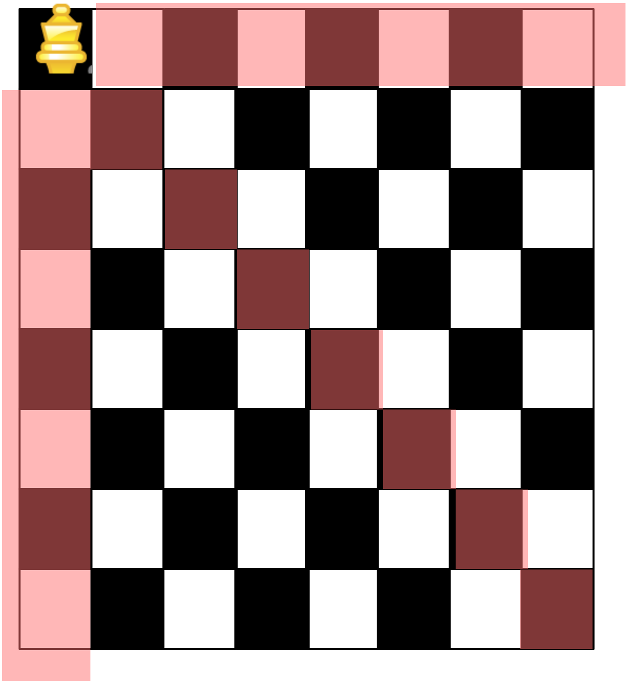
BACKTRACK SEARCH ALGORITHMS

Recursive Backtracking Search

- Recursion allows us to enumerate all **solutions** to some problem
- Backtracking algorithms are often used to solve **constraint satisfaction problems or optimization problems**
  - Find optimum solution that meet **some constraints**
- **Key property of backtracking search:**
  - Stop searching down a path at the first indication that constraints won't lead to a solution
- Knapsack problem
  - You have a set of products with a given weight and value. Suppose you have a knapsack that can hold N pounds, which subset of objects can you pack that **maximizes the value**.
  - Example:
    - Knapsack can hold 11 pounds
    - Product A: 7 pounds, $12 ea. Product B: 10 pounds, $18 ea.
    - Product C: 4 pounds, $7 ea. Product D: 2.4 pounds, $4 ea.
- Other examples:
  - Map Coloring, Satisfiability, Sudoku, N-Queens

N-Queens Problem

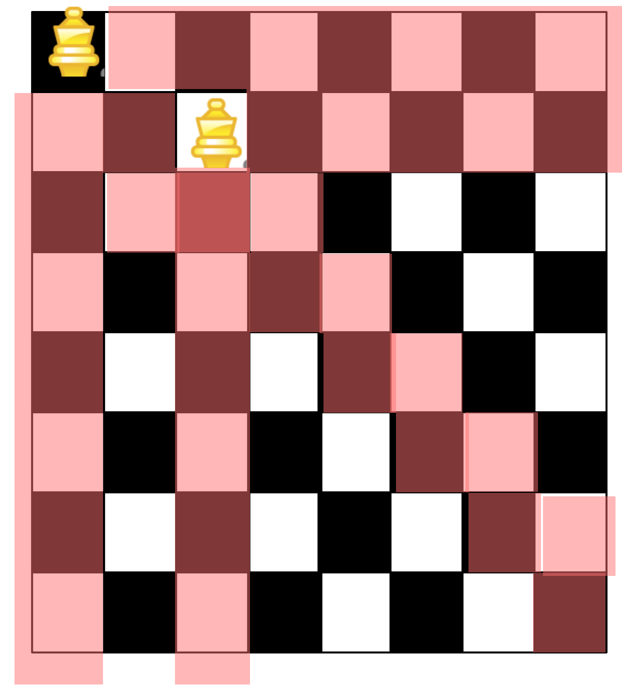
- Problem: How to place N queens on an NxN chess board such that no queens may attack each other
- Queens can attack at any distance vertically, horizontally, or diagonally
- Observation: Different queen in each row and each column
- Backtrack search approach:
  - Place 1st queen in a viable option then, then try to place 2nd queen, etc.
  - If no queen can be placed in row or there are no more options in row to try to place queen, backtrack to redo work in previous row



