

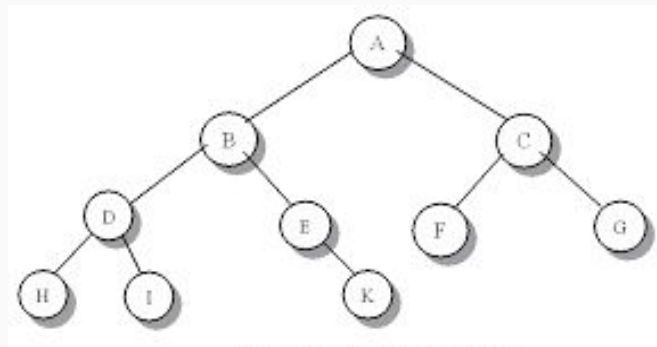
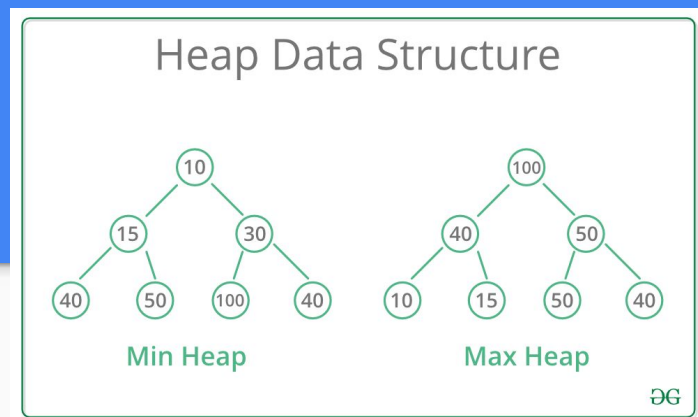
# Lab 11: BST and AVL

CSCI104



# REMEMBER: Heaps

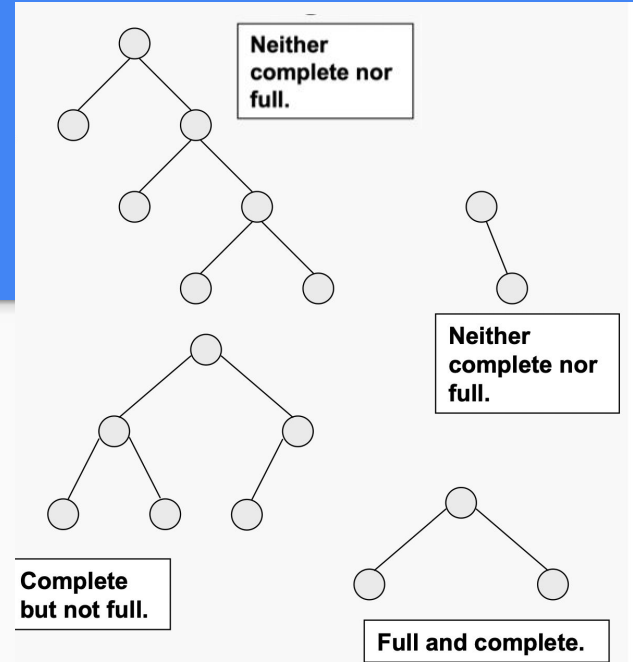
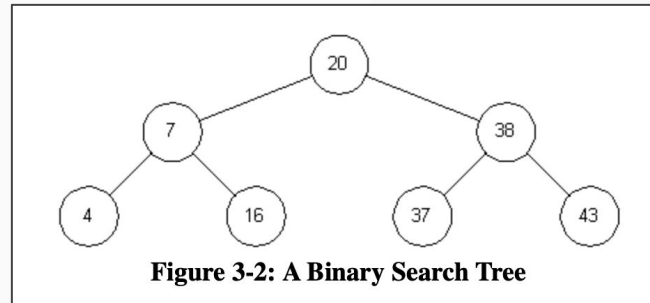
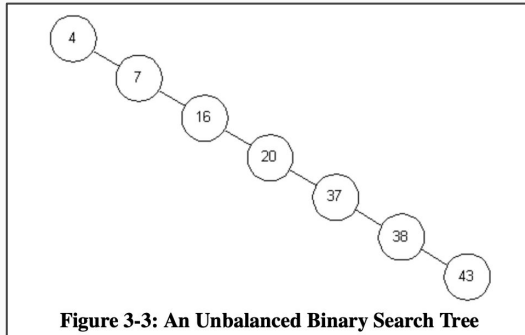
- COMPLETE d-ary tree
  - All levels except the last are completely filled
  - All leaves in last level are to the left side
- Every parent is “better” than both of its children
- Min Heap: node is less than or equal to all children
- Max Heap: node is greater than or equal to all children



