CS104: Data Structures & Object-Oriented Programming
Summer 2021 - Midterm Exam
06/30/21, 10:00 AM – 12:00 PM + 15 min. to upload
(Submit on Gradescope by 12:15 PM)

[Complete all the information in the box below.]

Name: Solutions

Student ID: _____ Email: _____@usc.edu

Lecture section (Circle One):

Redekopp		
9:30 a.m.		

Ques.	Your score	Max score	Time
1		8	10 min.
2		9	12 min.
3		8	13 min.
4		7	20 min.
5		8	25 min.
6		10	40 min.
Total		50	

Note: The last page is scratch paper. Submit it with your exam

Data structures and algorithms that may be used for certain problems

In each problem you may assume the use of the following:

- `std::sort(vector<T>& values)` which will perform an n*log(n) sort in ascending order'

- `std::find(iterator first, iterator last, T target)` which finds the target in Θ(n)

Assume the various STL containers are available:

- **vector<T>:** Traditional array-based list with O(1) operations:
  - push_back(), back(), operator[]
- **singly<T>:** A singly-linked list (with tail pointer) with O(1) operations:
  - back(), front(), push_back(), push_front(), and pop_front()
- **queue<T>:** Implements a queue with O(1) operations:
  - front(), push(), and pop()
- **stack<T>:** Implements a stack with O(1) operations:
  - top(), push(), and pop()
- **set<T>**: Standard set implementation with O(log n) operations:
  - insert(), erase(), find()
- **map<K,V>**: Standard map implementation with O(log n) operations:
  - insert(pair<K,V>), erase(key), find(key)

1. **(8 pts.) Abstract Data Types**: Choose the best abstract data type (List, Set, Map, PQ, Queue, Stack) given the description of the desired data structures below. Be as specific as possible (don't answer List if a Queue is applicable). Also show what types would be used as template arguments (e.g. `map<string, int>` or `stack<double>`). If multiple "best" options exist, choose either. Give a **SHORT** 1-2 sentence justification for your choice (don't waste time on a long explanation...it won't help you get any more credit than a short answer).
    1.1. A structure to support the following: an academic advisor wants to track the `students` she advises each day over the course of a semester. Given a day, she'd like to quickly check if she advised a particular student and also be able to quickly go from one day's students to the next or previous day's students.

map<Date, set<student>>`. It is fine to use an `int` as the key since we can use integers for the date. And that also means `List<set<student>>` =>
Explanation: map or list so the order of days is maintained and Set of students (i.e. strings of the student names or IDs) so she can quickly check if she did or did not advise that student.

    1.2. A structure for storing the percentage of people vaccinated in each US state (and quickly be able to update that percentage).

Map<state , percentage> (i.e. map<string, double>)

    1.3. A structure to store the answers on a website like StackOverflow.com. The answers should be displayed in order of "upvotes" from highest number of upvotes to least.

Priority_Queue<Answer>

    1.4. A structure to track who is in each checkout/cashier lane and their order for an entire grocery store where `customers` may choose a checkout line (given by an integer) to then wait in.

map<int, queue<Customer> >` or List<queue<Customer>> since a list can generally be accessed by an integer ordinal.
How "Customer" is maintained can be further specified (potentially a set or list of items the customer has ordered).

2. Heaps (9 pts.)

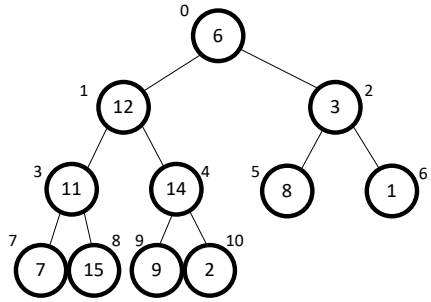
Given the following array numbers: `[6, 12, 3, 11, 14, 8, 1, 7, 15, 9, 2]` (the number `6` is in position `0` and `2` in position `10`), find the **min-heap** that is a result of calling the `make-heap` (`build-heap`) algorithm that uses *heapify* on the array and that runs in linear time.

