CSCI104

Week 6: Backtracking + BST and AVL

CSCI104

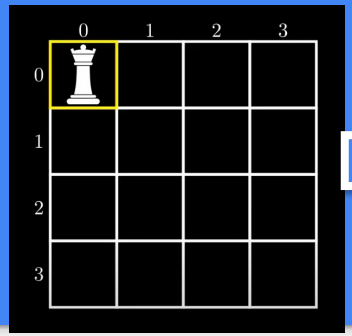
Lab 9: Backtracking

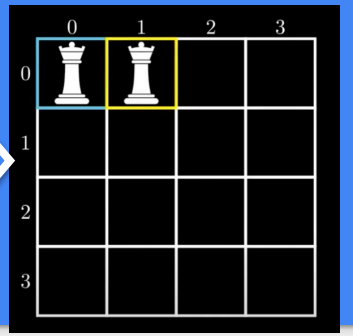
Backtracking

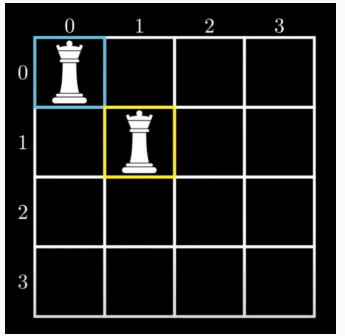
- A recursive way of searching the solution space for a problem.
- We are a solution incrementally
  - If partial solutions don't satisfy constraints, remove path from search space (dead ends)
  - Makes it more efficient than brute-force in practice.
- 3-digit number lock!

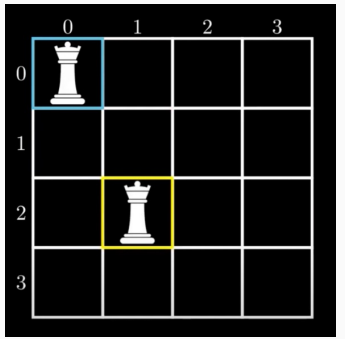
  - You start by setting the first digit to 0.
  - You move to the second digit and set to 0.
  - You move to the third digit and try all 10 combinations.
  - If none worked, you go back to the second digit and set to 1.
  - …

Backtracking









- N-Queens:
  - Try to place queens on a nxn board
  - Queens shouldn't be able to capture each other!
- How does it work?
  - Fix the position of the 1st queen,
  - Explore all possible solutions,
  - If none, move the first queen to the next location!
- Good step-by-step visualization:
  https://www.youtube.com/watch?v=XL8O5P3S0RU

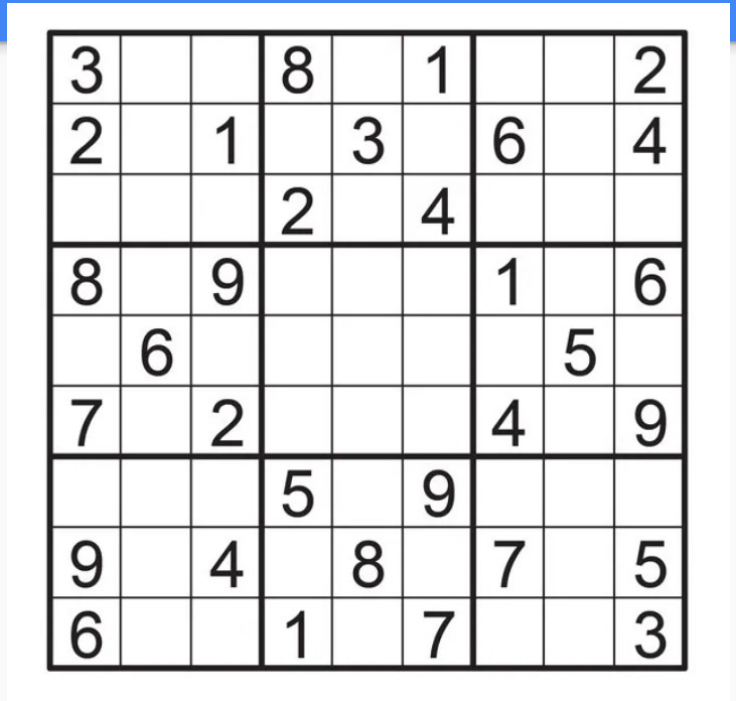
Backtracking

- What does a recursive backtracking solution look like generally?

