# CS 188: Artificial Intelligence
# Fall 2011

Lecture 18: HMMs: Intro and Filtering
11/2/2011

Dan Klein --- UC Berkeley
Presented by Woody Hoburg

## Announcements

- Midterm back today
  - solutions online
  - grades also in glookup
- P4 out Thursday

2

# Reasoning over Time

- Often, we want to reason about a sequence of observations
  - Robot localization
  - Medical monitoring
  - Speech recognition
  - Vehicle control

- Need to introduce time into our models
- Basic approach: hidden Markov models (HMMs)

[VIDEO]

3

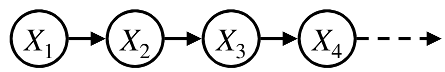
# Outline

- Markov Models
  (last lecture)
- Hidden Markov Models (HMMs)
  - Representation
  - Inference
    - Forward algorithm (special case of variable elimination)
    - Particle filtering (next lecture)

4

# Markov Models: recap

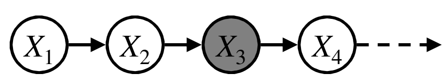
- A Markov model is a chain-structured BN
  - Each node is identically distributed (stationarity)
  - Value of X at a given time is called the state
  - As a BN:




$P(X_1)$ $\qquad$ $P(X|X_{-1})$

  - Parameters: called transition probabilities or dynamics, specify how the state evolves over time (also, initial probs)

# Conditional Independence

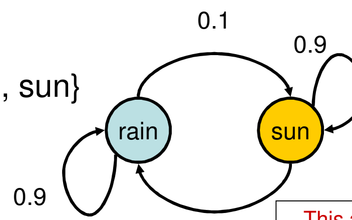


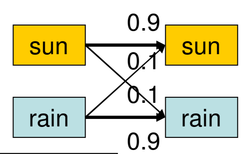

- Basic conditional independence:
  - Past and future independent of the present
  - Each time step only depends on the previous
  - This is called the (first order) Markov property
- Note that the chain is just a (growing) BN
  - We can always use generic BN reasoning on it if we truncate the chain at a fixed length

6

# Example: Markov Chain

- Weather:
  - States: X = {rain, sun}
  - Transitions:




This are two new representations of a CPT, not BNs!

  - Initial distribution: 1.0 sun
  - What's the probability distribution after one step?

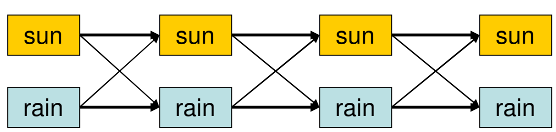
$$P(X_2 = \text{sun}) = P(X_2 = \text{sun}|X_1 = \text{sun})P(X_1 = \text{sun}) + P(X_2 = \text{sun}|X_1 = \text{rain})P(X_1 = \text{rain})$$

$$0.9 \cdot 1.0 + 0.1 \cdot 0.0 = 0.9$$



# Mini-Forward Algorithm

- Question: What's P(X) on some day t?
  - An instance of variable elimination! (In order $X_1$, $X_2$, … )




$$P(x_t) = \sum_{x_{t-1}} P(x_t|x_{t-1})P(x_{t-1})$$

$$P(x_1) = \text{known}$$

*Forward simulation*

# Example

- From initial observation of sun

$$\left\langle \begin{matrix} 1.0 \\ 0.0 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.9 \\ 0.1 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.82 \\ 0.18 \end{matrix} \right\rangle \quad \Rightarrow \quad \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle$$

$$\mathrm{P}(X_1) \qquad \mathrm{P}(X_2) \qquad \mathrm{P}(X_3) \qquad\qquad \mathrm{P}(X_\infty)$$

- From initial observation of rain

$$\left\langle \begin{matrix} 0.0 \\ 1.0 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.1 \\ 0.9 \end{matrix} \right\rangle \quad \left\langle \begin{matrix} 0.18 \\ 0.82 \end{matrix} \right\rangle \quad \Rightarrow \quad \left\langle \begin{matrix} 0.5 \\ 0.5 \end{matrix} \right\rangle$$

$$\mathrm{P}(X_1) \qquad \mathrm{P}(X_2) \qquad \mathrm{P}(X_3) \qquad\qquad \mathrm{P}(X_\infty)$$