4x4 Example of N-Queens

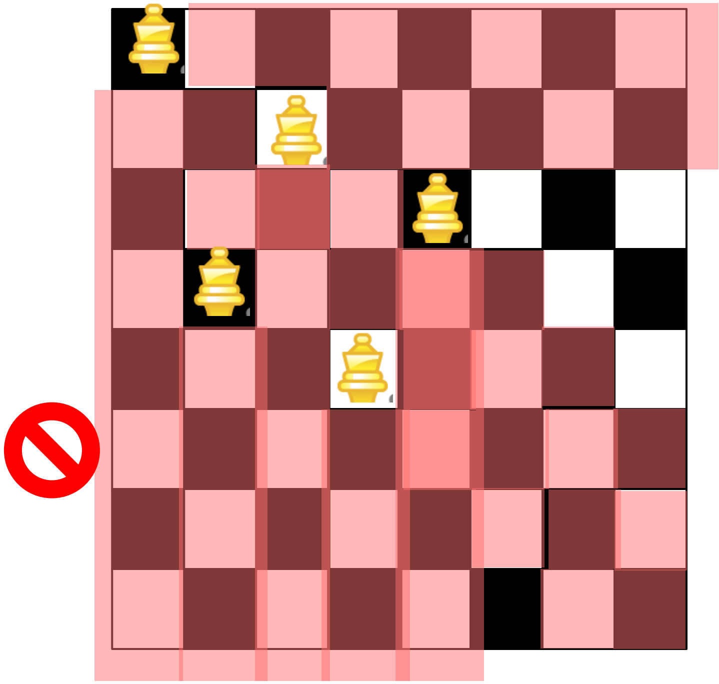
8x8 Example of N-Queens

- Now place 2nd queen



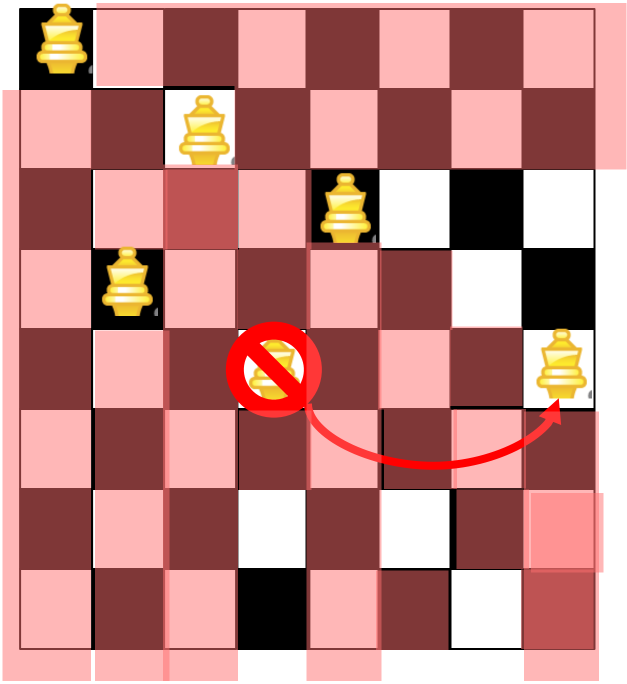
8x8 Example of N-Queens

- Now place others as viable
- After this configuration here, there are no locations in row 6 that are not under attack from the previous 5
- BACKTRACK!!!



