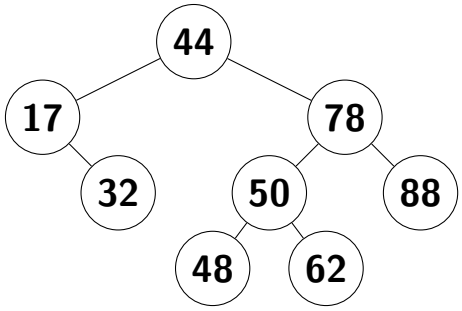
CSCI 104L Lecture 23 : AVL Trees



Insert Operations

Question 1. What would happen if we were to insert the value 14 into the above tree? Is it still an AVL tree?

Question 2. What if we insert the value 34 into it instead? If we follow the standard binary search tree insertion algorithm, is it still an AVL tree?

After inserting a node to a binary search tree, we apply the following procedure:

Start at the newly-inserted node and walk up to the root, checking if each node is balanced (the height-balance rule applies to this node). If a node is unbalanced, rotate the subtree rooted at that node. Rotate the following three nodes:

1. Let z be (a pointer to) the first unbalanced node on the way up.
2. Let y be the child of z with greater height (hint: this is always an ancestor of the node you inserted. Why?). Why are ties impossible?
3. Let x be the child of y with greater height (this is always an ancestor of the node you inserted, or the node itself. Why?). Why are ties impossible?

When x, y, z form a zig-zag pattern, we do a **double rotation**. Otherwise we do a **single rotation**. The rotations are pictured on the last page.

Question 3. After doing a rotation, the tree is guaranteed to be balanced, and we can stop. Why?

Question 4. Starting from the original tree, insert the value 34. What is the resulting AVL tree?

Question 5. Starting from the original tree, insert the value 54. What is the resulting AVL tree?

Deleting from an AVL Tree

To delete from an AVL tree, follow the same procedure as removal from a binary search tree. Then, starting at the node that was *removed*, move up to the root, recalculating heights if necessary. For each node z, if it is unbalanced, rotate the subtree rooted at that node:

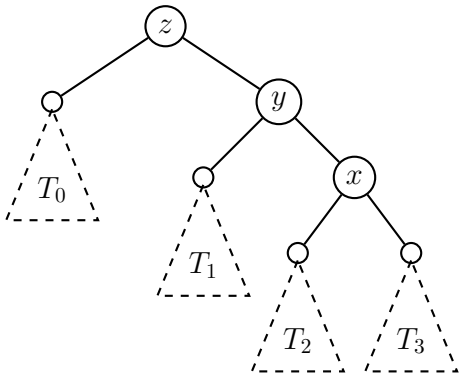
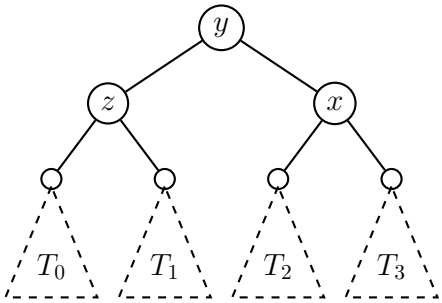
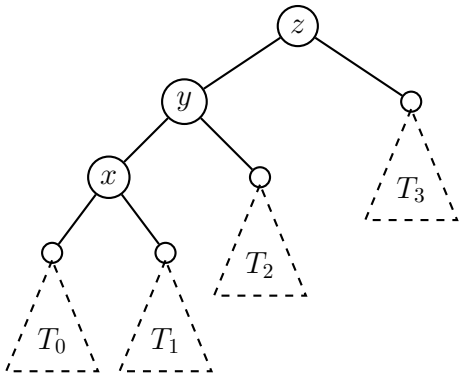
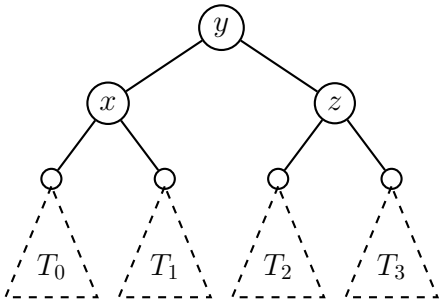
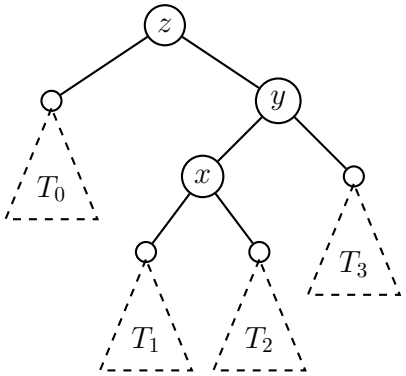
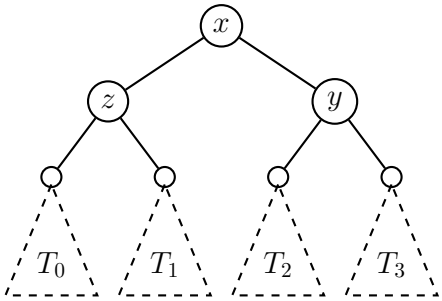
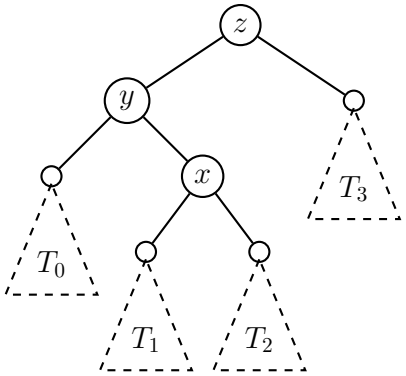
1. Let z be (a pointer to) the unbalanced node we found.
2. Let y be the child of z with greater height (hint: this is never an ancestor of the node you deleted. Why?). Why are ties impossible?
3. Let x be the child of y with greater height. In the event of a tie, choose x so that we'll have a single rotation instead of a double rotation

Then perform the same rotation you would do for that formation on insertion. Unlike insertion, however, this only fixes the problem *locally* – it might be unbalanced higher up. You need to continue this until the tree is balanced *globally*.

Question 6. Starting from the original tree, what does it look like if we delete 88?

Question 7. Starting from the original tree, what does it look like if we remove 32?

Question 8. Starting with an empty AVL tree, do the following operations, in sequence:
INSERT: $1, 2, 3, 12, 9, 13, 7, 4, 6, 5, 8$
DELETE: $4, 1$
INSERT: $1, 14, 11$
DELETE: $3, 13, 12, 11, 14, 2, 3, 7, 8, 9$

Before	After
	**Left rotate on z** 
	**Right rotate on z** 
	**Right rotate on y, then left rotate on z** 
	**Left rotate on y, then right rotate on z**