R)(1-\beta)]}\int_{s_R}^{\infty} y(s) - w(s)dF(s) + (1-\beta)\frac{(r+k)\gamma}{q[1-F(s_R)(1-\beta)]} \tag{15}$$

The loan repayment is a constant in each period and in its form reflects the familiar outcome of a Nash bargaining problem: the bank captures part of the firm's profits, while the firm captures part of the return to the bank in the form of the covered recruiting costs.

2.5 Wages and Firm Profits

In this section I characterize the skill dependent wage that solves the Nash bargaining problem between the worker and the firm-bank block. A full derivation of the wage is presented in Appendix B. I then show that, given this wage structure, net profits to the firm are increasing in worker skill.

2.5.1 Wages

Formally, the wage satisfies

$$\arg\max[W_t(s) - U_t(s)]^{\alpha}[S_{G,t}(s)]^{1-\alpha}$$

where $S_{G,t}(s) = J_{G,t}(s) + B_{G,t}$ is the total surplus for the firm-bank block in the goods market and $\alpha \in (0,1)$ is the bargaining power of the worker relative to the firm-bank block. The wage that solves this problem satisfies the usual Nash bargaining sharing rule

$$\alpha S_{G,t}(s) = (1-\alpha)[W_t(s) - U_t(s)] \tag{16}$$

As the Bellman equations for the value of the firm and the bank in the goods market are both stationary environments, we can drop the time subscripts and solve for $J_G(s)$ and B_G

from (3) and (6) giving us

$$J_G(s) = \pi(s)\left(\frac{1+r}{r+k}\right) \tag{17}$$

$$B_G = \psi\left(\frac{1+r}{r+k}\right) \tag{18}$$

The sum of (17) and (18) gives us the value for the total surplus in the goods market for the firm-bank block

$$S_G(s) = [\pi(s) - \psi]\left(\frac{1+r}{r+k}\right) = [y(s) - w(s)]\left(\frac{1+r}{r+k}\right) \tag{19}$$

From (7) and (8) we can solve for the values of employment and unemployment for workers with a skill level at or above the reservation skill. This gives us

$$W(s) = \left(\frac{1+r}{r+k}\right) w(s) + \left(\frac{k}{1+r}\right) U(s) \tag{20}$$

and

$$U(s) = \left(\frac{1+r}{r+f}\right) b + \left(\frac{f}{r+f}\right) W(s) \tag{21}$$

Together (20) and (21) give us

$$W(s) = \left(\frac{(1+r)(r+f)}{r(r+f+k)}\right) w(s) + \left(\frac{k(1+r)}{r(r+f+k)}\right) b \tag{22}$$

and

$$U(s) = \left(\frac{(1+r)(r+k)}{r(r+f+k)}\right) b + \left(\frac{f(1+r)}{r(r+f+k)}\right) w(s) \tag{23}$$

Plugging (19), (22), and (23) into the sharing rule in equation (16) and solving for the wage leaves us with

$$w(s) = \alpha\left[\frac{r+f+k}{r+\alpha f+k}\right] y(s) + (1-\alpha)\left[\frac{r+k}{r+\alpha f+k}\right] b \tag{24}$$

The equilibrium wage is therefore a weighted average of the worker's output in the goods market and their value of nonemployment. With greater bargaining power (higher α), workers' wages reflect to a greater degree their productivity in the goods market. With lower bargaining power, workers' wages reflect to a greater degree their value of nonemployment (their outside option to employment).

2.5.2 Firm Profits

Given the equilibrium wage characterised in Section 2.5.1, we can represent profits as a function of goods market output which we know is increasing in worker skill

$$\pi(s) = y(s) - \alpha\left[\frac{r+f+k}{r+\alpha f+k}\right]y(s) - (1-\alpha)\left[\frac{r+k}{r+\alpha f+k}\right]b - \psi$$

The derivative of total net profits with respect to worker skill is then

$$\begin{aligned}\pi'(s) &= y'(s) - \alpha\left[\frac{r+f+k}{r+\alpha f+k}\right]y'(s)\\ &= y'(s)\left[1-\alpha\left(\frac{r+f+k}{r+\alpha f+k}\right)\right]\\ &= y'(s)\left[\frac{(r+k)(1-\alpha)}{r+\alpha f+k}\right] > 0\end{aligned}$$

since $y'(s) > 0$.

This result reflects the fact that, through the wage bargaining process, and with less than complete bargaining power, workers' wages do not capture their full marginal productivity. For firms, the increase in output from a higher skilled worker outpaces the increase in their wage bill.[7]

2.6 The Reservation Skill Level

In this section I characterize the reservation skill level defined as the skill level of the least skilled worker a firm is willing to hire. A full derivation is presented in Appendix C. I follow the convention in the literature that the firm chooses to hire a worker when indifferent.

Formally, the reservation skill level is the level of skill, s_R, such that

$$J_{G,t}(s_R) = J_{L,t} \tag{25}$$

Since (3) is a stationary problem we can drop the time subscripts, solve for $J_G(s)$, and combine with (2) and (25) to show that

$$\pi(s_R) = \left(\frac{q(\theta_t)}{r}\right)\int_{s_R}^{\infty}[1-F(s)]d\pi(s) \tag{26}$$

[7] We can also see this by looking at the derivative of the wage with respect to skill $\frac{\partial w(s)}{\partial s} = \alpha\left[\frac{r+f+k}{r+\alpha f+k}\right]y'(s)$ which is strictly less than $y'(s)$ since $\alpha\left[\frac{r+f+k}{r+\alpha f+k}\right] < 1$.

since $\pi(s)$ is strictly increasing in s (as shown in Section 2.5.2). In words, this expression tells us that the net profits to the firm from employing a worker of the reservation skill level are equal to the discounted expected increase in profits of waiting to hire a more skilled worker.

