CS 188 Spring 2011 | Introduction to Artificial Intelligence | Section 9 Solutions HMMs and Particle Filtering

Q1. HMMs: Tracking a Jabberwock

You have been put in charge of a Jabberwock for your friend Lewis. The Jabberwock is kept in a large tugley wood which is conveniently divided into a 10×10 grid. It wanders freely around the 100 possible cells. At each time step $t = 1, 2, 3, \ldots$, the Jabberwock is in some cell $X_t \in \{1, \ldots, 10\}^2$, and it moves to cell X_{t+1} randomly as follows: with probability 0.5, it chooses one of the (up to 4) valid neighboring cells uniformly at random; with probability 0.5, it uses its magical powers to teleport to a random cell uniformly at random among the 100 possibilities (it might teleport to the same cell). It always starts in $X_1 = (1, 1)$.

(a) Compute the probability that the Jabberwock will be in $X_2 = (2, 1)$ at time step 2. What about $\Pr(X_2 = (4, 4))$?

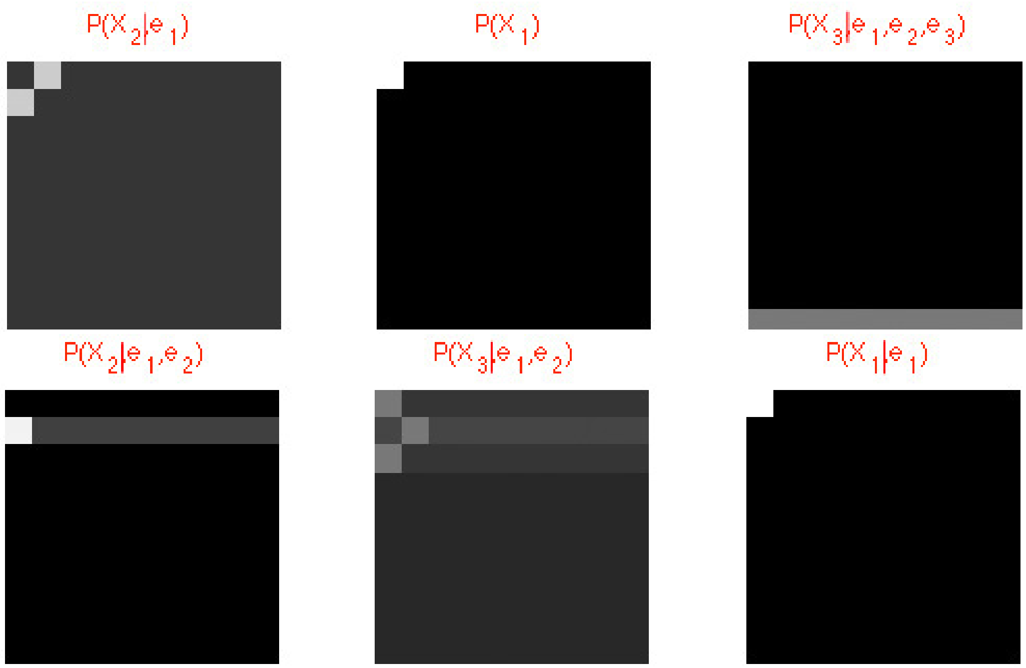
$P(X_2 = (2, 1)) = 1/2 \cdot 1/2 + 1/2 \cdot 1/100 = 0.255$

$P(X_2 = (4, 4)) = 1/2 \cdot 1/100 = 0.005$

(b) At each time step t, you dont see X_t but see E_t, which is the row that the Jabberwock is in; that is, if $X_t = (r, c)$, then $E_t = r$. You still know that $X_1 = (1, 1)$. Suppose we see that $E_1 = 1$, $E_2 = 2$ and $E_3 = 10$. Fill in the following table with the distribution over X_t after each time step, taking into consideration the evidence. Your answer should be concise. *Hint*: you should not need to do any heavy calculations.

t	$P(X_t \mid e_{1:t-1})$	$P(X_t \mid e_{1:t})$
1	(1,1) : 1.0, (others) : 0.0	(1,1): 1.0, (others): 0.0
2	(1,2), (2,1): 51/200, (others): 1/200	(2,1): 51/60, (2,2..10): 1/60

(c) These images correspond to probability distributions of the Jabberwock's location. Match them with the most appropriate probabilities from this list $P(X_1), P(X_1|e_1), P(X_2|e_1), P(X_2|e_1, e_2), P(X_3|e_1, e_2), P(X_3|e_1, e_2, e_3)$



Q2. Jabberwock in the wild

Lewis' Jabberwock is in the wild: its position is in a two-dimensional discrete grid, but this time the grid is not bounded. In other words, the position of the Jabberwock is a pair of integers $z = (x, y) \in \mathbb{Z}^2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \times \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.