Could this be a heap??

# Binary Search Trees (BST)

- Not necessarily a complete or full tree
- Left children (left subtree) hold values LESS THAN or equal to parent's values
- Right children (right subtree) hold values GREATER THAN parent's value



# Traversals: Pre-Order, In-Order, Post-Order

- All traversals operate on EVERY node eventually—just in different orders
  - “Pre” : visit the parent “pre-“ (before) visiting left and right sub-trees.
  - “In” : visit the parent “in”-between visiting left and right sub-trees.
  - “Post”: visit the parent “post-“ (after) visiting left and right sub-trees.

## Pre-Order Traversal

```
// Operate on current node
// Recurse left
// Recurse right
// return
```

## In-Order Traversal

```
// Recurse left
// Operate on current node
// Recurse right
// return
```

## Post-Order Traversal

```
// Recurse left
// Recurse right
// Operate on current node
// return
```

# Traversals in C++

For a BST, what is special about operating on elements using an in-order traversal? If we were printing integers using this traversal, what would the output look like?

```
void pre_order(Node* node) {  
    if (node == nullptr) return;  
    print(node);  
    pre_order(node->left);  
    pre_order(node->right);  
}
```

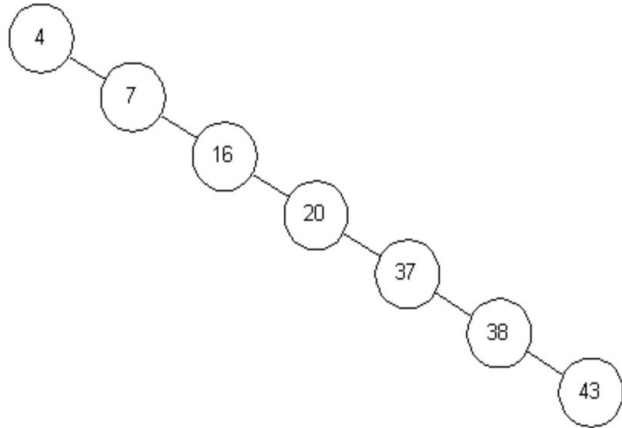
```
void in_order(Node* node) {  
    if (node == nullptr) return;  
    in_order(node->left);  
    print(node);  
    in_order(node->right);  
}
```

```
void post_order(Node* node) {  
    if (node == nullptr) return;  
    post_order(node->left);  
    post_order(node->right);  
    print(node);  
}
```

# Why BSTs? SEARCHING!

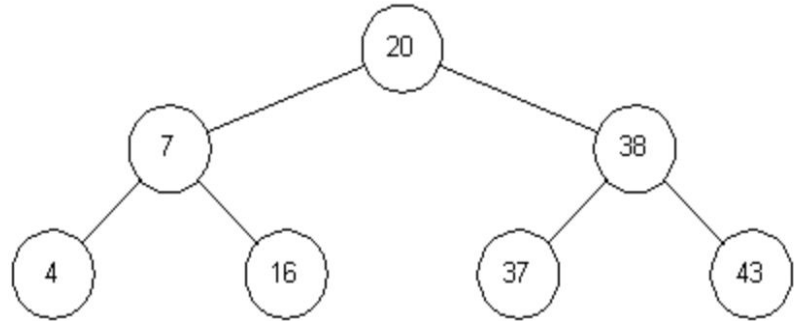
- Enable (potentially) faster searching
- Why do we say potentially? What is an example where the search is slow, even if it's a valid BST?