How does the reservation skill level change with frictions in the credit market? From equation (26) we see that profits to the firm at the reservation skill are a function of primitives, save for labour market tightness. As we saw in Section 2.3, labour market tightness is decreasing in credit frictions. Formally, differentiating the reservation skill with respect to labour market tightness leaves us with

$$\frac{\partial s_R}{\partial \theta_t^*} = \underbrace{(\pi^{-1})'(\cdot)}_{A} \times \underbrace{\left[\left(\frac{1}{r}\right)\int_{s_R}^{\infty}[1-F(s)]d\pi(s)\right]}_{B} \times \underbrace{[q'(\theta_t^*)]}_{C}$$

To sign this partial derivative, consider the sign of its multiplicative components: Component A is positive since

$$(\pi^{-1})'(\cdot) = \frac{1}{\pi'(\pi^{-1}(\cdot))} > 0$$

given that $\pi'(\cdot) > 0$. Component B is also positive since $r \geq 0$ and $\int_{s_R}^{\infty}[1-F(s)]d\Pi(s) > 0$. Finally, component C is negative since $q'(\theta_t^*) < 0$, meaning $\frac{\partial s_R}{\partial \theta_t^*} < 0$.

This result has an intuitive interpretation: as credit market frictions increase, fewer bank-firm matches form, and there are fewer vacancies posted in the labour market. This drives labour market tightness down. With fewer vacancies, the probability that a firm already in the labour market meets a random worker is higher. Therefore, the firm's chances of meeting a higher skilled worker in the future are also higher. This pushes the reservation skill up. In this way, credit market frictions affect the reservation skill through congestion in the labour market — a mechanism which finds support in Devereux (2002) and Quintini (2011).

3 Quantitative Exploration

In this section I introduce a baseline parameterization of the model discussed in Section 2 and solve for its equilibrium values, including the reservation skill. I then re-parameterize the model to reflect increased credit market frictions, solve, and show that the reservation skill is higher than in the baseline case.

3.1 Parameterization

The baseline parameterisation of the model closely follows Petrosky-Nadeau and Wasmer (2013) set to match the US labour and credit markets. The unit of time is taken to be one month. The discount rate is set to $r = 0.3\%$ to correspond (per month) to a 4% p.a. rate on a 3-month US T-bill. A number of credit market parameters will rely on parameters from the labour market so we start there. Normalizing the size of the labour force to 1, we can write the labour market matching function as a function of the unemployment rate u

$$m_L(\mathcal{V}_t, u_t) \equiv Z\mathcal{V}_t^{1-\eta}u_t^{\eta} \tag{27}$$

where $Z > 0$ is a constant and $\eta \in (0, 1)$ is the elasticity of worker-firm matches with respected to the unemployment rate.

To parameterize the labour market side of the model I choose values of q, u, k, η, and b and use these to impute values of f, θ, $\mathcal{V}$, Z, γ, and α. Panel A in Table 1 contains a summary of the parameter values chosen in the baseline specification; Panel B reports the values of the imputed parameters. Following Petrosky-Nadeau and Wasmer (2013), I set the rate at which workers and firms meet $q = 0.4$, the unemployment rate $u = 0.07$, the job destruction rate $k = 0.03$, the elasticity of the labour market matching function $\eta = 0.6$, and the value of nonemployment $b = 0.4$.[8] In Section 2.2.2, we showed that constant returns to scale exhibited by the labour market matching function means that the job filling rate for firms and the job finding rate for unemployed workers can be expressed as $q(\theta_t) = \frac{m_L(\theta_t)}{\theta_t}$ and $f(\theta_t) = m_L(\theta_t)$ respectively. Labour market tightness is then given by $\theta_t = \frac{f(\theta_t)}{q(\theta_t)}$. In a steady state equilibrium, the flow of workers in and out of unemployment are equal such that $f(\theta)[1 - F(s_R)]u = k(1 - u)$.

Full parameterization of the model requires taking a stand on the distribution of worker skill. The literature typically assumes that individual skill or ability is distributed normally (Rothschild and Stiglitz, 1982; Roy, 1950). Empirical work suggests that the distribution of ability may be better approximated by a log-normal distribution, and at the very least,

[8]The worker-firm meeting rate is based off of the estimates in den Haan et al. (2000). The unemployment rate is based off of the work by Gertler and Trigari (2009). The value of the job destruction rate is based off of findings of Davis et al. (2006) using JOLTS data. The value of the elasticity of the labour market matching function with respect to the unemployment rate is based off of the survey in Petrongolo and Pissarides (2001) and the recommendations provided in Brügemann (2008). The value of nonemployment is chosen to match that in Shimer (2005).

exhibits a clear bell shape (Mas and Moretti, 2009; Mayer, 1960; Roy, 1950). I assume that worker skill is distributed log-normally

$$s \sim \ln N(e^{\mu}, \sigma)$$

where $\ln(s) \sim N(\mu, \sigma)$. In the baseline specification I set $\mu = 0$ and $\sigma = 0.15$. The PDF and CDF of the assumed skill distribution are plotted in Figure 1.

The production function is assumed to take the following form

$$y(s) = (A \cdot s)^{\omega} \tag{28}$$

where $A > 0$ is a constant and $\omega \in (0, 1)$ is the elasticity of output with respect to labour. This function is a particular specification of the generalized Cobb-Douglas production function with heterogenous labour that appears in Iranzo et al. (2008). There, the labour input in the production function is the product of the number of employees and labour efficiency, itself an increasing function of worker skill.[9] I set $A = 10$ and assume the elasticity of output with respect to labour takes the standard value for the US economy of $\omega = 0.7$.

