

1. Write a set of inference rules that allow us to work through a simple addition example.

$$e ::= n \mid e_1 + e_2$$

$$\text{DoPlus} \frac{n' = n_1 + n_2}{n_1 + n_2 = n'}$$

$$\text{SearchPlus1} \frac{e_1 \rightarrow e_1'}{e_1 + e_2 \rightarrow e_1' + e_2}$$

$$\text{SearchPlus2} \frac{e_2 \rightarrow e_2'}{n_1 + e_2 \rightarrow n_1 + e_2'}$$

Example: $(4+2) + (3+6)$

$$\overset{\text{expr}}{e_1} : 4+2, \overset{\text{expr}}{e_2} : 3+6$$

$$\text{SearchPlus1} : 4+2 \rightarrow 6$$

$6 + (3+6)$

$$\overset{\text{number}}{n_1} : 6, \overset{\text{expr}}{e_2} : 3+6$$

$$\text{SearchPlus2} : 3+6 \rightarrow 9$$

$6 + 9$

$$\overset{\text{number}}{n_1} : 6, \overset{\text{number}}{e_2} : 9$$

$$\text{DoPlus} : 6+9 = 15$$

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Order of operations: e_1 must always be evaluated to a number first because of the n_1 term in SearchPlus2.

2. Write a set of inference rules such that the evaluation of e_1 / e_2 short-circuits if e_2 is zero.

$$e ::= n \mid x \mid e_1 + e_2 \mid e_1 / e_2 \mid e_1 - e_2$$

$$\text{DoZeroDiv} \frac{n_2 = 0}{e_1 / n_2 \rightarrow \text{undefined}}$$

$$\text{DoDiv} \frac{n' = n_1 / n_2 \quad n_2 \neq 0}{n_1 / n_2 \rightarrow n'}$$

$$\text{SearchDiv1} \frac{e_2 \rightarrow e_2'}{e_1 / e_2 \rightarrow e_1 / e_2'}$$

$$\text{SearchDiv2} \frac{e_1 \rightarrow e_1'}{e_1 / n_2 \rightarrow e_1' / n_2}$$

Example: $(4+2)/(3-3)$

$$\begin{array}{cc} \text{expr} & \text{expr} \\ e_1: 4+2, & e_2: 3-3 \end{array}$$

$$\text{SearchDiv1}: 3-3 \rightarrow 0$$

$$(4+2)/0$$

$$\begin{array}{cc} \text{expr} & \text{number} \\ e_1: 4+2, & n_2: 0 \end{array}$$

$$\text{DoZeroDiv}: n_2 = 0$$

undefined

Order of operations: e_2 must always be evaluated to a number before e_1 can start evaluating due to the n_2 term in SearchDiv 2.

Short-circuiting: If $n_2 = 0$, the evaluation can return early using DoZeroDiv and does not have to evaluate e_1 at all as seen in the example above.