1. `solve()` function : performs whatever necessary setup and calls the helper
2. `recursive_helper()` function : iterates over possibilities in some **sub domain**.
    - In N-Queens, this domain is a row. After checking if an option `is_valid()`, `recursive_helper()` calls itself on the next row after to test each option. It returns true if it has reached the end OR if its child call returns true. It returns false otherwise to tell the caller it has reached a dead end. If a child call returns false, undo whatever option was tested.
3. `is_valid()` function : Tells you whether or not an option is viable for a specific arrangement.

The Lab

- We'll solve a Sudoku using recursive backtracking!
  - Solution includes trying different numbers on each empty cell
  - **Remember:** A solution is only valid if the number appears only once in the row/column and block it is in
- Compile with '*make*' & run '*./sudoku*'

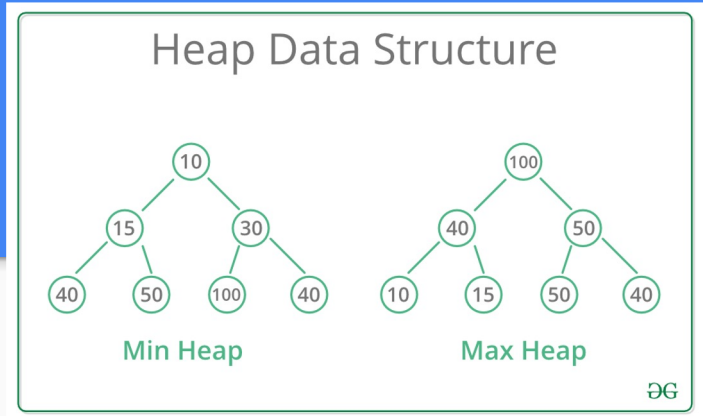


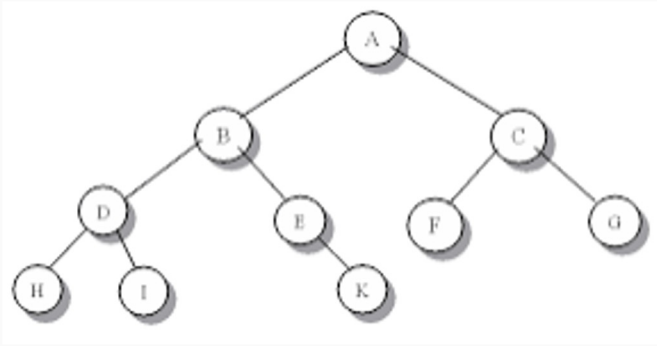
CSCI104

Lab 10: BST & AVL Trees

REMEMBER: Heaps

- COMPLETE d-ary tree
  - All levels except the last are completely filled
  - All leaves in last level are to the left side
- Every parent is “better” than both of its children
- Min Heap: node is less than or equal to all children
- Max Heap: node is greater than or equal to all children





Could this be a heap??

Binary Search Trees (BST)

- Not necessarily a complete or full tree
- Left children (left subtree) hold values LESS THAN or equal to parent's values
- Right children (right subtree) hold values GREATER THAN parent's value

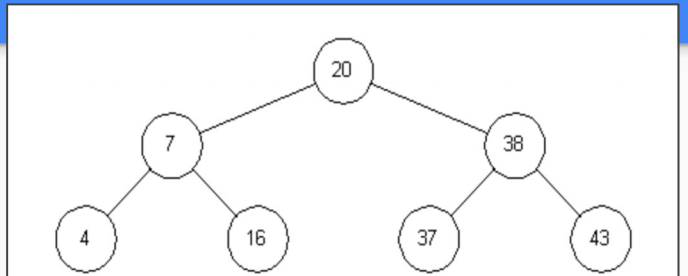


Figure 3-2: A Binary Search Tree

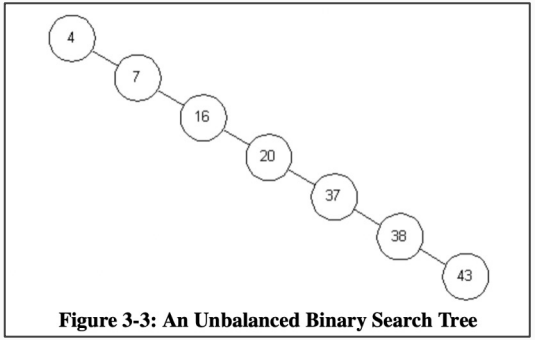


Figure 3-3: An Unbalanced Binary Search Tree

Traversals: Pre-Order, In-Order, Post-Order

- All traverals operate on EVERY node eventually–just in different orders
  - “Pre” : visit the parent “pre-“ (before) visiting left and right sub-trees.
  - “In” : visit the parent “in”-between visiting left and right sub-trees.
  - “Post”: visit the parent “post-“ (after) visiting left and right sub-trees.