To pin down the values of α and γ I apply the assumptions made in Petrosky-Nadeau and Wasmer (2013) to the worker with a skill level equal to the mean. Total recruiting costs are assumed to be 3.6% of the wage for the mean worker following Silva and Toledo (2007). The labour share of income for the mean worker is assumed to be two-thirds. Rearranging the wage equation in (24) allows us to solve for workers' bargaining power

$$\alpha = \frac{[r + k][b - w(s = \mu)]}{w(s = \mu)f - (r + f + k)y(s = \mu) + (r + k)b}$$

The credit market matching function takes the standard form

$$m_C(\mathcal{J}_t, \mathcal{B}_t) \equiv X \mathcal{J}_t^{1-\nu} \mathcal{B}_t^{\nu} \tag{29}$$

$\mathcal{J}_t$ and $\mathcal{B}_t$ are the the number of firms and banks in the credit market respectively. $X > 0$ is a constant and $\nu \in (0, 1)$ is the elasticity of firm-bank matches to the number of banks in the credit market. Parameterization of the credit market side of the model requires finding

[9] This approach to modelling the production function draws on a large empirical literature documenting the positive relationship between human capital and productivity at the individual level (Black and Lynch, 1996) and at the firm level (Abowd et al., 1999; Haltiwanger et al., 1999).

values for e, a, β, X, and ν. In the baseline parameterization I set each of these parameters equal to the values in the baseline case in Petrosky-Nadeau and Wasmer (2013).[10]

3.2 Solving for the Reservation Skill

As defined in Section 2.6, the reservation skill level is the skill level of the employee that sets the value of the firm in the goods market equal to the value of the firm in the labour market. In Section 2.3, I showed that the value of the firm in the labour market is a constant, assuming free entry in the credit market. To solve for the reservation skill we therefore need to solve for the value function of the firm in the goods market. I do so by using value function iteration over a discrete grid of possible skill levels ranging from 0 to 2 capturing more than 99.99% of the range of the theoretical skill distribution.

In practice, I begin with an initial guess of the value function, $J_{G,0}(s)$, and update using the Bellman equation for the firm in the goods market

$$J_{G,n+1}(s) = y(s) - w(s, s_R) - \psi(s_R) + \left(\frac{1-k}{1+r}\right) J_{G,n}(s)$$

for each iteration n. I continue to update using the Bellman equation until successive iterations of the value function are closer to one another than a pre-specified tolerance parameter. Note that both the repayment and the wage — via workers' bargaining power α and the meeting rate f — depend on the reservation skill. Solving for the value function iteratively therefore requires the reservation skill to be calculated for each iteration of the value function prior to updating. In addition, the repayment, captured in (15), contains an integral — the expected value of output less wages as a function of skill from the reservation skill and up. To solve the model I approximate the integral, integrating by parts to arrive at

$$\int_{s_R}^{\infty} y(s) - w(s)dF(s) = -[y(s) - w(s)][1 - F(s)]\Big|_{s_R}^{\infty} + \int_{s_R}^{\infty} [1 - F(s)][y'(s) - w'(s)]ds$$

and evaluate this expression over the discretized skill distribution.

[10] Petrosky-Nadeau and Wasmer (2013) parameterise the credit market side of the model as follows: first they take an agnostic approach and assume that $\nu = 0.50$. Comparatively, less work has been done on matching in credit markets than in labour markets. Liberati (2018) also assumes a value of 0.50 for the elasticity of the credit market matching function. Through calibration, Dong et al. (2016) compute a value of around 0.73. Next, the authors set $a = e$ and seek to calibrate β to match the size of the financial sector's share of aggregate value added observed in the data. Knowledge of β allows one to impute credit market tightness. The authors then assume that the average duration of search in the credit market is 4 months which allows one to pin down the constant in the credit market matching function. With the credit market matching function fully parameterized, and the value of equilibrium credit market tightness, one can obtain p, e, and a.

Figure 2, Panel (a) plots the value functions of the baseline specification from the first 20 successive iterations of the updating procedure described above. The figure shows that the updated value functions gradually converge. Panel (b) plots the reservation skill levels from successive iterations of the value function. The plot shows that the reservation skill converges within the first 5 iterations in the baseline parameterization.

This iterative solution method using the baseline parameterization leaves us with the values of the imputed parameters and the reservation skill level presented in Panel B of Table 1. In the baseline specification, the firm is willing to hire workers with a skill level of 0.91 or greater — or workers in the 26th percentile of the skill distribution. The firm's immediate problem in the goods market is depicted in Figure 3. As noted in Section 2.5.2, per-period profits to the firm are increasing in worker skill as the rise in output with skill outpaces the rise in the wage. The repayment due to the bank while the firm produces in the goods market means that it is not profitable for the firm to hire a worker of a skill level below the reservation skill.

In understanding the firm's hiring problem in this model it is also important to understand how the repayment varies with the reservation skill level. Figure 4 plots the repayment as a function of the reservation skill level for the baseline specification. As the reservation skill increases, the per-period repayment decreases in a non-linear manner. This is due to the fact that as the reservation skill increases, the expected value of the firms' profits decrease, decreasing the part of the firm's surplus that the bank could capture through bargaining.

3.3 Increased Credit Market Frictions

Next, I parameterize the model to match the specification with increased credit market frictions in Petrosky-Nadeau and Wasmer (2013). In their paper, the authors depart from their baseline parameterization by first increasing the bargaining power of the bank relative to the firm, β. This represents an increase in distortions in the credit market as it pushes β further away from the Hosios efficiency condition in the credit market requiring $\beta = \nu$.[11] A model parameterization further away from the Hosios condition increases the targeted rate

[11]The Hosios condition is an oft-cited condition for constrained efficiency in search and matching models. In the model presented here, the condition requires that the bargaining power of the bank be equal to the elasticity of the credit market matching function (the elasticity of credit market matches with respect to the number of banks). The condition ensures that banks are sufficiently compensated for their effect on match formation.

of unemployment, as well as the targeted share of value added of the financial sector. The imputed screening and search effort costs for banks and firms in the credit market, a and e, rise as a result.

The parameters of the model with increased credit market frictions are presented in Column 2 of Table 1; Panel A reports the chosen parameters while Panel B reports the imputed parameters following the same procedure outlined in Sections 3.1 and 3.2. The results presented confirm the model prediction presented in Section 2.6: increased frictions in the credit market result in an increased reservation skill. Moving from the baseline specification to the specification with increased credit market frictions, the reservation skill increases from 0.91 to 0.95. As a result, firms are only willing to hire workers in the 36th percentile of the skill distribution. Given the ad-hoc nature of the assumed distribution of worker skill, the results obtained here should be interpreted as illustrative of the mechanism discussed in Section 2.6 rather than a precise calculation of magnitudes.