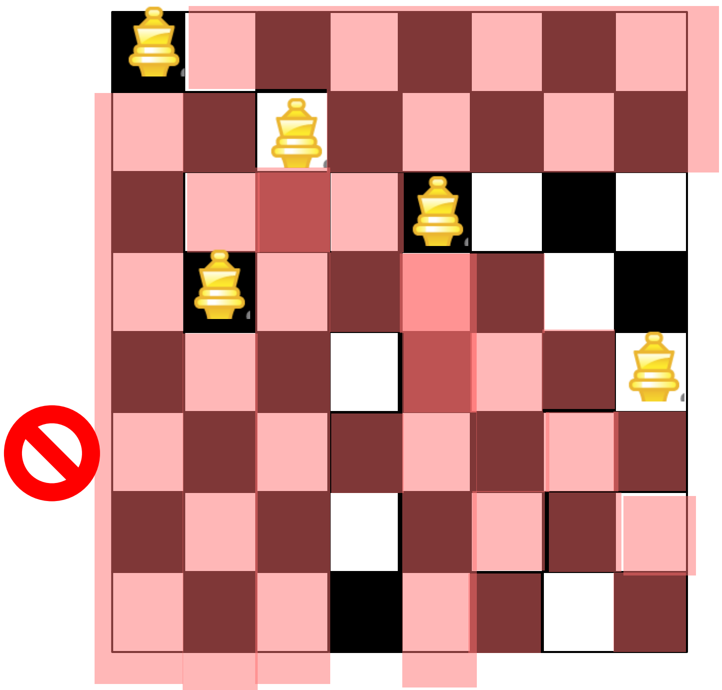
8x8 Example of N-Queens

- Backtrack: go back to row 5 and switch assignment to next viable option and progress back to row 6



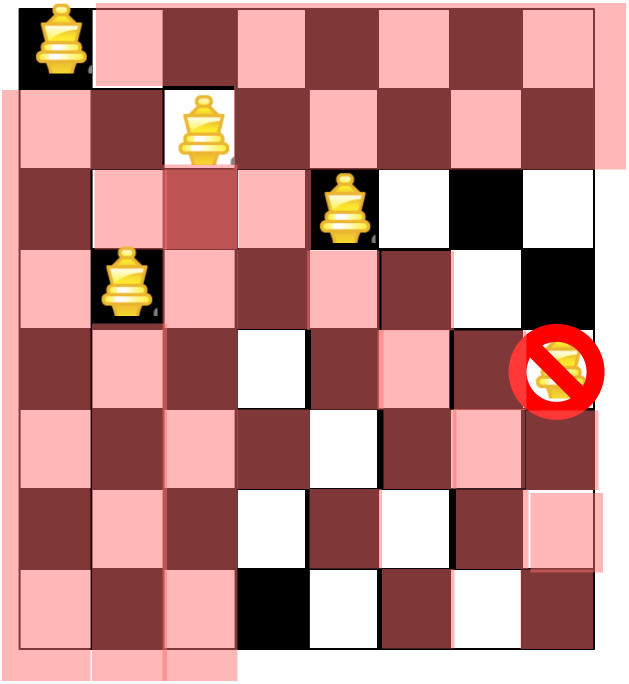
8x8 Example of N-Queens

- Still no location available in row 6, so BACKTRACK!
- Backtrack to row 5 to check for another location



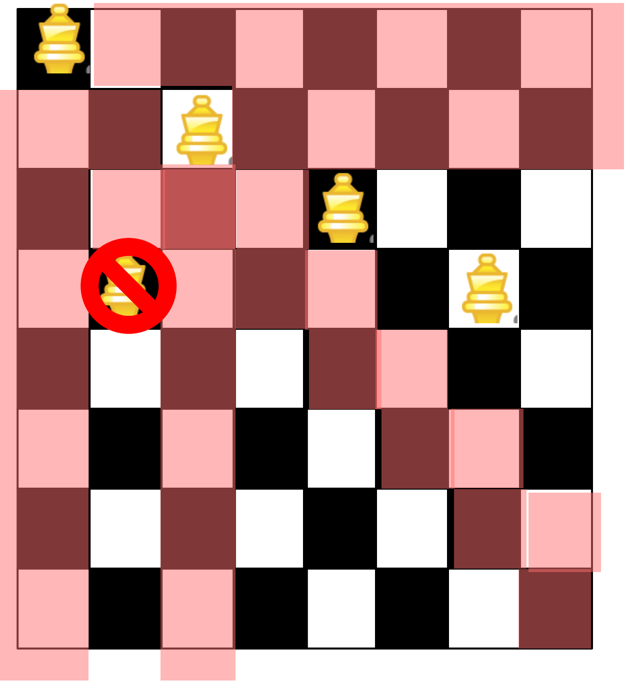
8x8 Example of N-Queens

- Backtrack: Check the next free location in row 5
- There are no more locations for queen in row 5 so BACKTRACK!