Pre-Order Traversal

```
// Operate on current node
// Recurse left
// Recurse right
// return
```

In-Order Traversal

```
// Recurse left
// Operate on current node
// Recurse right
// return
```

Post-Order Traversal

```
// Recurse left
// Recurse right
// Operate on current node
// return
```

Traversals in C++

For a BST, what is special about operating on elements using an in-order traversal? If we were printing integers using this traversal, what would the output look like?

```
void pre_order(Node* node) {
    if (node == nullptr) return;
    print(node);
    pre_order(node->left);
    pre_order(node->right);
}

void in_order(Node* node) {
    if (node == nullptr) return;
    in_order(node->left);
    print(node);
    in_order(node->right);
}

void post_order(Node* node) {
    if (node == nullptr) return;
    post_order(node->left);
    post_order(node->right);
    print(node);
}
```

Why BSTs? SEARCHING!

- Enable (potentially) faster searching
- Why do we say potentially? What is an example where the search is slow, even if it’s a valid BST?

Why BSTs? SEARCHING!

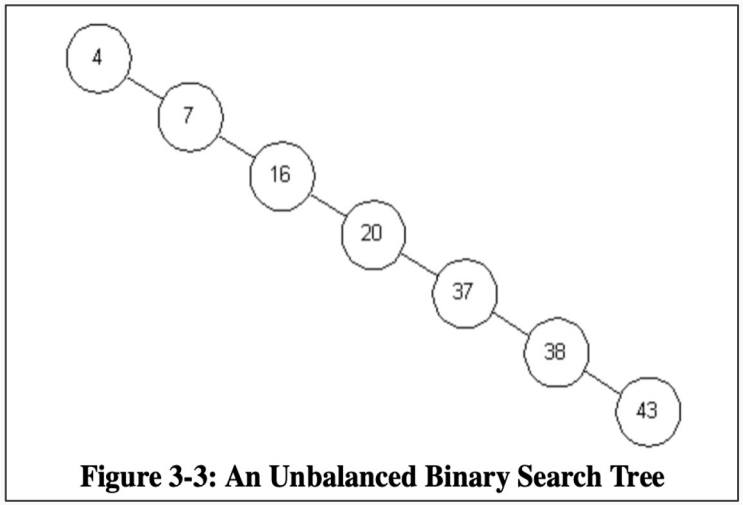


Figure 3-3: An Unbalanced Binary Search Tree

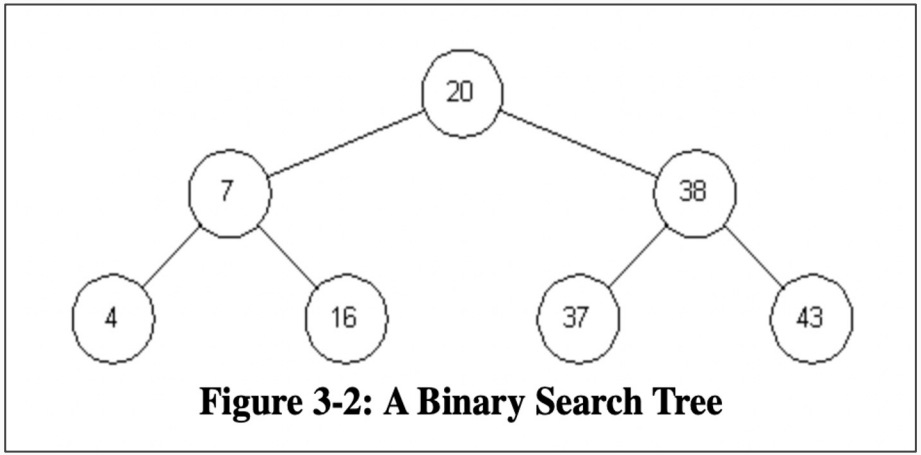


Figure 3-2: A Binary Search Tree

Slower search: O(n)
Basically like a linked list

Faster search: O(logn)

Search Function

- Can do it iteratively or recursively

To search for key `X` in a BST, we compare *X* to the current node.