The remaining parameters in Column 2 highlight the congestion mechanism through which credit market frictions affect the reservation skill. With greater frictions in the credit market, there are fewer firms searching for financing and credit market tightness falls from 0.47 to 0.19. As a result, vacancies in the labour market fall from 0.09 to 0.04 as does labour market tightness from 1.35 to 0.35. For firms with an open vacancy, the job filling rate rises from 0.40 to 0.89, increasing the likelihood that a firm will meet a worker of a higher skill. The result is that the reservation skill is pushed upwards.

4 Conclusion

In this paper, I seek to understand how frictions in credit markets may affect the demand for skilled labour. To this end, I build a model of search and matching in credit and labour markets. In the model, increased credit market frictions lead firms to increase their skill requirements. The mechanism highlighted in my model is one of congestion. Greater frictions in the credit market reduce equilibrium credit market tightness and labour market tightness. For a given firm in the labour market, this leads to a higher probability of encountering a worker of any skill as well as a higher probability of encountering more skilled workers. The result is that firms increase their skill requirements.

A better understanding of the effects of credit market frictions on firms' demand for skilled labour could help policy makers in determining which tools to implement in preserving matching efficiency in the labour market. The framework presented in this paper highlights the scope for policies aimed at reducing frictions in the credit market. When it comes to labour market policies, the framework suggests that the type of intervention may matter, a point also noted by Gavazza et al. (2018). Active labour market policies and training programs aimed at increasing the skill level of the least skilled in the labour market may help in pushing these workers above firms' reservation skill. Policies aimed at increasing the search effort of the unemployed, however, may push employer skill requirements higher by decreasing labour market tightness, further exacerbating skill mismatch in the labour market.

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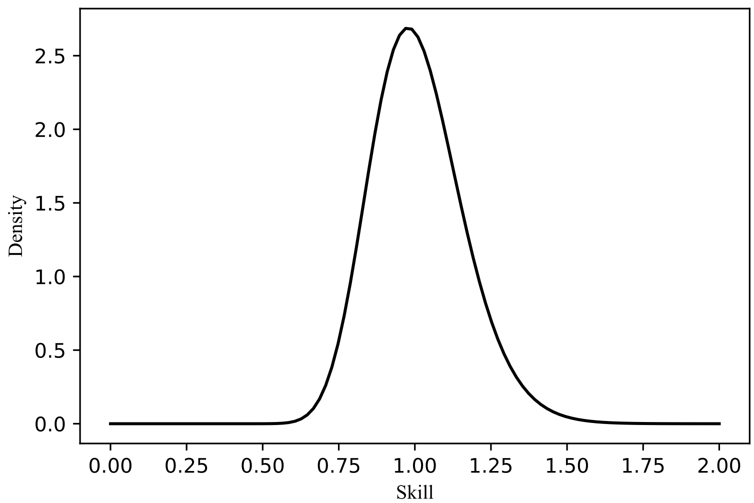
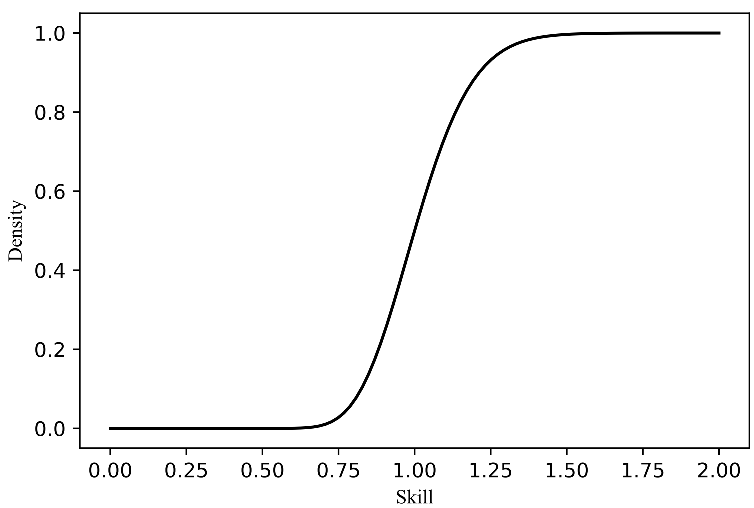
Figures