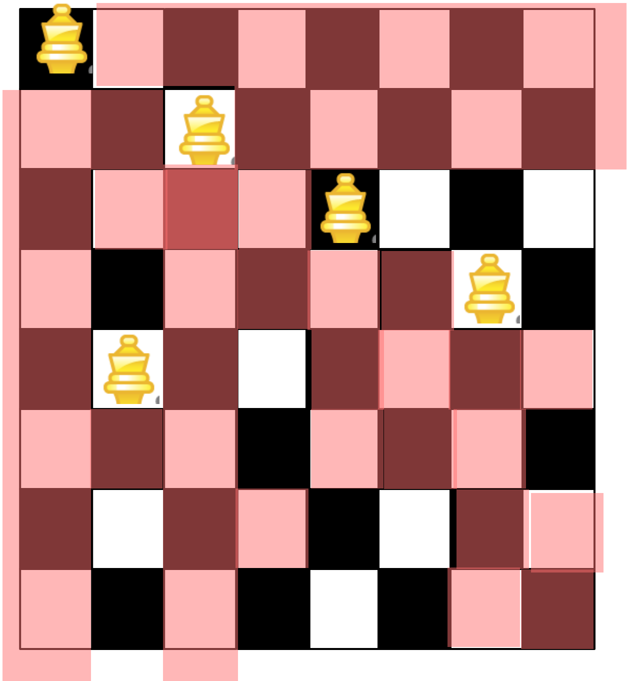
8x8 Example of N-Queens

- To backtrack, return back to row 4
- Move Queen in row 4 to another place in row 4 and restart row 5 exploration



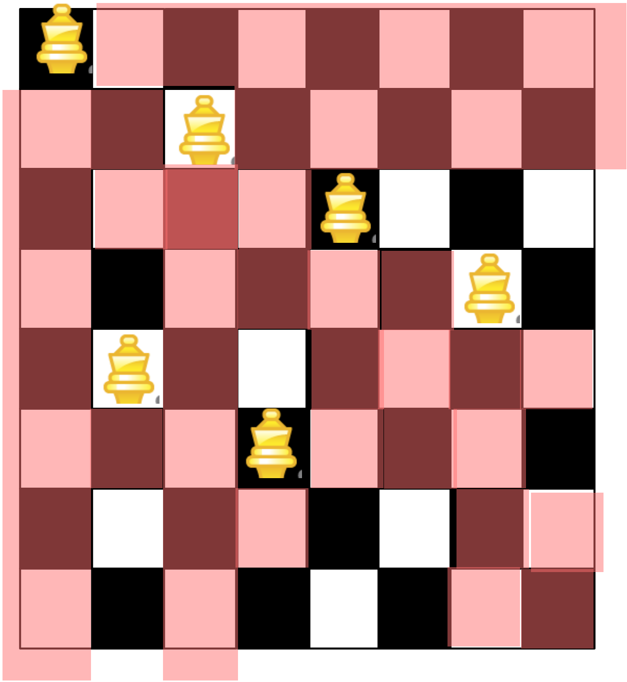
8x8 Example of N-Queens

- Move to another place in row 4 and restart row 5 exploration



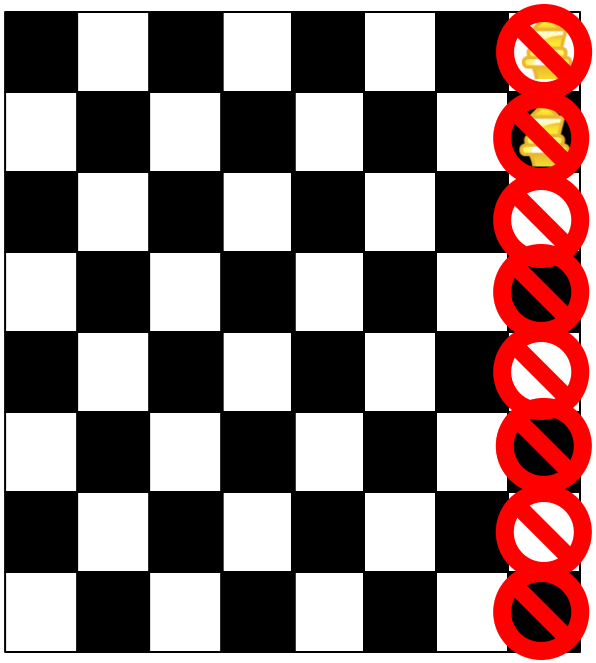
8x8 Example of N-Queens

- Now a viable location for a Queen exists for row 6
- Keep placing Queens until all rows including row 8 has Queen
- Return this configuration of Queens as solution
- What if no solution exists?



