

Linear Programming Model for Circular Economy Production Planning

- Old devices or components are processed into secondary products through remanufacturing, refurbishing, and recycling and reintegrated into the goods cycle.
- The availability of reusable factors $i \in I_A$ from old devices is uncertain (uncertain quantity and quality).
- The availabilities A_{it} are certain
- each factor $i \in I_A$ can be replaced by a primary product
- procurement cost of $i \in I_A$ are b_i per unit, while the costs of the corresponding primary product are $c_i > b_i$
- At the beginning of the period $t = 1$, $R_{i1} = R_i^a$ unit of $i \in I_A$ or the corresponding product are available
- v_{it} : Procurement quantity of factor $i \in I_A$ in period t
- w_{it} : Procurement quantity of primary product that replaces factor $i \in I_A$ in period t
- R_{it} : Capacity of factor i or the corresponding primary product in period t

$$\text{Max.} \quad \sum_{t=1}^T \sum_{j=1}^n (p_j z_{jt} - k_j y_{jt} - h_j x_{j(t+1)}) - \sum_{i \in I_A} (b_i v_{it} + c_i w_{it})$$

Subject to:

$$\sum_{j=1}^n a_{ij} y_{jt} \leq R_{it}, \quad (i \in \{1, \dots, m\} \setminus I_A; t = 1, \dots, T) \quad 1)$$

$$x_{j1} = x_j^a, \quad (j = 1, \dots, n) \quad 2)$$

$$x_{j(t+1)} = x_{jt} + y_{jt} - z_{jt}, \quad (j = 1, \dots, n; t = 1, \dots, T) \quad 3)$$

$$z_{jt} \leq d_{jt}, \quad (j = 1, \dots, n; t = 1, \dots, T) \quad 4)$$

$$x_{j(t+1)}, y_{jt}, z_{jt} \geq 0, \quad (j = 1, \dots, n; t = 1, \dots, T) \quad 5)$$

$$R_{i1} = R_i^a, \quad (i \in I_A) \quad 6)$$

$$R_{i(t+1)} = R_{it} + v_{it} + w_{it} - \sum_{j=1}^n a_{ij} y_{jt}, \quad (i \in I_A; t = 1, \dots, T) \quad 7)$$

$$v_{it} \leq A_{it}, \quad (i \in I_A; t = 1, \dots, T) \quad 8)$$

$$R_{i(t+1)}, v_{it}, w_{it} \geq 0, \quad (i \in I_A; t = 1, \dots, T) \quad 9)$$

1. resource constraint: ensure that the resources required for production do not exceed the available resource capacity at any given time.
2. initiate inventory: defines the initial level of inventory for each product at the start of the planning horizon.
3. Inventory dynamics : nventory dynamics describe how inventory evolves over time, considering production, sales, and possibly recycling or returns.
4. Demand Constraint : that the amount of product sold (or used) in each period does not exceed the demand.
5. Non-negativity : that no variables can take negative values, which is a common requirement in production planning models.
6. Initial Ressource Allocation : defines the initial allocation of resources at the start of the planning horizon.
7. Ressource Dynamics : defines how resources evolve over time, based on procurement and usage.
8. Avaibility constraints : limits the availability of resources, considering production and consumption in each period.
9. Non-negativity of Ressources : ensures that the resources do not become negative, which would not be physically or economically possible.