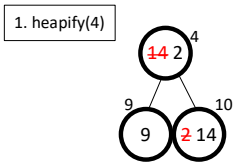
In the area below show the final array contents (similar to the format we showed above). For full credit, you must also show your work at each step in the algorithm by drawing the complete **tree** representation (not array form, but tree form) after each heapify step until the algorithm is complete. **Hint:** You don't need to call heapify on EVERY node but can start a certain location. For each call, you need only show the relevant part of the tree that heapify operats on. Gradescope: Upload a picture or PDF of your work in the file upload area below.

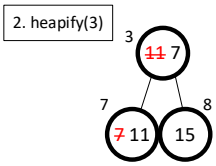
Final array contents after the algorithm completes:

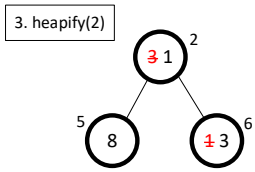
`[_1_, _2_, _3_, _7_, _9_, _8_, _6_, _11_, _15_, _12_, _14_]`

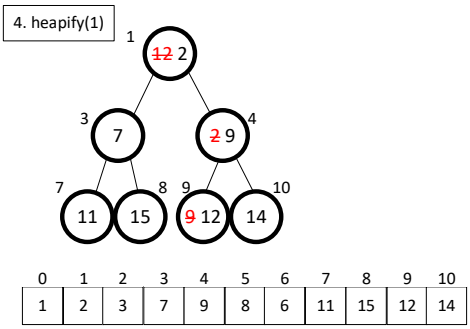
Diagrams of intermediate trees after each heapify call **(indicate what location you start your heapify call on)**.

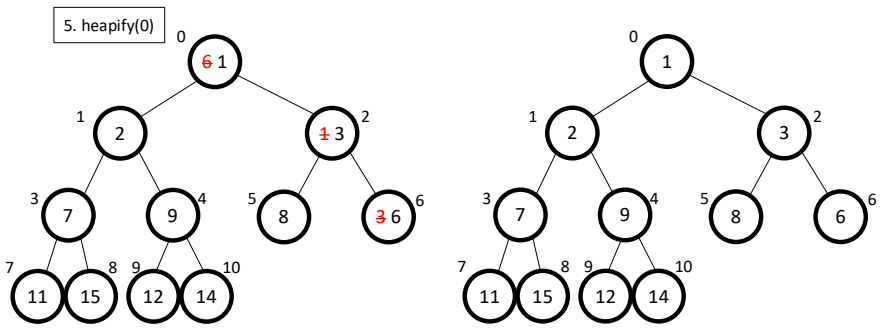












3. Linked Lists and Recursion (8 pts.)

Given the code in the image below answer the following questions regarding a call to **`llmystery(head, 2)`** assuming `head` points to the linked list of values:

1 2 3 4 5

Answer the following questions:

3.1. **True/False:** Will the original linked list be modified (values or pointers)? __**NO**_____

3.2. On the next page, show what will be printed by the code, and further show a call tree (box diagram) of the recursive calls (with relevant arguments) made during execution.