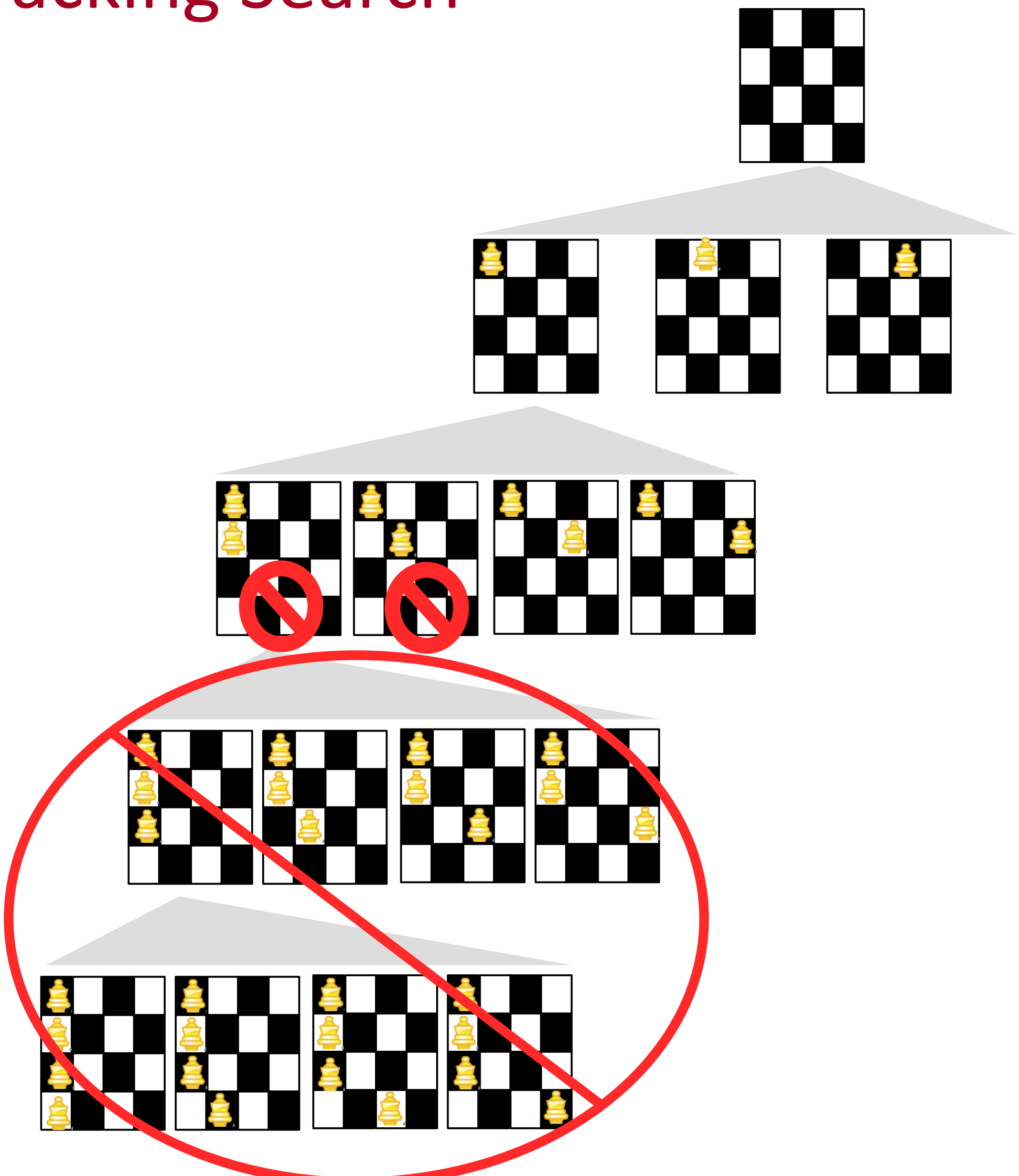
8x8 Example of N-Queens

- What if no solution exists?
  - Row 1 queen would have exhausted all her options and still not find a solution