# Why BSTs? SEARCHING!



**Figure 3-3: An Unbalanced Binary Search Tree**

Slower search:  $O(n)$   
Basically like a linked list



**Figure 3-2: A Binary Search Tree**

Faster search:  $O(\log n)$

# Search Function

- Can do it iteratively or recursively

To search for key  $X$  in a BST, we compare  $X$  to the current node.

- If the current node is null,  $X$  must not reside in the tree.
- If  $X$  is equal to the current node, simply return the current node.
- If it is less than the current node, we check the left subtree.
- Else, it must be greater than the current node, so we check the right subtree.

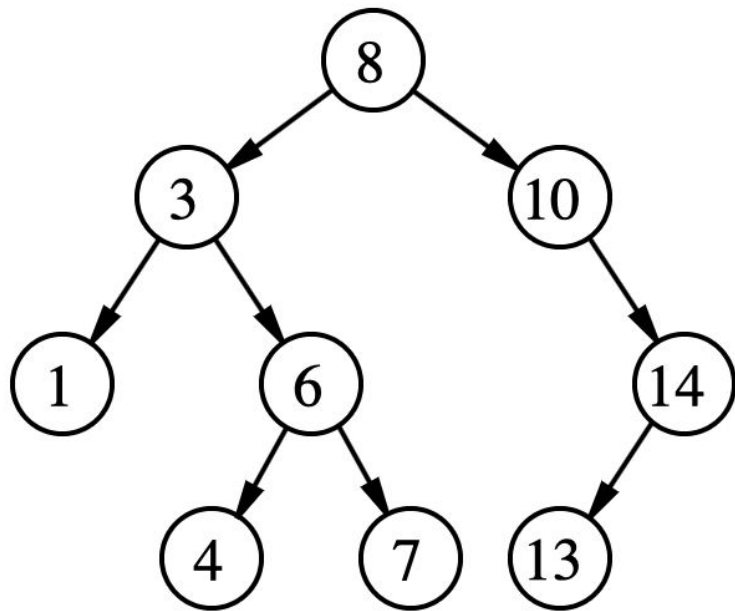
Or, in code:

```
// Finds the node with value == val inside the bst. Returns nullptr if not found
Node* find(Node* root, int val) {
    if (root == nullptr) return nullptr;
    if (root->val == val) return root;
    if (root->val > val) return find(root->left, val);
    return find(root->right, val);
}
```

Recursive  
example



# Search Example



Operation: `find(6)` // We begin at the root

Let's walk through this:

- Current node = 8,  $6 < 8$ , therefore go left.
- Current node = 3,  $6 > 3$ , therefore go right.
- Current node = 6,  $6 = 6$ , we've found the node.

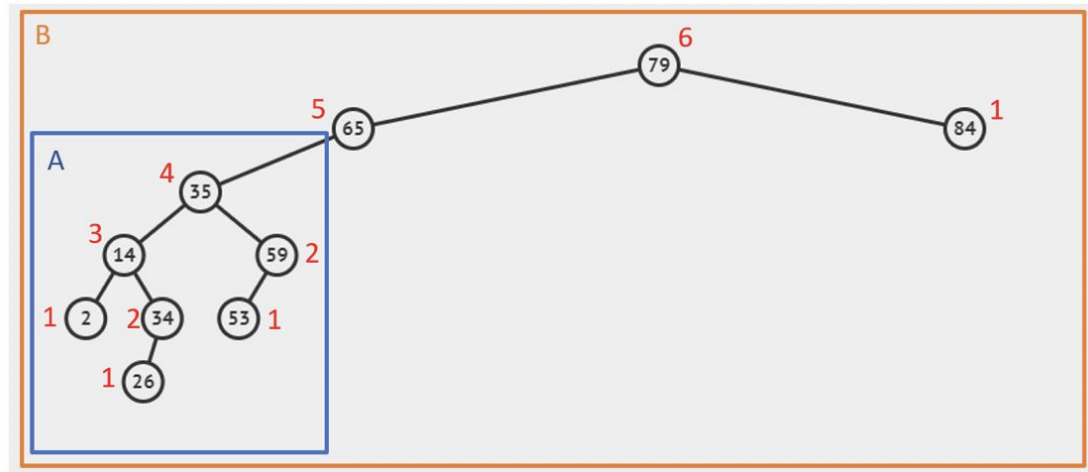
Operation: `find(0)` // We begin at the root

Let's walk through this one too:

- Current node = 8,  $0 < 8$ , therefore go left.
- Current node = 3,  $0 < 3$ , therefore go left.
- Current node = 1,  $0 < 1$ , therefore go left.
- Current node = null. 0 is not in the tree.