Figure 1: The Distribution of Worker Skill

(a) PDF



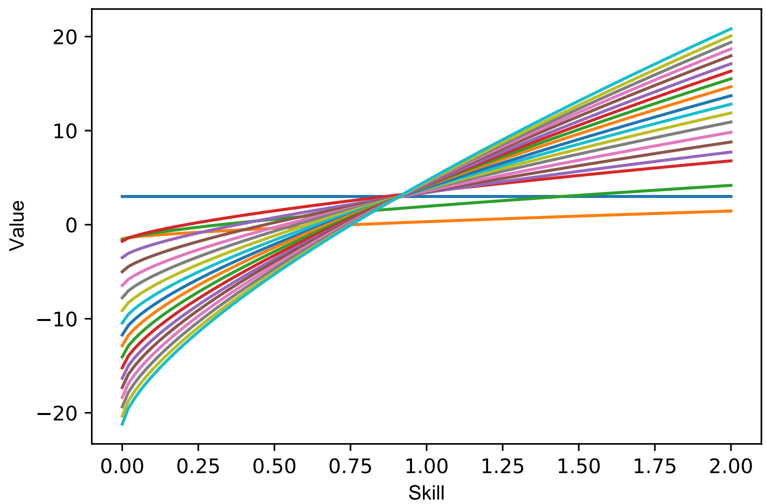
(b) CDF



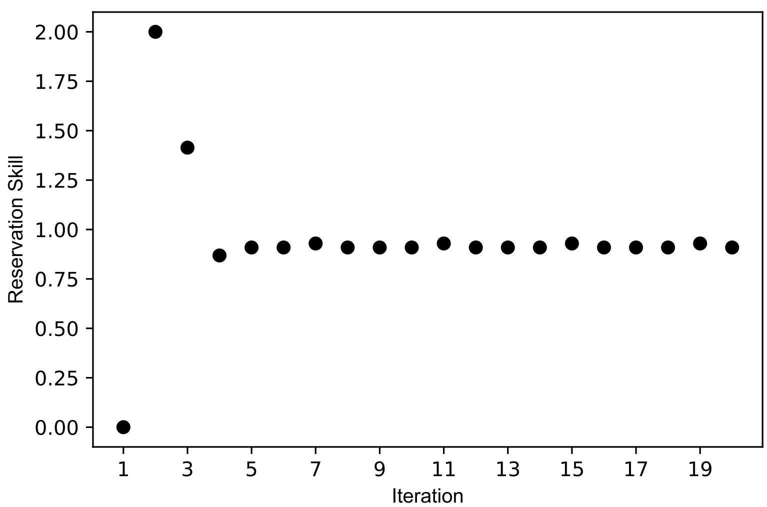
Notes: Panel (a) plots the probability density function of the assumed skill distribution where skill is distributed log-normally $s \sim \ln N(e^{\mu}, \sigma)$ with $\ln(s) \sim N(\mu, \sigma)$. I set $\mu = 0$ and $\sigma = 0.15$. Panel (b) plots the cumulative distribution function of the same distribution.

Figure 2: Value Function Iteration

(a) Value Function of the Firm in the Goods Market

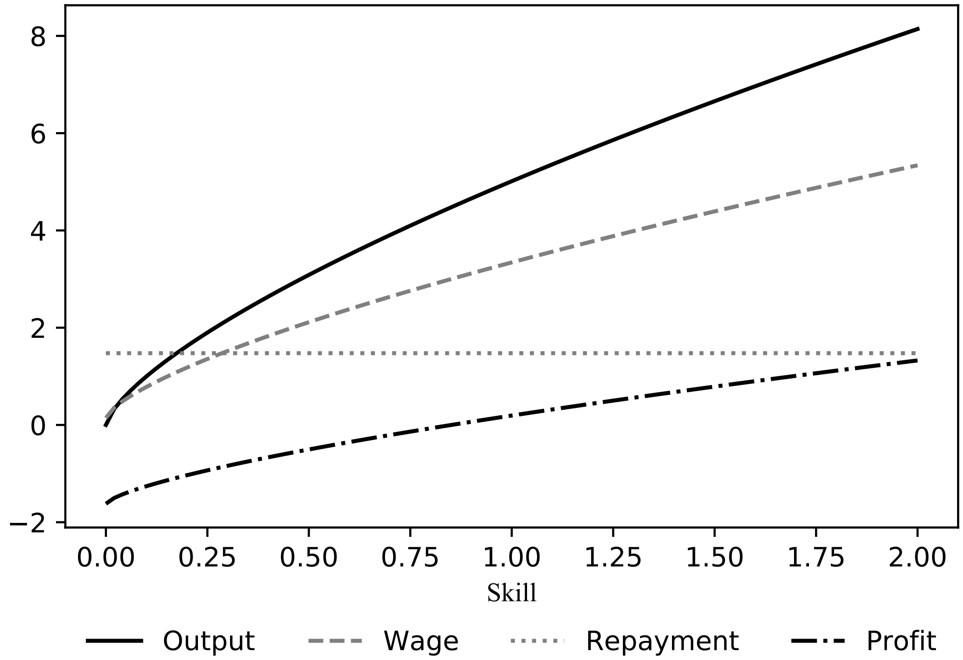


(b) The Reservation Skill Level



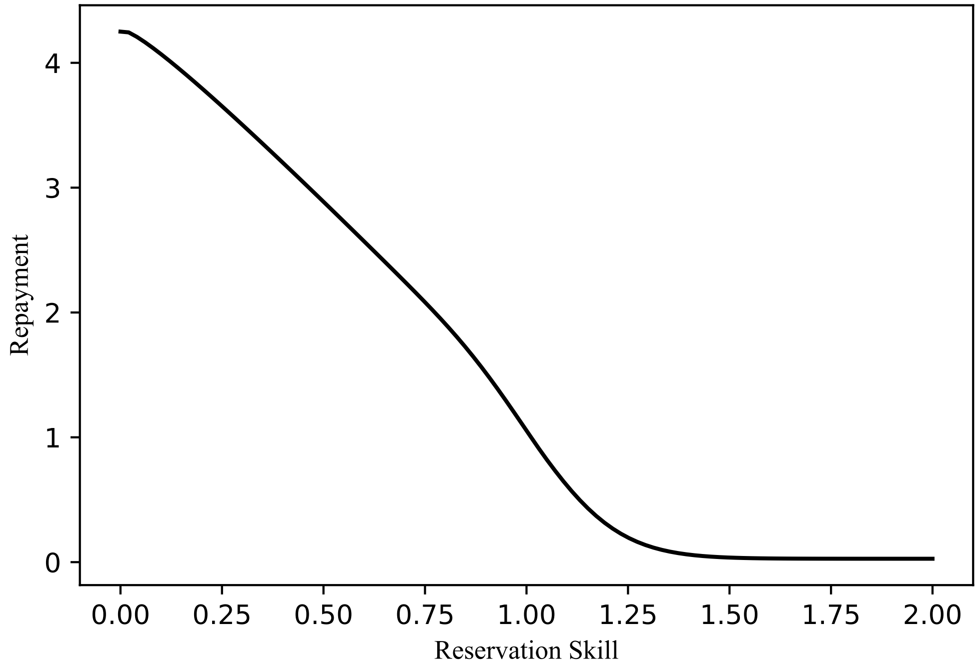
Notes: Panel (a) plots the value function of the firm in the goods market for successive iterations of the solution method given the baseline specification. Panel (b) plots reservation skill calculated from successive iterations of the value function solution method.

