CSCI 104L Lecture 2: Recursion

Analyzing recursive functions

Exercise 1. How can you analyze something like this?

```
void recurse (int *A, int size) {
  if (size <= 1) return;
  //do stuff (taking O(1) time)
  recurse(first half of A, size/2);
  recurse(second half of A, size/2);
}
```

Exercise 2. Find a recurrence relation, and analyze the runtime:

```
int binarySearch(int t, int *b, int lo, int hi) {
    if (hi < lo) return -1; //nothing to search, it's not in the array.
    else {
        int mid = (hi+lo)/2;  //the middle of the array, rounded down.
        if (t == b[mid]) return mid;  //found it!
        else if (t < b[mid]) return binarySearch(t, b, lo, mid-1); //search left.
        else return binarySearch(t, b, mid+1, hi); //search right.
    }
}
```

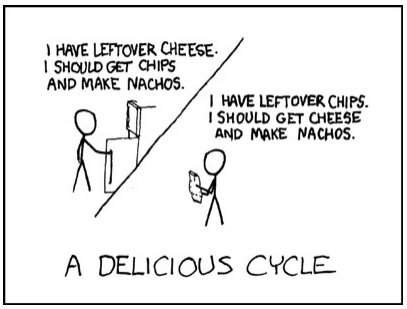
Exercise 3. Find a recurrence relation, and analyze the runtime for foo(0, n-1, 0):

```
int *a;
void foo(int left, int right, int digit) {
  for (int i = left; i <= right; i++) a[i] += digit;
  if (right > left) {
    foo(left, (left+right)/2, 0);
    foo((left+right)/2+1, right, 1);
  }
}
```

Exercise 4. Analyze the runtime for the following recurrence relation:
$f(n) = 2f(\frac{n}{2}) + \Theta(n \log n)$

Exercise 5. Analyze the runtime for the following recurrence relation:
$f(n) = 4f(\frac{n}{2}) + \Theta(n)$

Exercise 6. You have an n dollar debt. Every day, you can either pay 1 or 3 dollars towards that debt. How many different ways are there to pay off your debt?



XKCD # 140 **Delicious**: I'm currently in the I Have Cheese phase of this cycle.

Recursive Definitions

You can define other things recursively, not just functions.

- A string of lower-case letters is either: (1) the empty string, or (2) a letter 'a'-'z' followed by a string of lower-case letters.
- A non-negative integer is either: (1) the number 0, or (2) n+1, where n is a non-negative integer.
- A palindrome is either: (1) the empty string, or (2) a single letter 'a'-'z', or (3) a string xPx, where x is a single letter 'a'-'z', and P is a palindrome.
- A simple algebraic expression is either:
    1. A number.
    2. A variable.
    3. (A+B), where A and B are simple algebraic expressions.
    4. (A*B), where A and B are simple algebraic expressions.