CSCI 104L Lecture 28: Red-Black Trees

2-3-4 Trees

The natural extension of trinary search trees is **4-ary search trees**. All of the original ideas from 2-3 trees apply, except now you are allowed to have a 4-node with 3 values and 4 children.

When inserting, you only have a problem if you now have a "5-node" with 4 values and 5 children.

3-4-5 Trees

If you continue extending the maximum number of children, you also typically increase the minimum number of children. Let b' be the maximum number of children, and b be the minimum number of children. To ensure that a tree can always be formed, there are a few rules:

- The root is always allowed to have a minimum of 2 children.
- $b' \geq 2b - 1$

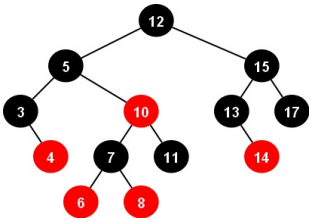
B-Trees

This is the generalization of the above idea, where you can plug in your choice of b' and b subject to the above constraints. These were used for indexing in very old computers (with very limited amounts of RAM) such that a single node with b' values could just barely fit into working memory.

Red-Black Trees

A Red-Black Tree is exactly a 2-3-4 tree that has been broken up into a binary search tree.

- Each node is either red or black.
- A red node can only have black children.
- For any path from root to leaf, there is an equal number of black nodes.
- The root is black.



Insert

Always insert a node x as a red leaf node. Note that this is just like adding a value to a 2-3-4 node. If its parent y is also red, this needs to be fixed.

1. If x and y are red, then y's parent z is black. The important part to note is whether z has a red sibling u or not.
2. If u doesn't exist, or isn't red, then you do exactly a rotation from AVL Trees (single or double, based on whether x, y, and z form a zig-zag or not). The new root of this subtree should become black, and its children should become red.
    - Note that no change is necessary, nor happens in the 2-3-4 tree analogue.
3. If u is red, then recolor y and u as black, and recolor z as red. If z's parent is black, then you're done.
    - This is exactly analogous to unzipping the tree.
4. If z is the root, just recolor it black, and you're done.
5. Otherwise, restart the process with z as x, either rotating or recoloring as needed.

Delete has about 10 rules, depending on which ruleset you use. These rules are all analogous to how you delete in a 2-3-4 tree.