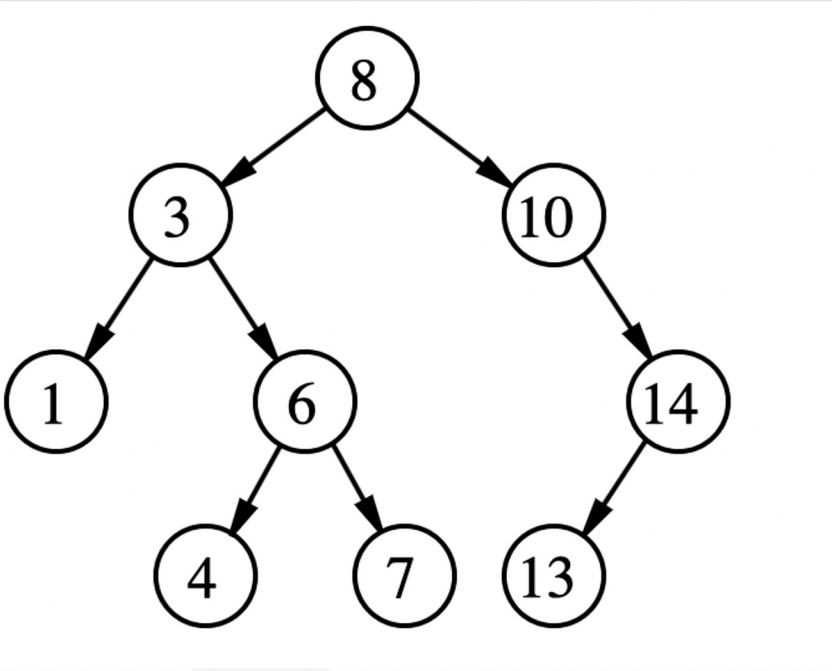
- If the current node is null, `X` must not reside in the tree.
- If `X` is equal to the current node, simply return the current node.
- If it is less than the current node, we check the left subtree.
- Else, it must be greater than the current node, so we check the right subtree.

Or, in code:

```
// Finds the node with value == val inside the bst. Returns nullptr if not found
Node* find(Node* root, int val) {
    if (root == nullptr) return nullptr;
    if (root->val == val) return root;
    if (root->val > val) return find(root->left, val);
    return find(root->right, val);
}
```

Recursive example

Search Example



Operation: `find(6)` // We begin at the root

Let's walk through this:

- Current node = 8, 6 < 8, therefore go left.
- Current node = 3, 6 > 3, therefore go right.
- Current node = 6, 6 = 6, we've found the node.

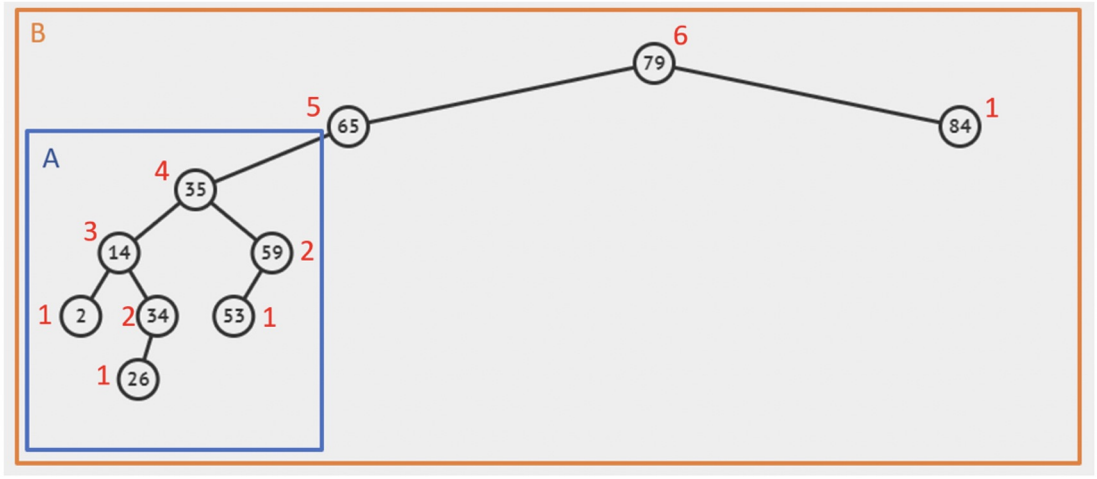
Operation: `find(0)` // We begin at the root

Let's walk through this one too:

- Current node = 8, 0 < 8, therefore go left.
- Current node = 3, 0 < 3, therefore go left.
- Current node = 1, 0 < 1, therefore go left.
- Current node = null. 0 is not in the tree.