# Balanced Binary Tree

- **Height-balancing property:** heights of each subtree differ by no more than 1
- Avoids the slower search times!
- Keeps the height of the tree  $\log(n)$



A is balance, B is not

# Maintaining BST Property

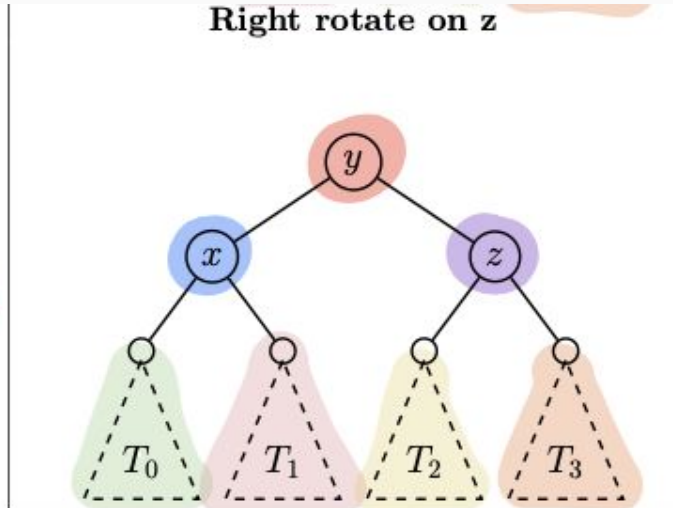
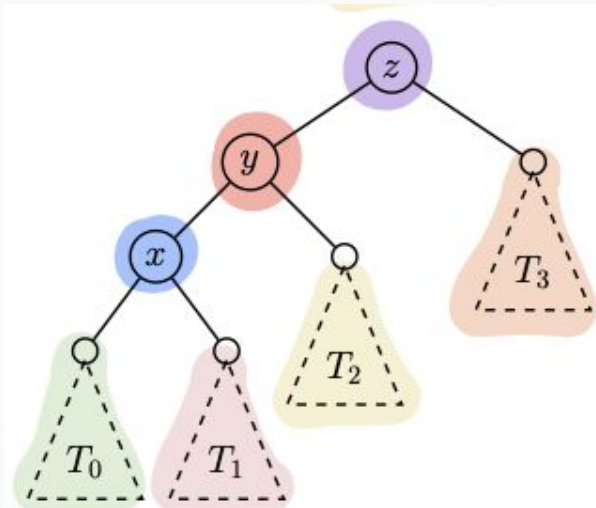
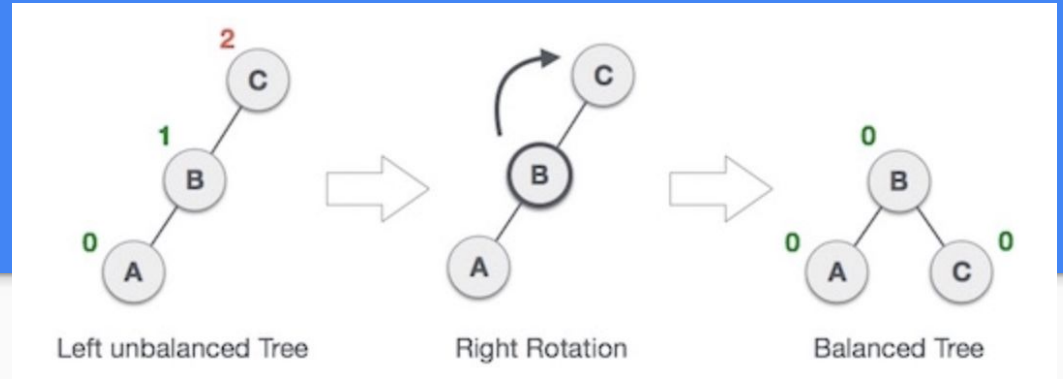
- REMEMBER: BST Property = left subtree node keys less than parent's and right subtree node keys greater than parent's
- Maintained by **smart** insertion and deletion
- Insert function
  - Traverse the tree based on key to be inserted
  - Insert once you encounter a situation where you cannot traverse further
- Remove function
  - Need to choose which node to promote
  - If node you want to remove has 0 children: just remove it
  - If node you want to remove has 1 child: promote the child of the node
  - If node you want to remove has 2 children: swap with its predecessor OR successor