Backtracking Search

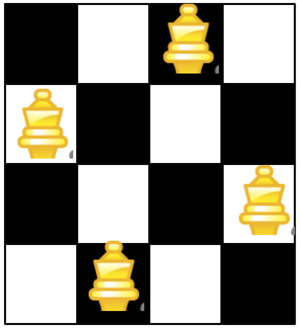
- Recursion can be used to generate all options
  - 'brute force' / test all options approach
  - Test for constraint satisfaction only at the bottom of the 'tree'
- But backtrack search attempts to 'prune' the search space
  - Rule out options at the partial assignment level



Brute force enumeration might test only when a complete assignment is made (i.e. all 4 queens on the board)

N-Queens Solution Development

- Let's develop the code
- 1 queen per row
  - Use an array where index represents the queen (and the row) and value is the column
- Start at row 0 and initiate the search [i.e. search(0)]
- Base case:
  - Rows range from 0 to n-1 so STOP when row == n
  - Solution found!
- Recursive case
  - Recursively try all column options for that queen



Index = Queen i in row i

q[i] = column of queen i

0	1	2	3
2	0	3	1

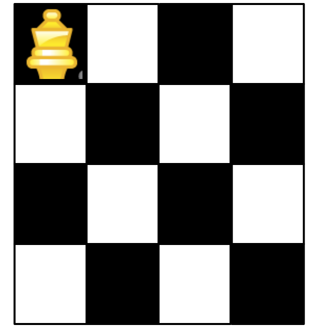
```
int *q;   // pointer to array storing
          // each queens location
int n;    // number of board / size

void search(int row)
{
  if(row == n)
    printSolution(); // solved!
  else {
   for(q[row]=0; q[row]<n; q[row]++){
     search(row+1);
  }
}
```

N-Queens Solution Development

- To check whether it is safe to place a queen in a column, keep a threat 2-D array indicating the threat level at each square on the board
  - Threat level of 0 means SAFE
  - Update squares under threat of queen placement