Balanced Binary Tree

- **Height-balancing property: heights of each subtree differ by no more than 1**
- Avoids the slower search times!
- Keeps the height of the tree log(n)



A is balance, B is not

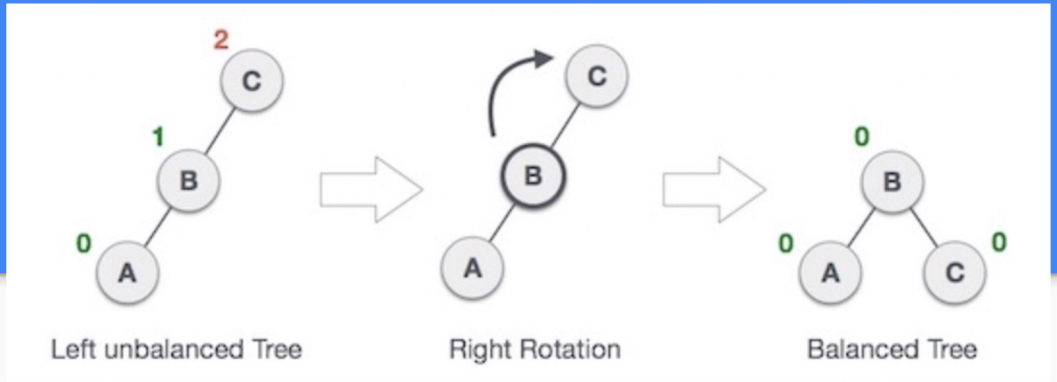
Maintaining BST Property

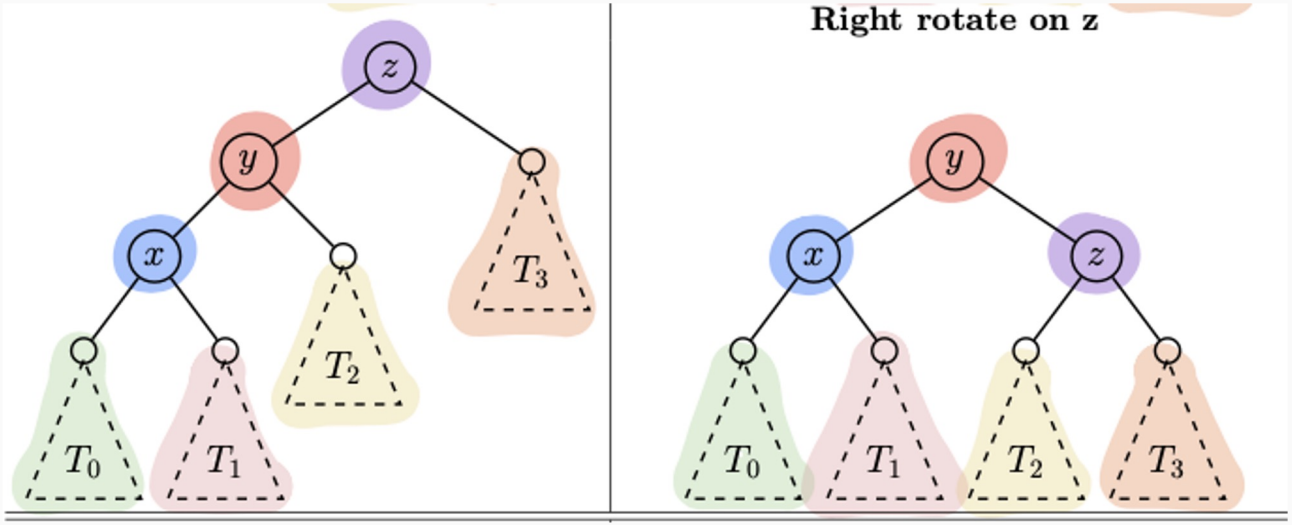
- REMEMBER: BST Property = left subtree node keys less than parent's and right subtree node keys greater than parent's
- Maintained by **smart** insertion and deletion
- Insert function
  - Traverse the tree based on key to be inserted
  - Insert once you encounter a situation where you cannot traverse further
- Remove function
  - Need to choose which node to promote
  - If node you want to remove has 0 children: just remove it
  - If node you want to remove has 1 child: promote the child of the node
  - If node you want to remove has 2 children: swap with its predecessor OR successor