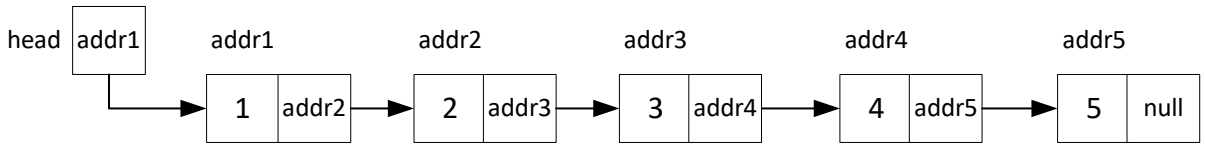
```
struct IntItem {
  int val;
  IntItem *next;
  IntItem(int v, IntItem* n) { val = v; next = n; }
};

void llmystery(IntItem* head, size_t loc);
int llmysteryHelper(IntItem* head, size_t loc, IntQueue& v);

void llmystery(IntItem* head, size_t loc)
{
  IntQueue q;
  int t = llmysteryHelper(head, loc, q);
  cout << t << endl; // *** COUT PRINT HERE ***
  while(!q.empty()) {
    cout << q.front() << " ";  // *** COUT PRINT HERE ***
    q.pop_front();
  }
  cout << endl;
}

int llmysteryHelper(IntItem* head, size_t loc, IntQueue& q)
{
  if(head == nullptr) {  return 0; }
  else if(loc > 0) {
    int t = llmysteryHelper(head->next, loc-1, q);
    q.push_back(head->val);
    return t;
  }
  else {
    q.push_back(head->val);
    return head->val + llmysteryHelper(head->next, loc, q) ;
  }
}
```

Show what will be printed by the call to `llmystery(head, 2)`

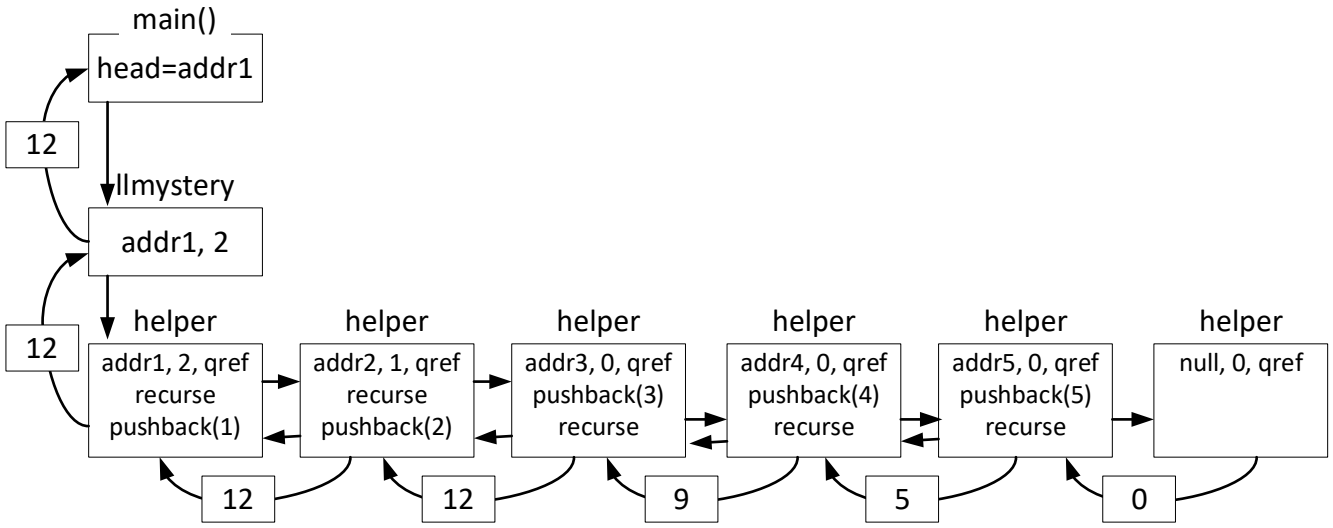
3 4 5 2 1

Upload a diagram of the call tree (box diagram) of the recursive calls made during execution.

Linked List



Call tree



Prints: 3 4 5 1 2

4. Class Organization, Heaps, and BSTs (7 pts.)

Complete your code in the provided `bst-heap.h` at the bottom of the file. You will **NOT be able to compile and test this code because we do not provide the implementation of the base `BST<Key>` class.** Assume it is provided and works. We will visually grade your MaxHeap<Key> class assuming the BST class works.