Figure 3: Firm Outcomes as a Function of Employee Skill



Notes: This figure plots output, the wage, the financing repayment, and firm profits as a function of the skill level of a hired worker in the baseline specification.

Figure 4: The Repayment as a Function of the Reservation Skill



Notes: This figure plots the financing repayment as a function of the reservation skill level in the baseline specification.

Tables

Table 1: Chosen and Imputed Parameters

		Baseline	Increased Credit Market Frictions
Panel A: Chosen Parameters			
Discount rate	r	0.3%	0.3%
Production function constant	A	10.00	10.00
Production function elasticity	ω	0.70	0.70
Credit Market			
Firms' per period search cost	e	1.58	2.48
Banks' per period screening cost	a	1.58	2.48
Banks' relative bargaining power	β	0.68	0.84
Matching function constant	X	0.37	0.57
Matching function elasticity	ν	0.50	
Labour Market			
Job filling rate	q	0.40	0.89
Unemployment rate	u	0.07	0.11
Job destruction rate	k	0.03	0.03
Matching function elasticity	η	0.60	0.60
Workers' value of nonemployment	b	0.40	0.40
Panel B: Imputed Parameters			
Credit Market			
Credit market tightness	ϕ	0.47	0.19
Labour Market			
Workers' meeting rate	f	0.54	0.33
Labour market tightness	θ	1.35	0.37
Vacancies	$\mathcal{V}$	0.09	0.04
Matching function constant	Z	0.48	0.49
Firms' per period recruiting cost	γ	1.28	1.25
Workers' relative bargaining power	α	0.09	0.10
Reservation skill	s_R	0.91	0.95
Reservation skill, %ile of the skill distribution	$F(s_R)$	0.26	0.36

Notes: This table provides an overview of the baseline parameters values chosen as described in Section 3.1. Unless otherwise noted, the parameters values are the same ones chosen in Petrosky-Nadeau and Wasmer (2013). The value of r is chosen to correspond (per month) to a 4% p.a. rate on a 3-month US T-bill. The value of ω is chosen to match the value typically used for the US economy. The value of b is chosen to match that in Shimer (2005). The value of k is chosen to match that in Davis et al. (2006). The value of q is chosen to match that in den Haan et al. (2000). The value of u is chosen to match that in Gertler and Trigari (2009). The value of η is chosen to match that in Brügemann (2008) and Petrongolo and Pissarides (2001). The value of ϕ is imputed using the equilibrium expression $\phi = \frac{(1-\beta)}{\beta}\left(\frac{a}{e}\right)$. The value of f is imputed using the equilibrium condition $f = \frac{k(1-u)}{[1-F(s_R)]u}$. The value of γ is imputed using the assumption that $\frac{\gamma\mathcal{V}}{w} = 0.036$ following Silva and Toledo (2007). The value of α is imputed by solving the wage equation in (29). The value of Z is imputed using the expression $Z = q\theta^{\eta}$. The value of θ is imputed using the expression $\theta = \frac{f}{q}$.

Appendix

A The Repayment

Start with the Nash bargaining condition between the firm and the bank

$$\beta J_{L,t} = (1-\beta)B_{L,t} \quad \Rightarrow \quad B_{L,t} = \beta(J_{L,t} + B_{L,t})$$

Substituting in for $B_{L,t}$ and $J_{L,t}$ using (5) and (2) we have

$$\begin{aligned}
-\gamma + \left(\frac{1}{1+r}\right)[q[1-F(s_R)]B_{G,t+1} + (1-q)B_{L,t+1}] = \\
\beta\Bigg[\left(\frac{1}{1+r}\right)\left[q\int_0^{\infty} \max\{J_{L,t+1}, J_{G,t+1}(s)\}dF(s) + (1-q)J_{L,t+1}\right] - \\
\gamma + \left(\frac{1}{1+r}\right)[q[1-F(s_R)]B_{G,t+1} + (1-q)B_{L,t+1}]\Bigg]
\end{aligned}$$

Simplifying gives us

$$\begin{aligned}
-\frac{(1+r)\gamma}{q} + [1-F(s_R)]B_{G,t+1} + \frac{(1-q)}{q}B_{L,t+1} = \\
\beta\int_{s_R}^{\infty} J_{G,t+1}(s)dF(s) + \beta F(s_R)J_{L,t+1} + \beta\frac{(1-q)}{q}J_{L,t+1} - \\
\beta\frac{(1+r)\gamma}{q} + \beta[1-F(s_R)]B_{G,t+1} + \beta\frac{(1-q)}{q}B_{L,t+1}
\end{aligned}$$

Rearranging terms

$$\begin{aligned}
[1-F(s_R)]B_{G,t+1} = \\
(1-\beta)\frac{(1+r)\gamma}{q} + \beta\int_{s_R}^{\infty} J_{G,t+1}(s)dF(s) + \beta F(s_R)J_{L,t+1} + \beta[1-F(s_R)]B_{G,t+1} + \\
+\beta\frac{(1-q)}{q}(J_{L,t+1} + B_{L,t+1}) - \frac{(1-q)}{q}B_{L,t+1} \qquad (30)
\end{aligned}$$

Given the Nash bargaining condition, the last two terms on the right hand side disappear. Substituting in for $J_{G,t+1}(s)$ and $B_{G,t+1}$ using (3) and (6) we have

$$\begin{aligned}
[1-F(s_R)]\left(\psi_{t+1} + \left(\frac{1-k}{1+r}\right)B_{G,t+2}\right) = \\
(1-\beta)\frac{(1+r)\gamma}{q} + \beta\int_{s_R}^{\infty} y(s) - w(s) - \psi_{t+1} + \left(\frac{1-k}{1+r}\right)J_{G,t+2}(s)dF(s) + \beta F(s_R)J_{L,t+1} + \\
\beta[1-F(s_R)]\left(\psi_{t+1} + \left(\frac{1-k}{1+r}\right)B_{G,t+2}\right)
\end{aligned}$$