Self-Balancing BSTs

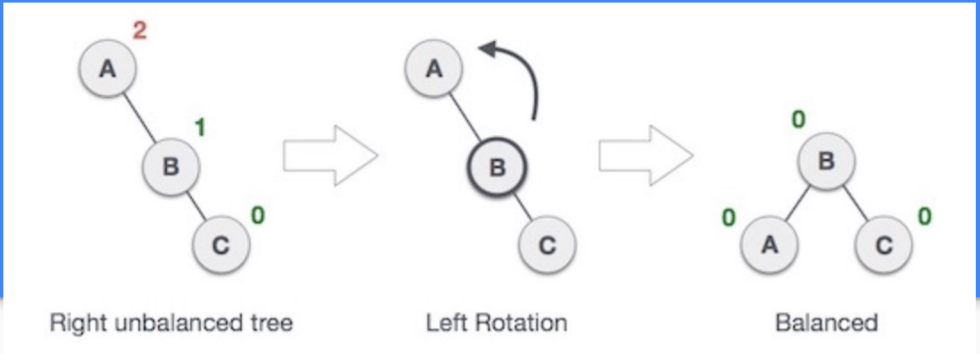
- We will be focusing on AVL trees
- You keep the tree balanced even after insertions or deletions
- This involves using rotations!
  - Foundation of AVL trees