Suppose you are provided a complete, templated binary search tree (BST) class: **`BST<Key>`.** It does not necessarily guarantee balance. We will assume type `Key` supports all the basic comparison operators: `<`, `>`, `==`, `!=`, etc.

You are now asked to write a **`MaxHeap<Key>`** class to implement a max heap (priority queue). It should use the **`BST<Key>`** class to implement it (though how to structure the relationship between these two is part of the question and left to you to decide). You may assume no duplicate keys are added to the heap.

Your `MaxHeap<Key>` must implement the following public interface using the standard definitions of the `push`, `pop`, and `top` operations. If **top()** or **pop()** is called on an empty heap, **throw std::out_of_range exception.**

```
template <typename Key>
class Heap /* your choice */
{
public:
    Heap();
    ~Heap();
    void push(const Key& newKey);
    void pop();
    const Key& top() const;  // throws std::out_of_range if empty
private:
    // add any data members or helper functions as necessary
};
```

4.1. Finish the implementation of the class / functions in the provided `bst-heap.h`.

- Your implementation should utilize/re-use as much (as reasonable) of the `BST<Key>` implementation to avoid unnecessary code. (There may be some duplication in your Heap implementation, but be judicious...if a BST operation can already accomplish a task, try to use it)
- Your runtime does not need to match that of a traditional Heap

```
template <typename Key>
class Heap : public BST<Key> {
public:
    Heap();
    ~Heap();
    void push(const Key& newKey);
    void pop();
    const Key& top() const;  // throws std::out_of_range if empty
    bool empty() const;
private:
    // no data members necessary since we inherit BST
};

template<typename Key>
Heap<Key>::Heap()  // BST's constructor will be called automatically here
{
}

template<typename Key>
Heap<Key>::~Heap()
{
}// BST's destructor will be called automatically here

template<typename Key>
void Heap<Key>::push(const Key& newKey)
{  this->insert(newKey);  }
}

template<typename Key>
void Heap<Key>::pop()
{  this->remove(this->top());  }
}

template<typename Key>
const Key&
Heap<Key>::top() const
{
    if(nullptr == this->root_) {
        throw std::out_of_range("heap is empty");
    }
    Node<Key> *start = this->root_;
    while(start->right_ != nullptr)
    {
        start = start->right_;
    }
    return start->key_;
}

template<typename Key>
bool Heap<Key>::empty() const
{ return this->empty(); }
```

4.2. Analyze the runtime of your `top()` implementation (show a very short justification or work).

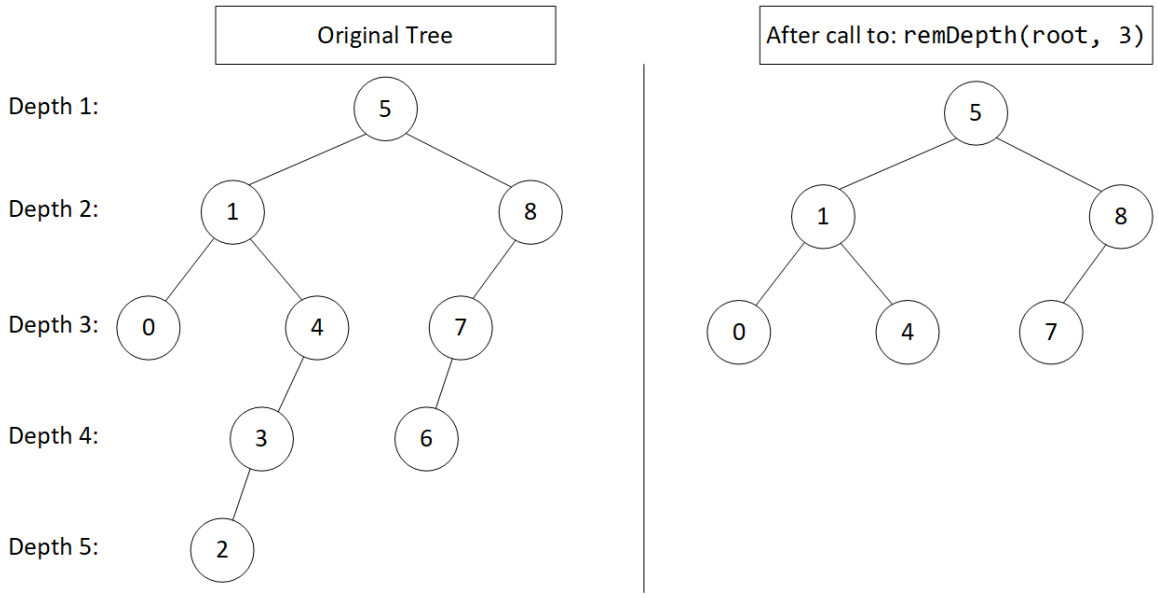
Answer: Theta(n)
Justification: Top() must move to the bottom right node of the BST. Since the BST does not guarantee balance this will be a linear runtime => $\sum_{i=1}^{height} 1 = \sum_{i=1}^{n} 1 = \theta(n)$