0
1
2
3

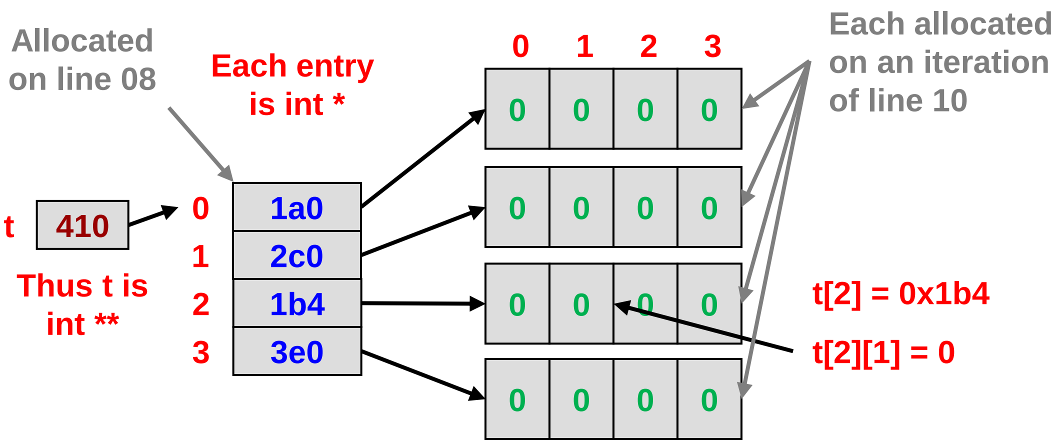


0	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1

Index = Queen i in row i

q[i] = column of queen i

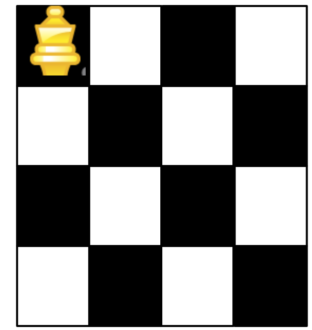
0	1	2	3
0			




```
00  int *q;   // pointer to array storing
01            // each queens location
02  int n;    // number of board / size
03  int **t; // thread 2D array
04
05  int main()
06  {
07    q = new int[n];
08    t = new int*[n];
09    for(int i=0; i < n; i++){
10      t[i] = new int[n];
11      for(int j = 0; j < n; j++){
12        t[i][j] = 0;
13      }
14    }
15    search(0); // start search
16    // deallocate arrays
17    return 0;
18  }
```

N-Queens Solution Development

- Check if queen placed is safe
- Update the threats (+1) due to this new queen placement
- Recurse to next row
- If return, no solution existed for given placement, so Backtrack!
- To Backtrack: i) remove threats and ii)iterate to try the next location for this queen



Index = Queen i in row i

q[i] = column of queen i

0	1	2	3
0			

```
int *q;  // pointer to array storing
         // each queens location
int n;   // number of board / size
int **t; // n x n threat array
void search(int row)
{
  if(row == n)
    printSolution(); // solved!
  else {
   for(q[row]=0; q[row]<n; q[row]++){
     // check that col: q[row] is safe
     if(t[row][q[row]] == 0){
       // if safe place and continue
       addToThreats(row, q[row], 1);
       search(row+1);
       // if return, remove placement
       addToThreats(row, q[row], -1);
} } }
```

t	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Safe to place queen in upper left

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	0	1	0
3	1	0	0	1

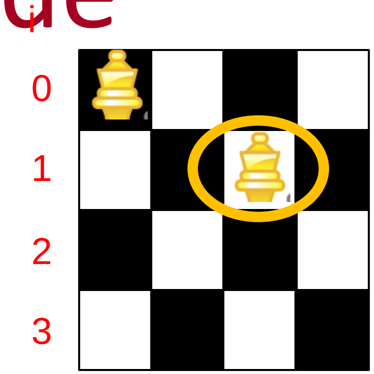
Now add threats

t	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Upon return, remove threat and iterate to next option

addToThreats Code

- Observations
  - Already a queen in every higher row so addToThreats only needs to deal with positions lower on the board
    - Iterate row+1 to n-1
  - Enumerate all locations further down in the same column, left diagonal and right diagonal
  - Add or remove a threat by passing in **change**
  - **Change** is +1 to add threats and -1 to remove threats



Index = Queen i in row i
q[i] = column of queen i

0	1	2	3
0			

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	0	1	0
3	1	0	0	1

t	0	1	2	3
0	0	1	1	1
1	1	1	0	0
2	1	1	2	1
3	2	0	1	1

```
void addToThreats(int row, int col, int change)
{
  for(int j = row+1; j < n; j++){
    // go down column
    t[j][col] += change;
    // go down right diagonal
    if( col+(j-row) < n )
       t[j][col+(j-row)] += change;
    // go down left diagonal
    if( col-(j-row) >= 0)
       t[j][col-(j-row)] += change;
  }
}
```

N-Queens Solution

```
int *q;   // queen location array
int n;    // number of board / size
int **t; // n x n threat array

int main()
{
  q = new int[n];
  t = new int*[n];
  for(int i=0; i < n; i++){
    t[i] = new int[n];
    for(int j = 0; j < n; j++){
      t[i][j] = 0;
    }
  }

  search(0);

  // deallocate arrays
  return 0;
}
```

```
void addToThreats(int row, int col, int change)
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    if( col-(j-row) >= 0)
       t[j][col-(j-row)] += change;
  }
}

void search(int row)
{
  if(row == n){
    printSolution(); // solved!
    return true;
  }
  else {
   for(q[row]=0; q[row]<n; q[row]++){
     // check that col: q[row] is safe
     if(t[row][q[row]] == 0){
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       addToThreats(row, q[row], 1);
       search(row+1);

       // if return, remove placement
       addToThreats(row, q[row], -1);
     }
    }

} }
```