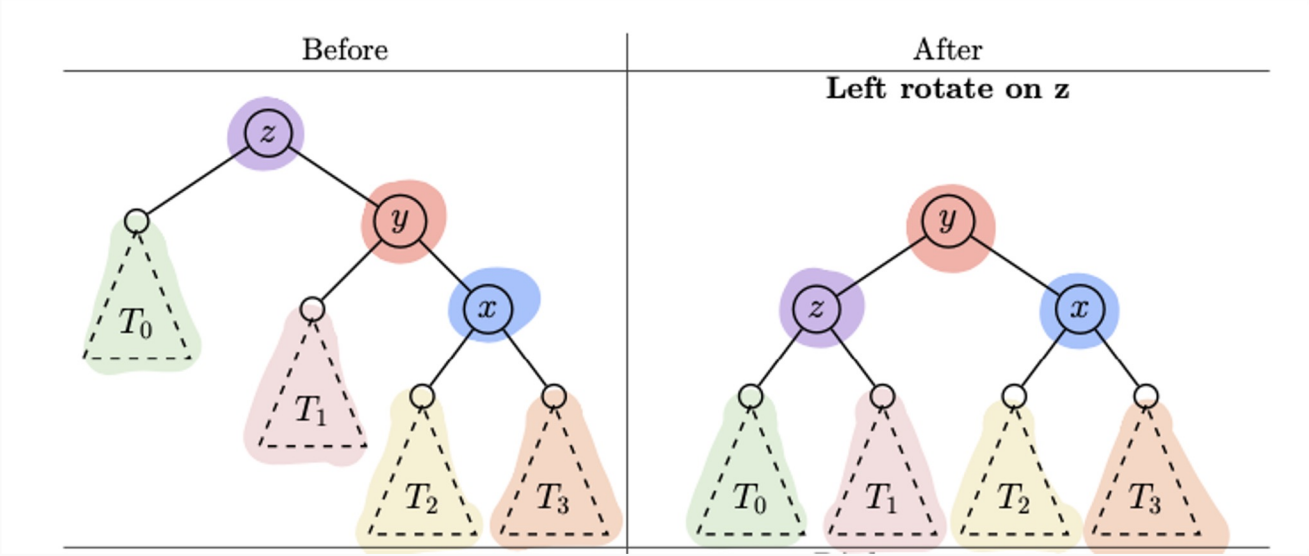
Single Rotations RIGHT



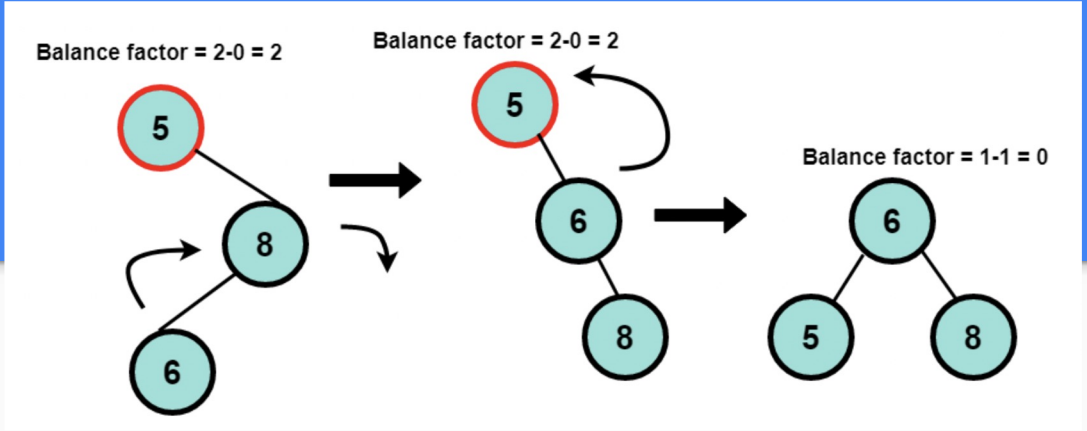


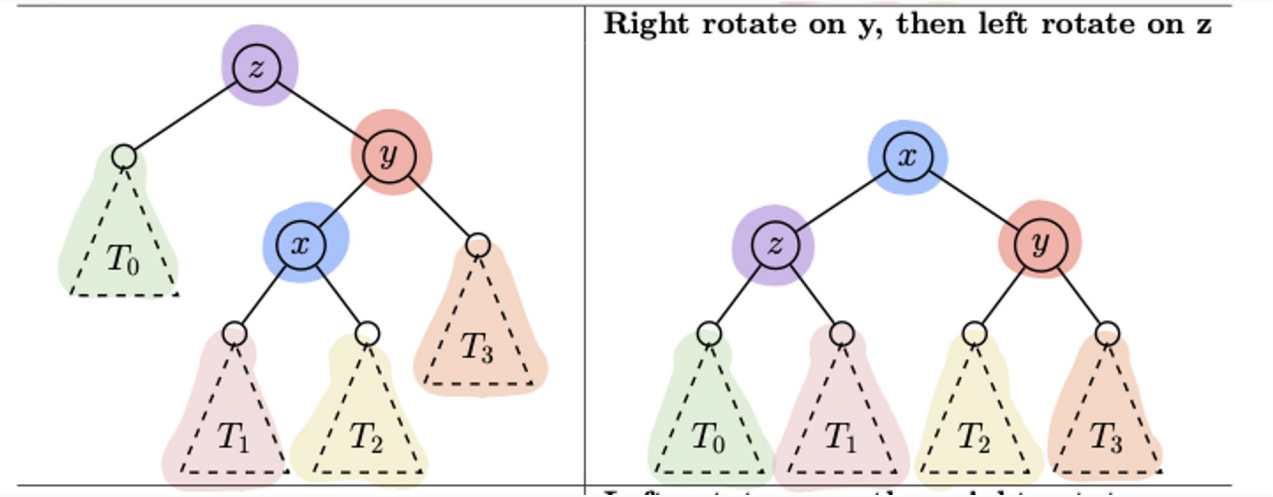
Single Rotations
LEFT



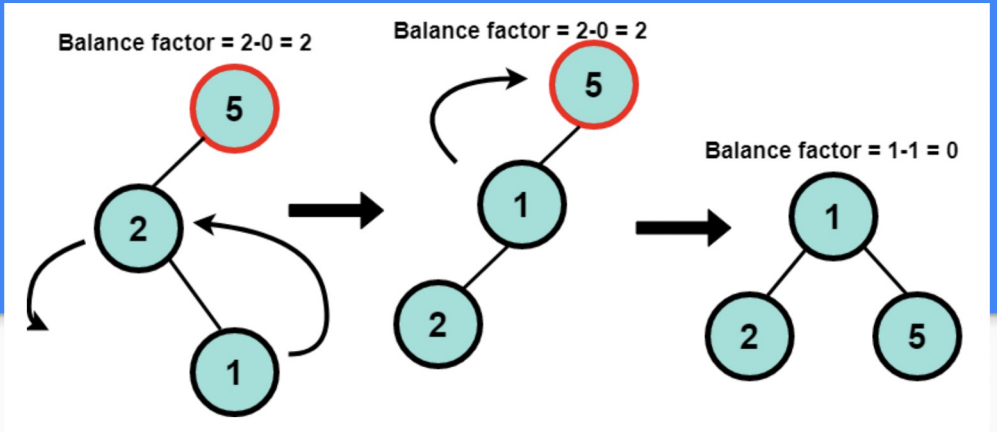


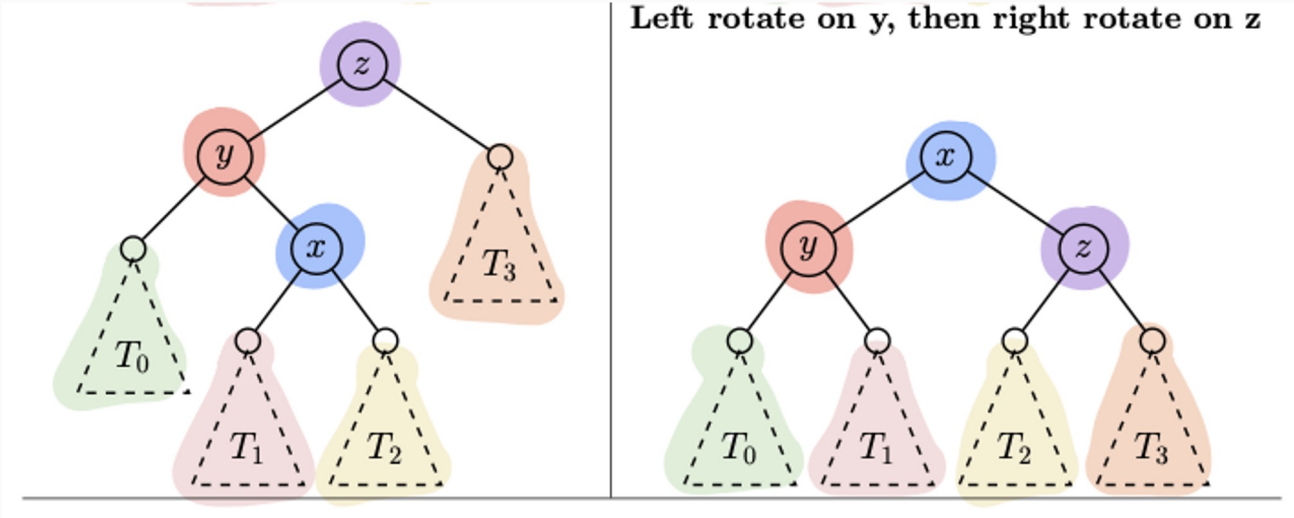
Double Rotations RIGHT LEFT





Double Rotations
LEFT RIGHT





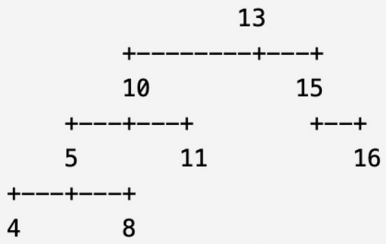
AVL Insert and Remove

- Insert
  - Insert as you would in a BST
  - Fix the tree if it is unbalanced after inserting the node (ROTATION)
    - Need at most 1 rotation (either a single or double rotation)
- Remove
  - Remove as you would in a BST
  - Keep traversing up the tree and fixing tree if unbalanced (ROTATIONS)
    - You may need multiple rotations to fully fix the tree

The Lab

- Exercise 1
  - Draw or type out operations on tree
- Exercise 2
  - Write isBalanced function
  - **NOT NEEDED TO GET CHECKED OFF**
  - This function is also part of the PA
  - We encourage you to use a single traversal for this function
  - Cannot work in groups, we will not be going over this solution

Initial Tree



Insert 14

Insert 3

Remove 3

Remove 4

Write a function to determine whether a binary tree is height-balanced or not.

- A binary tree in which the depth of the two subtrees of every node never differs by more than 1.

```
bool isBalanced(Node *root)
```