Simplifying and rearranging terms we have

$$\psi_{t+1} + \left(\frac{1-k}{1+r}\right) B_{G,t+2} =$$
$$(1-\beta)\frac{(1+r)\gamma}{q[1-F(s_R)]} + \frac{\beta}{[1-F(s_R)]}\int_{s_R}^{\infty} y(s) - w(s) - \psi_{t+1} + \left(\frac{1-k}{1+r}\right) J_{G,t+2}(s)dF(s) +$$
$$\frac{\beta F(s_R)}{[1-F(s_R)]} J_{L,t+1} + \beta\left(\psi_{t+1} + \left(\frac{1-k}{1+r}\right) B_{G,t+2}\right)$$

and

$$\psi_{t+1}\left(\frac{1-F(s_R)(1-\beta)}{[1-F(s_R)]}\right) =$$
$$\frac{\beta}{[1-F(s_R)]}\int_{s_R}^{\infty} y(s) - w(s)dF(s) +$$
$$(1-\beta)\frac{(1+r)\gamma}{q[1-F(s_R)]} + \frac{\beta}{[1-F(s_R)]}\left(\frac{1-k}{1+r}\right)\int_{s_R}^{\infty} J_{G,t+2}(s) + [1-F(s_R)]B_{G,t+2}dF(s)$$
$$\frac{\beta F(s_R)}{[1-F(s_R)]} J_{L,t+1} - \left(\frac{1-k}{1+r}\right) B_{G,t+2}$$

Going back a number of steps we see that we can simplify this expression by plugging in (30)

$$\psi_{t+1}\left(\frac{1-F(s_R)(1-\beta)}{[1-F(s_R)]}\right) =$$
$$\frac{\beta}{[1-F(s_R)]}\int_{s_R}^{\infty} y(s) - w(s)dF(s) + (1-\beta)\frac{(1+r)\gamma}{q[1-F(s_R)]} +$$
$$\left(\frac{1-k}{1+r}\right)\left(B_{G,t+2} - (1-\beta)\frac{(1+r)\gamma}{q[1-F(s_R)]}\right) - \left(\frac{1-k}{1+r}\right) B_{G,t+2}$$

$$\psi_{t+1}\left(\frac{1-F(s_R)(1-\beta)}{[1-F(s_R)]}\right) =$$
$$\frac{\beta}{[1-F(s_R)]}\int_{s_R}^{\infty} y(s) - w(s)dF(s) + (1-\beta)\frac{(1+r)\gamma}{q[1-F(s_R)]} - (1-\beta)\frac{(1-k)\gamma}{q[1-F(s_R)]}$$

Simplifying gives us

$$\psi =$$
$$\frac{\beta}{[1-F(s_R)(1-\beta)]}\int_{s_R}^{\infty} y(s) - w(s)dF(s) + (1-\beta)\frac{(r+k)\gamma}{q[1-F(s_R)(1-\beta)]}$$

B Wages

Solve for $J_G(s)$ and B_G from (3) and (6)

$$
\begin{aligned}
J_G(s) = \pi(s) + \left(\frac{1}{1+r}\right)[kJ_C + (1-k)J_G(s)] \\
J_G(s)\left(1 - \frac{1-k}{1+r}\right) = \pi(s) \\
J_G(s) = \pi(s)\left(\frac{1+r}{r+k}\right)
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
B_G = \psi + \left(\frac{1}{1+r}\right)[kB_C + (1-k)B_G] \\
B_G\left(1 - \frac{1-k}{1+r}\right) = \psi \\
B_G = \psi\left(\frac{1+r}{r+k}\right)
\end{aligned}
\tag{32}
$$

Sum (31) and (32)

$$
S_G(s) = [\pi(s) + \psi]\left(\frac{1+r}{r+k}\right) = [y(s) - w(s)]\left(\frac{1+r}{r+k}\right) \tag{33}
$$

From (7) and (8) solve for the values of employment and unemployment for the worker

$$
\begin{aligned}
W(s) = w(s) + \left(\frac{1}{1+r}\right)[kU(s) + (1-k)W(s)] \\
W(s)\left(1 - \frac{1-k}{1+r}\right) = w(s) + \left(\frac{k}{1+r}\right)U(s) \\
W(s) = \left(\frac{1+r}{r+k}\right)w(s) + \left(\frac{k}{1+r}\right)U(s)
\end{aligned}
\tag{34}
$$

and

$$
\begin{aligned}
U(s) = b + \left(\frac{1}{1+r}\right)[fW(s) + (1-f)U(s)] \\
U(s)\left(1 - \frac{1-f}{1+r}\right) = b + \left(\frac{f}{1+r}\right)W(s) \\
U(s) = \left(\frac{1+r}{r+f}\right)b + \left(\frac{f}{r+f}\right)W(s)
\end{aligned}
\tag{35}
$$

Together (34) and (35) give us

$$\begin{aligned}
W(s) &= \left(\frac{1+r}{r+k}\right) w(s) + \left(\frac{k}{1+r}\right)\left[\left(\frac{1+r}{r+f}\right) b + \left(\frac{f}{r+f}\right) W(s)\right] \\
W(s)\left(1 - \frac{kf}{(r+k)(r+f)}\right) &= \left(\frac{1+r}{r+k}\right) w(s) + \left(\frac{k(1+r)}{(r+k)(r+f)}\right) b \\
W(s)\left(\frac{r(r+f+k)}{(r+k)(r+f)}\right) &= \left(\frac{1+r}{r+k}\right) w(s) + \left(\frac{k(1+r)}{(r+k)(r+f)}\right) b \\
W(s) &= \left(\frac{(1+r)(r+f)}{r(r+f+k)}\right) w(s) + \left(\frac{k(1+r)}{r(r+f+k)}\right) b \qquad (36)
\end{aligned}$$

and

$$\begin{aligned}
U(s) &= \left(\frac{1+r}{r+f}\right) b + \left(\frac{f}{r+f}\right)\left[\left(\frac{1+r}{r+k}\right) w(s) + \left(\frac{k}{1+r}\right) U(s)\right] \\
U(s)\left(1 - \frac{kf}{(r+k)(r+f)}\right) &= \left(\frac{1+r}{r+f}\right) b + \left(\frac{f(1+r)}{(r+k)(r+f)}\right) w(s) \\
U(s)\left(\frac{r(r+f+k)}{(r+k)(r+f)}\right) &= \left(\frac{1+r}{r+f}\right) b + \left(\frac{f(1+r)}{(r+k)(r+f)}\right) w(s) \\
U(s) &= \left(\frac{(1+r)(r+k)}{r(r+f+k)}\right) b + \left(\frac{f(1+r)}{r(r+f+k)}\right) w(s) \qquad (37)
\end{aligned}$$