# Self-Balancing BSTs

- We will be focusing on AVL trees
- You keep the tree balanced even after insertions or deletions
- This involves using rotations!
  - Foundation of AVL trees

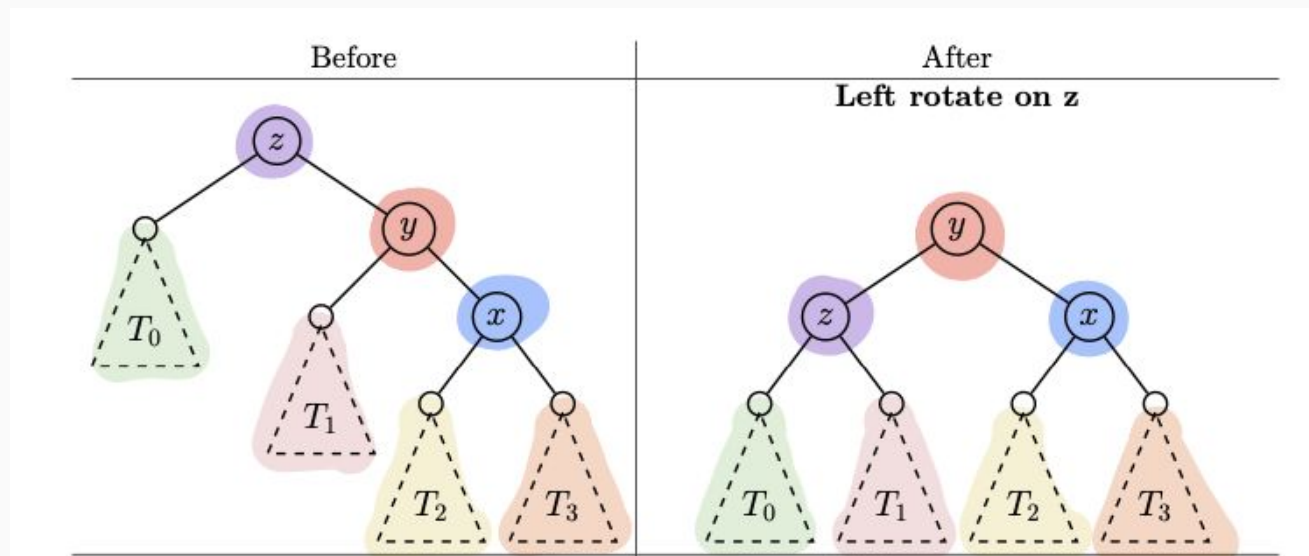
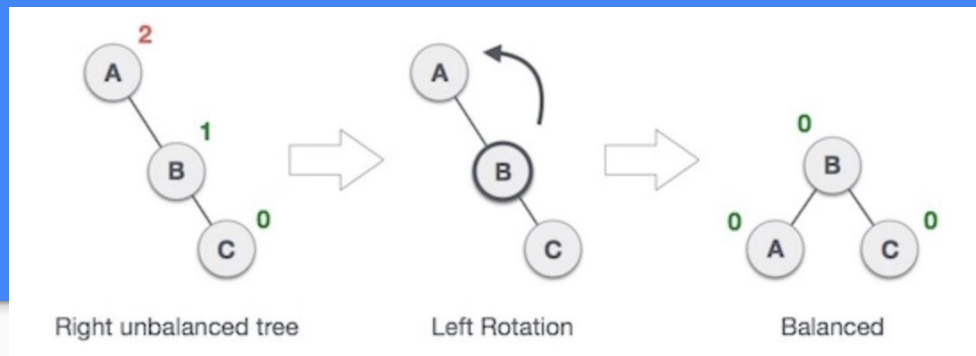
# Single Rotations