5. Binary Trees and Recursion (8 pts.)

Use the provided skeleton file `remdepth.cpp` to complete the code.

Write a recursive function: **`Node* remDepth(Node* root, int depth)`** to delete all nodes **`BELOW`** a certain depth from a **binary** tree (not necessarily BST), and return the pointer to the (potentially now NULL) root. Your implementation **MAY NOT** use loops anywhere. You may define helper function(s), as necessary.

An example, showing how we define depth and how the function should work is illustrated below.



Other examples: A call to **`remdepth(root, 0)`** would cause the entire tree to be deleted, in which case you should return **`nullptr`**. A call to **`remdepth(root, 1)`** would cause all nodes **except the root** to be deleted.

If no nodes are below the specified depth, simply do nothing (i.e. do not alter the tree in any way).

Your code should run in **Theta(n)**, where **n** is the number of nodes in the tree.

```
Node* remDepth(Node* root, int depth)
{
    if(root == nullptr) return nullptr;

    root->left = remDepth(root->left, depth-1);
    root->right = remDepth(root->right, depth-1);
    if(depth <= 0) {
        delete root;
        return nullptr;
    }
    else {
        return root;
    }
}
```

6. STL and ADTs (10 pts.)

Use the provided skeleton file `travel.cpp` to complete the code.

In this problem you may only use: `map<K,V>`, `set<K>`, `stack<T>`, and `queue<T>`. **No vector, deque, or list**.

Tommy, the forgetful tourist, enjoys travelling around the country. You will write a class **`Tourist`**, to help him plan and track his travels. This class will implement 4 primary functions:

```
addPlaceToVisit()
visitNextLocation()
currentLocation()
previousVisit()
```

Tommy talks to people who tell him about fun **locations** he should visit. Locations are represented by the name of the place and its x,y coordinates (e.g. latitude and longitude). He adds them to his "places to visit itinerary" via a call to **`addPlaceToVisit()`** which should quickly check if they are already on his itinerary of places to visit, and if not, store the location. When he's ready to travel to the next location he calls **`visitNextLocation()`** which chooses a new place to visit, removes it from his places to visit, and updates his current location. By default you can choose any place on his itinerary to go next, but for bonus points, you should ensure that he visits places on his itinerary in order from **west to east** (i.e. **`visitNextLocation()`** should always give the *west-most (lowest x-value)* unvisited location.

Further, he wants to track where he's visited because he often leaves one of his belongings somewhere he visited and needs to keep going back to previous locations to find his lost belongings. This process is accomplished by (potentially many) calls to **`previousVisit()`** which should retrace the locations he visited before his current location from **most-to-least** recently visited. To keep things simple, once he finds his item at a location, the next call to **`visitNextLocation()`** should resume his travels by going to the next **new** place to visit (rather than returning back through the same locations he was just at looking for his items).