Plug in (33), (36), and (37) into the sharing rule in equation (16) and solve for the wage

$$\begin{aligned}
&\alpha[y(s) - w(s)]\left(\frac{1+r}{r+k}\right) = \\
&(1-\alpha)\left[\left(\frac{(1+r)(r+f)}{r(r+f+k)}\right) w(s) + \left(\frac{k(1+r)}{r(r+f+k)}\right) b - \left(\frac{(1+r)(r+k)}{r(r+f+k)}\right) b + \left(\frac{f(1+r)}{r(r+f+k)}\right) w(s)\right]
\end{aligned}$$

$$\begin{aligned}
&\alpha[y(s) - w(s)]\left(\frac{1}{r+k}\right) = (1-\alpha)\left(\frac{(r+f)w(s) + kb - (r+k)b - fw(s)}{r(r+f+k)}\right) \\
&\alpha[y(s) - w(s)]\left(\frac{1}{r+k}\right) = (1-\alpha)\left(\frac{rw(s) - rb}{r(r+f+k)}\right) \\
&\alpha[y(s) - w(s)](r+f+k) = (1-\alpha)[w(s) - b](r+k) \\
&\alpha(r+f+k)w(s) + (1-\alpha)(r+k)w(s) = \alpha(r+f+k)y(s) + (1-\alpha)(r+k)b \\
&w(s)(r+k+\alpha f) = \alpha(r+f+k)y(s) + (1-\alpha)(r+k)b \\
&w(s) = \alpha\left[\frac{r+f+k}{r+\alpha f+k}\right] y(s) + (1-\alpha)\left[\frac{r+k}{r+\alpha f+k}\right] b \qquad (38)
\end{aligned}$$

C The Reservation Skill Level

The reservation skill level is the level of skill, s_R, such that

$$J_{G,t}(s_R) = J_{L,t} \tag{39}$$

Drop the time subscripts and solve for $J_G(s)$

$$J_G(s) = \pi(s) + \left(\frac{1}{1+r}\right)(1-k)J_G(s) \quad \Rightarrow \quad J_G(s) = \frac{(1+r)\pi(s)}{(r+k)} \tag{40}$$

Next, rearrange (2) to get

$$\begin{aligned} J_L = \left(\frac{1}{1+r}\right)\left[q(\theta_t)\int_0^\infty \max\{J_L, J_G(s)\}dF(s) + (1-q(\theta_t))J_L\right] \\ \Rightarrow \quad J_L = \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\int_0^\infty \max\{J_L, J_G(s)\}dF(s) \end{aligned}$$

Together with (39) and (40), we have

$$\begin{aligned} \frac{(1+r)\pi(s_R)}{(r+k)} &= \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\int_0^\infty \max\{\frac{(1+r)\pi(s_R)}{(r+k)}, \frac{(1+r)\pi(s)}{(r+k)}\}dF(s) \\ \pi(s_R) &= \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\int_0^\infty \max\{\pi(s_R), \pi(s)\}dF(s) \end{aligned}$$

Subtract $\left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\pi(s_R)$ from both sides

$$\begin{aligned} \pi(s_R) - \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\pi(s_R) &= \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\int_0^\infty \max\{\pi(s_R)-\pi(s_R), \pi(s)-\pi(s_R)\}dF(s) \\ \pi(s_R)\left(1-\left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\right) &= \left(\frac{q(\theta_t)}{r+q(\theta_t)}\right)\int_0^\infty \max\{0, \pi(s)-\pi(s_R)\}dF(s) \\ \pi(s_R) &= \left(\frac{q(\theta_t)}{r}\right)\int_{s_R}^\infty [\pi(s)-\pi(s_R)]dF(s) \end{aligned}$$

since $\pi(s)$ is strictly increasing in s. Integrating by parts, we can simplify the Riemann-Stieltjes integral on the right hand side of the expression as follows

$$
\begin{aligned}
\pi(s_R) &= \left(\frac{q(\theta_t)}{r}\right)\left[\int_{s_R}^{\infty}\pi(s)dF(s) - \int_{s_R}^{\infty}\pi(s_R)dF(s)\right] \\
&= \left(\frac{q(\theta_t)}{r}\right)\left[\left(\infty F(\infty) - \pi(s_R)F(s_R) - \int_{s_R}^{\infty}F(s)d\pi(s)\right) - (\pi(s_R)F(\infty) - \pi(s_R)F(s_R))\right] \\
&= \left(\frac{q(\theta_t)}{r}\right)\left[\infty - \pi(s_R) - \int_{s_R}^{\infty}F(s)d\pi(s)\right] \\
&= \left(\frac{q(\theta_t)}{r}\right)\left[\int_{s_R}^{\infty}d\pi(s) - \int_{s_R}^{\infty}F(s)d\pi(s)\right] \\
&= \left(\frac{q(\theta_t)}{r}\right)\int_{s_R}^{\infty}[1-F(s)]d\pi(s)
\end{aligned}
$$

and the reservation skill level is then

$$
s_R = \pi^{-1}\left(\left(\frac{q(\theta_t)}{r}\right)\int_{s_R}^{\infty}[1-F(s)]d\pi(s)\right)
$$