## RIGHT



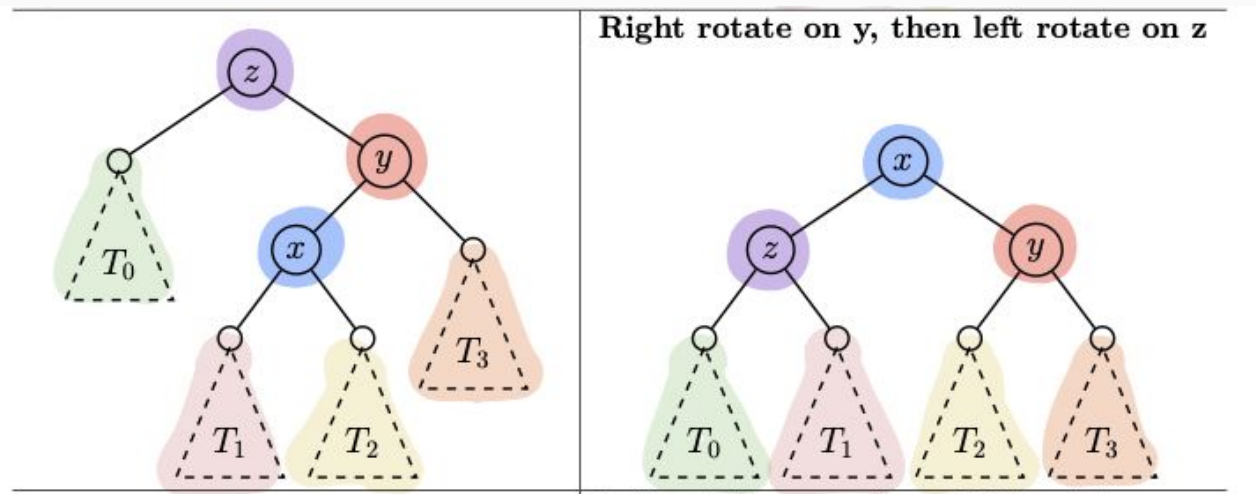
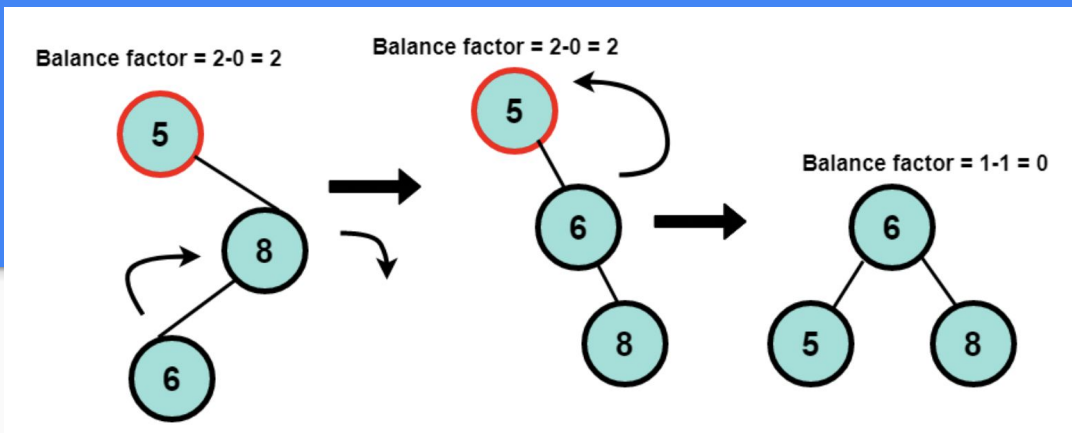
# Single Rotations

