CSCI 104L Lecture 12: Probability

Discrete Probability

- An **experiment** is a procedure that yields one of a set of possible outcomes; e.g., flipping a coin
- The **sample space** is the set of possible outcomes for the experiment; e.g., {heads, tails, on edge}
- An **event** is a subset of the sample space (such as heads or on edge).

Question 1. You roll two dice, and you're wondering what the probability is that you'll get snake eyes.

What is the experiment? What is the sample space?

What is the event in question? What is the probability of this event?

The **Uniform Distribution** assigns the probability $\frac{1}{n}$ to each element of S. For example, an unbiased coin assigns $\frac{1}{2}$ to both heads and tails. An unbiased die assigns $\frac{1}{6}$ to each side. Two unbiased dice assign $\frac{1}{36}$ to every possible result.
If S is a finite sample space of equally likely outcomes, and E is an event of S, then the probability of E is $p(E) = \frac{|E|}{|S|}$.

Question 2. What is the probability that the sum of two dice rolls is 7?

Question 3. How many different 5-card poker hands are there?

Question 4. How many different 5-card poker hands are there which contain a 4 of a kind?

Question 5. What is the probability that you get 4 of a kind in a 5-card poker hand?

Question 6. What is the probability that you get a full house?

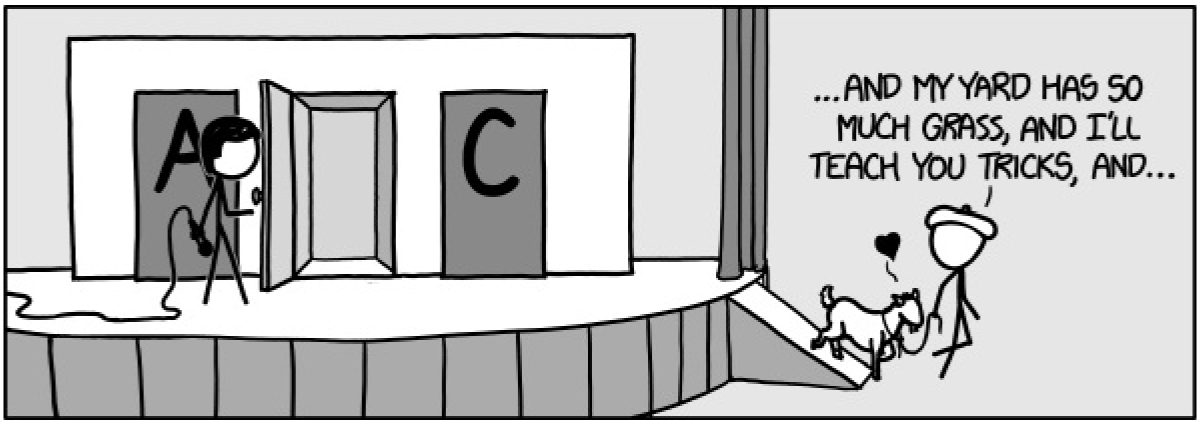
Question 7. There is a sequence of 10 randomly generated bits. What is the probability that at least one of the bits is 0?

Given an event E in a sample space S, the probability of event $\overline{E} = S - E$ is $p(\overline{E}) = 1 - p(E)$

Question 8. There are a line of 100 people. Everyone at an even position in the line is a pirate. Everyone at a position divisible by 5 is a ninja. If you choose someone randomly from the line, what is the probability that they are either a pirate or a ninja?

$p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$.

Question 9. The Monty Hall Three-Door Puzzle. There is a car behind one of three doors, and a goat behind each of the other doors. You choose a door, but do not open it. The game show host knows what is behind each door, and chooses one of the two other doors which contain a goat. He then opens it, showing you that it has no prize. You now have the option to open your current door, or switch to the other unopened door. Should you switch? Should you stay? Does it not matter?



XKCD #1282: Monty Hall. A few minutes later, the goat from behind door C drives away in the car.

Question 10. A biased coin is twice as likely to come up heads as it is tails. With what probability will it come up heads?

Note that we are now departing from the uniform distribution.

If $S = \{x_1, ...x_n\}$ is the set of possible outcomes, then $\sum_{i=1}^{n} p(x_i) = 1$, and $0 \leq p(x_i) \leq 1, \forall i$.

Question 11. A biased die has a 3 appear twice as often any other individual number, but each of the other five outcomes are equally likely. What is the probability that an odd number is rolled?

$$p(E) = \sum_{s \in E} p(s)$$

Question 12. For the same biased die, we know that an odd number was rolled. What is the probability that a 3 was rolled?

Let E and F be events with $p(F) > 0$. The **conditional probability** of E, given F is $p(E|F) = \frac{p(E \cap F)}{p(F)}$
This should make sense: we're forcing F to happen, so we're limiting the number of possible outcomes to $p(F)$. If E happens, we've still forced F to happen, so really we're looking at $E \cap F$.

Question 13. There are 4 people in line, and each person is equally likely to be a pirate or a ninja. What is the probability that there are two consecutive ninja, given that the first person in the line is a ninja?

Question 14. A family has two children, the probability of each being a boy or girl is uniformly generated. E is the event that the family has two boys. F is the event that the youngest child is a boy. What is $P(E|F)$?

Question 15. A family has two children, the probability of each being a boy or girl is uniformly generated. E is the event that the family has two boys. F is the event that the family has at least one boy. What is $P(E|F)$?