CSCI 104
Priority Queues / Heaps

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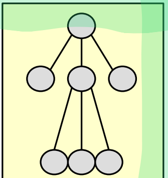
A graph with restrictions

TREES

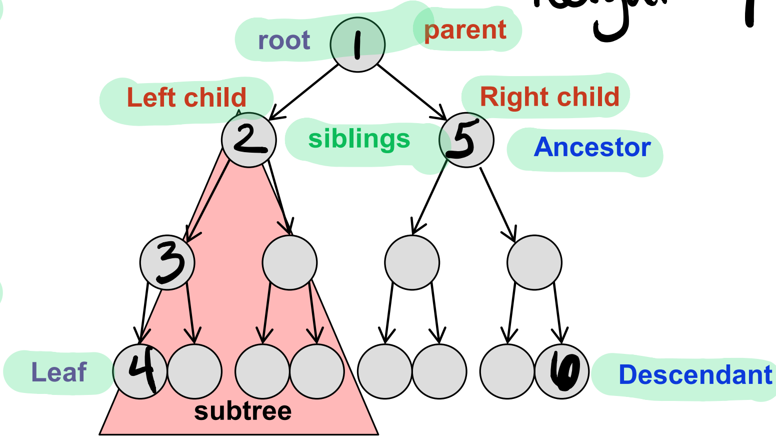
Tree Definitions – Part 1

- **Definition**: A connected, acyclic (no cycles) graph with:
  - A root node, r, that has 0 or more subtrees
  - Exactly one path between any two nodes
- In general:
  - Nodes have exactly one parent (except for the root which has none) and 0 or more children
- d-ary tree
  - Tree where each node has at most d children
  - Binary tree = d-ary Tree with d=2



A 3-ary (trinary) tree

height = 4



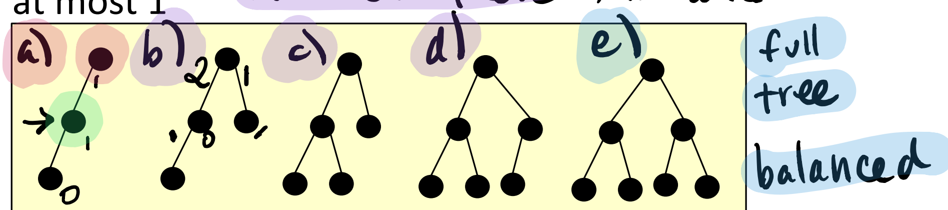
Terms:

- **Parent(i)**: Node directly above node i
- **Child(i)**: Node directly below node i
- **Siblings**: Children of the same parent
- **Root:** Only node with no parent
- **Leaf:** Node with 0 children
- **Height:** Number of nodes on longest path from root to leaf
- **Subtree(n)**: Tree rooted at node n
- **Ancestor(n)**: Any node on the path from n to the root
- **Descendant(n):** Any node in the subtree rooted at n

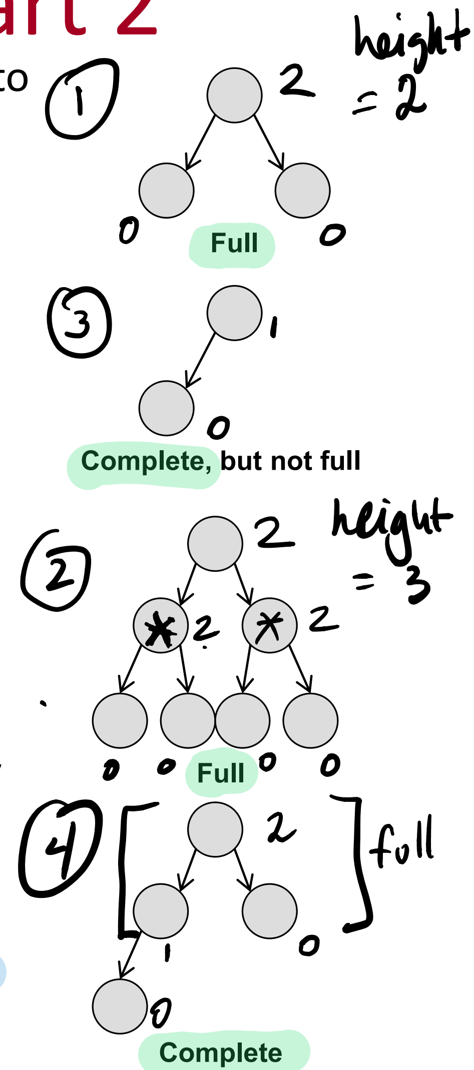
Tree Definitions – Part 2

- Tree height: maximum # of nodes on a path from root to any leaf
- Full d-ary tree, T, where
  - Every vertex has 0 or d children and all leaf nodes are at the same level
  - If height h>1 and both subtrees are full binary trees of height, h
  - If height h==1, then it is full by definition
- Complete d-ary tree
  - Each level is filled left-to-right and a new level is not started until the previous one is complete
- Balanced d-ary tree b, c, d not full
  - Tree where subtrees from EVERY node differ in height by at most 1 but complete, balanced

a) Not full, not complete not balanced



DAPS, 6th Ed. Figure 15-8



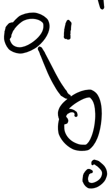
Tree Height

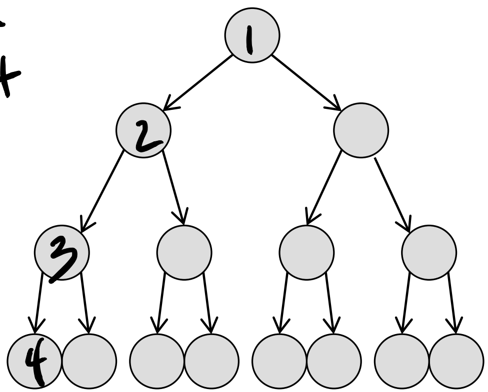
If full or complete → balanced

if full → complete

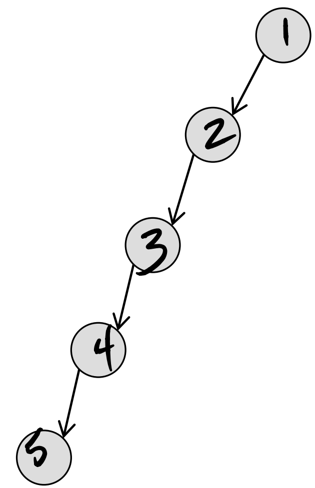
- A full binary tree of n nodes has height, $\lceil log_2(n+1) \rceil$
  - This implies the minimum height of any tree with n nodes is $\lceil log_2(n+1) \rceil$
- The maximum height of a tree with n nodes is, n

Balanced tree, not full nor complete





15 nodes => height $log_2(16)$ = 4



5 nodes => height = 5