## LEFT



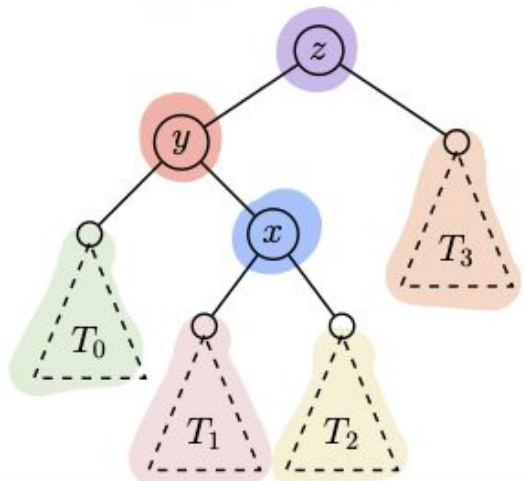
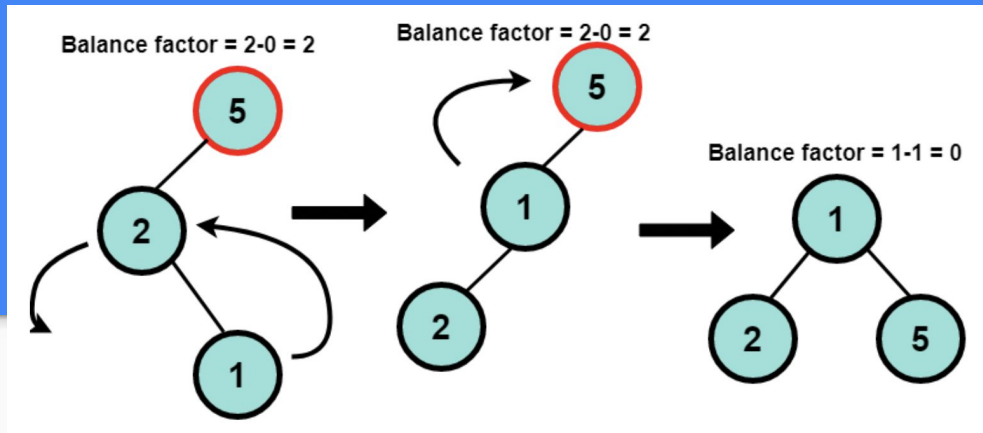
# Double Rotations

## RIGHT LEFT

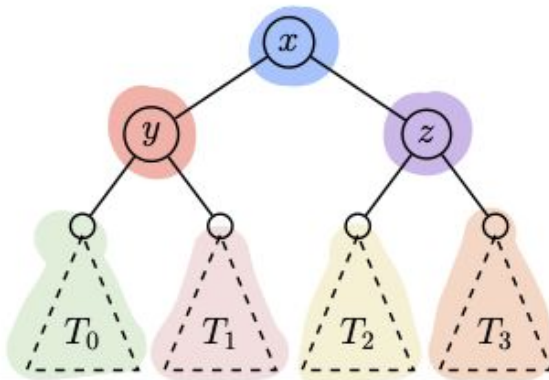


# Double Rotations

## LEFT RIGHT



Left rotate on y, then right rotate on z





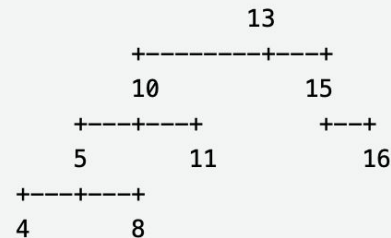
# AVL Insert and Remove

- Insert
  - Insert as you would in a BST
  - Fix the tree if it is unbalanced after inserting the node (ROTATION)
    - Need at most 1 rotation (either a single or double rotation)
- Remove
  - Remove as you would in a BST
  - Keep traversing up the tree and fixing tree if unbalanced (ROTATIONS)
    - You may need multiple rotations to fully fix the tree

# The Lab

- Draw or type out operations on tree
- Try our best to not do it with your neighbors; you will **need to personally understand this** for the next PA!

Initial Tree



Insert 14

Insert 3

Remove 3

Remove 4