As an example, suppose Tommy hears about locations: **B, A, C, A, E, D** and adds them to his places to visit (removing the duplicate **A**). By making multiple calls to **visitNextLocation** he visits to **A, B, C**, then **D** (let's assume these are in west-to-east order). At location **D** he realizes he left his USC hat somewhere. By making calls to **previousVisit()** he will retrace his steps to **C** and then **B** at which point he finds his hat. From there a call to **visitNextLocation** would yield the only unvisited location that remains on his places to visit: **E**. But if he arrives at E and realizes he also lost his earbuds, a new sequence of calls to **previousVisit()** should yield **D**, **C**, **B**, and so on. Thus, when we return to previously visited locations we cannot forget that order but must restore those locations when we start traveling to new locations via calls to **visitNextLocation**.

Any other behavior not specified here is left to your discretion. (You need not add any features we have not specified.) However, you must adhere to the runtime requirements given in the function prototypes in the **Tourist** class declaration.

```
// struct to store an x,y "geo"-location of a place to visit
struct Location {
    double x,y;
    string name;
    Location() : name("Home"), x(0), y(0) {}
    Location(string n, double myx, double myy) :
        name(n), x(myx), y(myy) {}
    bool operator<(const Location rhs) const
    {
        return x < rhs.x
            || (x == rhs.x && y < rhs.y)
            || (x == rhs.x && y == rhs.y && name < rhs.name);
    }
};
ostream& operator<<(ostream& os, const Location loc)
{
    os << loc.x << "," << loc.y << " " << loc.name;
}

// convenience typedef's for Stack and Queue
typedef stack<Location> Stack;
typedef queue<Location> Queue;

class Tourist {
public:
    // Default constructor -- should start from 0,0 "Home" location
    // (i.e. that should be your current location)
    Tourist();
    // Adds a location to visit if it does not already exist
    // Returns true if it existed already, false otherwise
    // Must run in O(log n)
    bool addPlaceToVisit(const Location& loc);
    // Returns the current Location
    // Must run in O(1)
    Location current() const;
    // Returns the new location visited just before the current location
    // if no previous location exists, returns the current location
    // Must run in O(1)
    Location previousVisit();
    // Goes to the next location.  Must run in O(log n)+O(r)
    // (r is the number of locations revisited
    //  since the last call to this function)
    Location visitNextLocation();
private:
```

```
    // Modify this as needed
    typedef set<Location> PlacesToVisitSet;
    PlacesToVisitSet toVisit_;
    // Add more data members as necessary...but no vector, deque, list
    Stack visited_;
    Stack retraced_;
};


// Add your implementation
Tourist::Tourist()

{
    visited_.push(Location());
}

// Add your implementation
bool Tourist::addPlaceToVisit(const Location& loc)
{
    PlacesToVisitSet::iterator it = toVisit_.find(loc);
    if(it == toVisit_.end()) {
        toVisit_.insert(loc);
        return false;
    }
    else {
        return true;
    }
}

// Add your implementation
Location Tourist::current() const
{
    return visited_.top();
}
```

```
// Add your implementation
Location Tourist::previousVisit()
{
    // If more than 1 previous location, go to previous but save it
    //  so we can retrace our steps
    if(visited_.size() > 1){
        retraced_.push(visited_.top()); // visited.top() = current()
        visited_.pop();
    }
    // If only 1 previous location, leave it (can't go to previous)
    return visited_.top(); //  visited.top() = current()

}

// Add your implementation
Location Tourist::visitNextLocation()
{
    // if we were retracing our steps, now add them all back
    while(!retraced_.empty()) {
        visited_.push(retraced_.top()); // top()
        retraced_.pop();
    }
    // get western most location
    PlacesToVisitSet::iterator it = toVisit_.begin();
    visited_.push(*it);
    toVisit_.erase(*it);
    return *it;
}
```

Intentionally blank for scratch work. Please turn it in with your exam:
Name: _____ Section time: _____