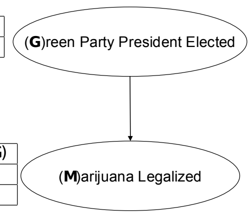
CS188 Fall 2013 Section 6: Bayesian Networks

1 Green Party President

It's election year again! In a parallel universe the Green Party is running for presidency. Pundits believe that Green Party presidents are more likely to legalize marijuana than candidates from other parties, but legalization could occur under any administration. Armed with the power of probability, the analysts model the situation with the Bayes Net below.

	+g	-g
P(G)	0.1	0.9



	P(+m | G)	P(-m | G)
+g	0.667	0.333
-g	0.25	0.75

1. Fill in the joint probability table over G and M.

G	M	P(G, M)
+g	+m	1/15
+g	-m	1/30
-g	+m	9/40
-g	-m	27/40

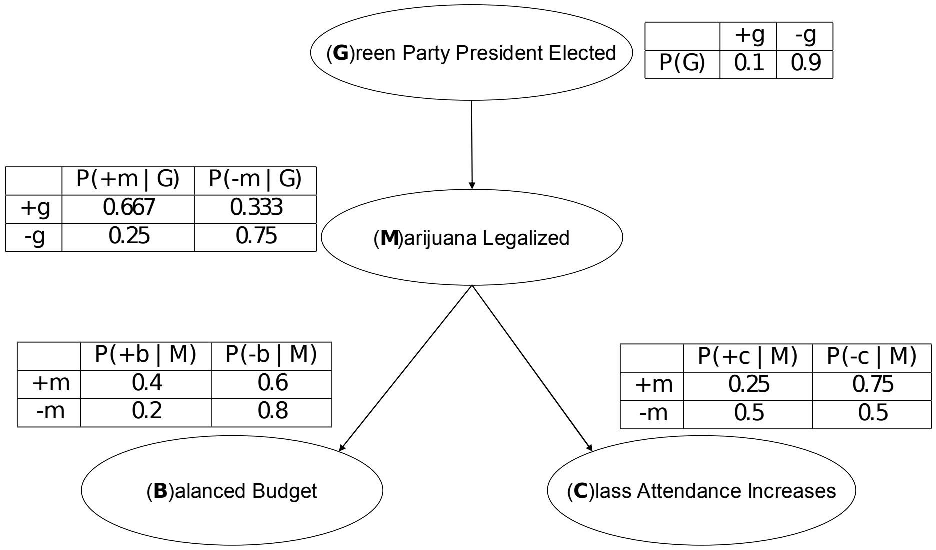
2. What is P(+m), the marginal probability that marijuana is legalized?

$P(+m) = P(+m, +g) + P(+m, -g) = P(+m \mid +g)P(+g) + P(+m \mid -g)P(-g) = \frac{2}{3} \cdot \frac{1}{10} + \frac{1}{4} \cdot \frac{9}{10} = \frac{7}{24}$

3. News agencies air 24/7 coverage of the recent legalization of marijuana (+m), but you can't seem to find out who won the election. What is the conditional probability P(+g | +m) that a Green Party president was elected?

$$P(+g \mid +m) = \frac{P(+g, +m)}{P(+m)} = \frac{P(+m \mid +g)P(+g)}{P(+m)} = \frac{\frac{2}{3} \cdot \frac{1}{10}}{\frac{7}{24}} = \frac{8}{35}$$

We can make better inferences if we observe more evidence. On the next page, we will expand on the model (Bayes net) by introducing two new random variables: whether the budget is balanced (B), and whether class attendance increases (C). The expanded Bayes net and conditional distributions are shown below.



4. The full joint distribution is given below. Fill in the missing values.

G	M	B	C	$P(G, M, B, C)$	G	M	B	C	$P(G, M, B, C)$
+	+	+	+	1/150	-	+	+	+	9/400
+	+	+	-	1/50	-	+	+	-	27/400
+	+	-	+	1/100	-	+	-	+	27/800
+	+	-	-	3/100	-	+	-	-	81/800
+	-	+	+	1/300	-	-	+	+	27/400
+	-	+	-	1/300	-	-	+	-	27/400
+	-	-	+	1/75	-	-	-	+	27/100
+	-	-	-	1/75	-	-	-	-	27/100

1. Compute the following quantities. You may use either the full joint distribution or the conditional tables, whichever is more convenient.

2. P(+b | +m) = $\frac{4}{10}$ (directly from conditional)

3. P(+b | +m, +g) = $\frac{4}{10}$ (also directly from conditional, since $B \perp\!\!\!\perp G | M$)

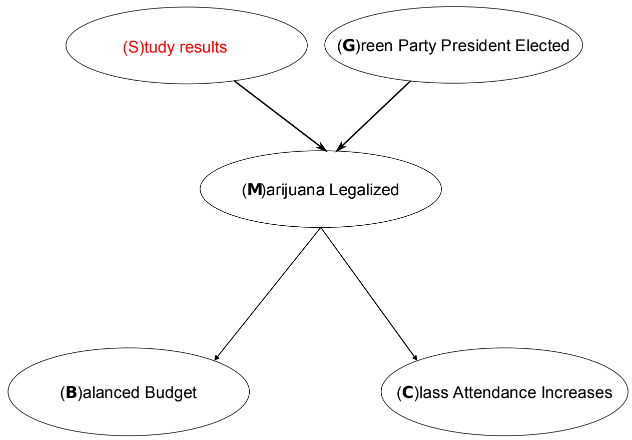
4. P(+b) = $\sum_{G,M,C}$ P(G,M,+b,C) = $\frac{8}{1200} + \frac{24}{1200} + \frac{4}{1200} + \frac{4}{1200} + \frac{27}{1200} + \frac{81}{1200} + \frac{81}{1200} + \frac{81}{1200} = \frac{31}{120}$

   (summed from joint)

5. P(+c | +b) = $\frac{\text{P(+b, +c)}}{\text{P(+b)}} = \frac{\sum_{G,M} \text{P(G,M,+b,+c)}}{31/120} = \left(\frac{8}{1200} + \frac{4}{1200} + \frac{27}{1200} + \frac{81}{1200}\right) \cdot \frac{120}{31} = \frac{12}{31}$

   (summed from joint)

1. Now, add a node S to the Bayes net that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not directly influence B or C. Draw the new Bayes net below. Which CPT or CPT's need to be modified?



$P(M|G)$ will become $P(M|G, S)$, and will contain 8 entries instead of 4.

2. Consider your augmented model. Just based on the structure, which of the following are guaranteed to be true?

3. $B \perp\!\!\!\perp G$ Not indicated by structure

4. $C \perp\!\!\!\perp G|M$ True

5. $G \perp\!\!\!\perp S$ True

6. $G \perp\!\!\!\perp S|M$ Not indicated by structure

7. $G \perp\!\!\!\perp S|B$ Not indicated by structure

8. $B \perp\!\!\!\perp C$ Not indicated by structure

9. $B \perp\!\!\!\perp C|G$ Not indicated by structure

Note that just based on the structure (but without knowing the CPT values), we can assert independence but we cannot assert dependence.