At each time step $t = 1, 2, 3, \ldots$, the Jabberwock is in some cell $Z_t = z \in \mathbb{Z}^2$, and it moves to cell Z_{t+1} randomly as follows: with probability $1/2$, it stays where it is, otherwise, it chooses one of the four neighboring cells uniformly at random (no teleportation is allowed).

(a) Write the transition probabilities.

$$P(Z_{t+1} = (x', y') | Z_t = (x, y)) = \begin{cases} \frac{1}{2} & \text{if } x' = x, y' = y \\ \frac{1}{8} & \text{if } |x - x'| + |y - y'| = 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) Use a particle filter to track the Jabberwock.

As a source of randomness use values in order from the following sequence $\{a_i\}_{1 \le i \le 14}$ of numbers generated independently and uniformly at random from $[0, 1)$:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}
0.142	0.522	0.916	0.792	0.703	0.231	0.036	0.859	0.677	0.221	0.156	0.249	0.934	0.679

At each time step t you get an observation of the x coordinate R_t in which the Jabberwock sits, but it is a *noisy observation*. Given that the true position is $Z_t = (x, y)$, you observe the correct value $R_t = x$ according to the following probability:

$$P(R_t = r | Z_t = (x, y)) \propto 0.5^{|x-r|}$$

To sample transitions from a random number use the following table:

$[0; 0.5)$	Stay
$[0.5; 0.625)$	Up
$[0.625; 0.75)$	Left
$[0.75; 0.875)$	Right
$[0.875; 1)$	Down

Suppose that you know that half of the time, the Jabberwock starts at $z_1 = (0, 0)$, and the other half, at $z_1 = (1, 1)$. Now, you get the following observations: $R_1 = 1, R_2 = 0, R_3 = 1$. Fill in the following table:

To get you started, since $a_1 \in [0; 0.5)$ and $a_2 \in [0.5; 1)$ the two particles get initialized at $z_1 = (0, 0)$ and $z_1 = (1, 1)$ respectively.

$t = 1$	Prior $P(Z_1)$	Weights $\propto P(R_1 = 1 \vert Z_1)$	Resampling $P(Z_1 \vert R_1 = 1)$
Particle 1:	$z_1 = (\ 0\ ,\ 0\ )$	$w_1 = \propto 1/2 = 1/3$	$z_1 = (1, 1)$
Particle 2:	$z_1 = (\ 1\ ,\ 1\ )$	$w_2 = \propto 1 \quad = 2/3$	$z_1 = (1, 1)$
Used random samples:	a_1, a_2		a_3, a_4

$t = 2$	Transition $P(Z_2 \mid Z_1)$	Weights $\propto P(R_2 = 0 \mid Z_2)$	Resampling $P(Z_2 \mid Z_1, R_2 = 0)$
Particle 1:	$z_2 = (0, 1)$"Left"	$w_1 = \propto 1 \quad = 2/3$	$z_2 = (0, 1)$
Particle 2:	$z_2 = (1, 1)$"Stay"	$w_2 = \propto 1/2 = 1/3$	$z_2 = (1, 1)$
Used random samples:	a_5, a_6		a_7, a_8

$t = 3$	Transition $P(Z_3 \mid Z_2)$	Weights $\propto P(R_3 = 1 \mid Z_3)$	Resampling $P(Z_3 \mid Z_2, R_3 = 1)$
Particle 1:	$z_3 = (-1, 1)$"Left"	$w_1 = \propto 1/4 = 1/5$	$z_3 = (-1, 1)$
Particle 2:	$z_3 = (1, 1)$"Stay"	$w_2 = \propto 1 \quad = 4/5$	$z_3 = (1, 1)$
Used random samples:	a_9, a_{10}		a_{11}, a_{12}

(c) Use your samples (the unweighted particles in the last step) to evaluate the posterior probability that the x-coordinate of Z_3 is different than the column of Z_3, i.e. $X_3 \neq Y_3$.

Exactly one of the two paths satisfies this property, so the estimate is 1/2.

(d) What is the problem of using the elimination algorithm instead of a particle filter for tracking Jabberwock?

Factors of infinite size would need to be stored.