9

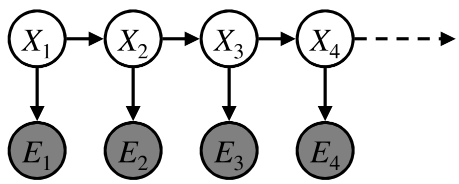
# Outline

- Markov Models
  - (last lecture)
- Hidden Markov Models (HMMs)
  - Representation
  - Inference
    - Forward algorithm (special case of variable elimination)
    - Particle filtering (next lecture)

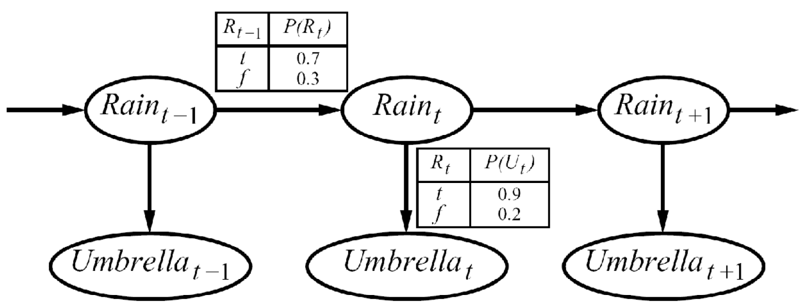
10

# Hidden Markov Models

- Markov chains not so useful for most agents
  - Eventually you don't know anything anymore
  - Need observations to update your beliefs
- Hidden Markov models (HMMs)
  - Underlying Markov chain over states S
  - You observe outputs (effects) at each time step
  - As a Bayes' net:



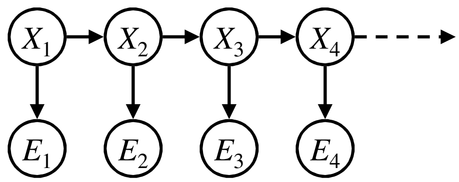

# Example




- An HMM is defined by:
  - Initial distribution: $P(X_1)$
  - Transitions: $P(X|X_{-1})$
  - Emissions: $P(E|X)$

# Conditional Independence

- HMMs have two important independence properties:
  - Markov hidden process, future depends on past via the present
  - Current observation independent of all else given current state




- Quiz: does this mean that observations are independent given no evidence?
  - [No, correlated by the hidden state]

# Real HMM Examples

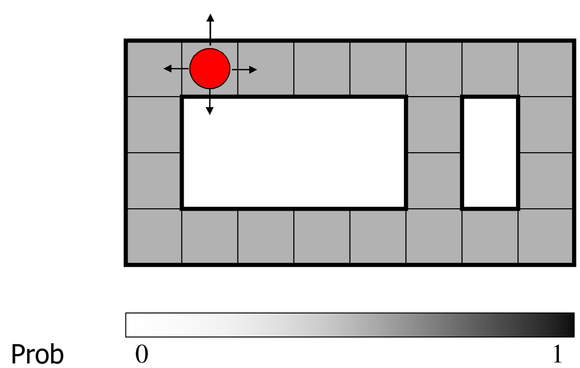
- Speech recognition HMMs:
  - Observations are acoustic signals (continuous valued)
  - States are specific positions in specific words (so, tens of thousands)
- Machine translation HMMs:
  - Observations are words (tens of thousands)
  - States are translation options
- Robot tracking:
  - Observations are range readings (continuous)
  - States are positions on a map (continuous)

# Filtering / Monitoring

- Filtering, or monitoring, is the task of tracking the distribution B(X) (the belief state) over time
- We start with B(X) in an initial setting, usually uniform
- As time passes, or we get observations, we update B(X)
- The Kalman filter was invented in the 60's and first implemented as a method of trajectory estimation for the Apollo program

# Example: Robot Localization

*Example from Michael Pfeiffer*

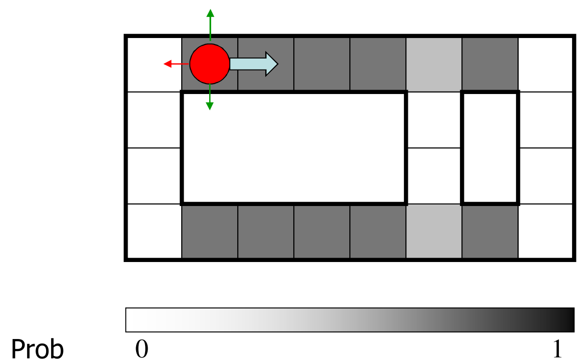



t=0

Sensor model: can read in which directions there is a wall, never more than 1 mistake

Motion model: may not execute action with small prob.

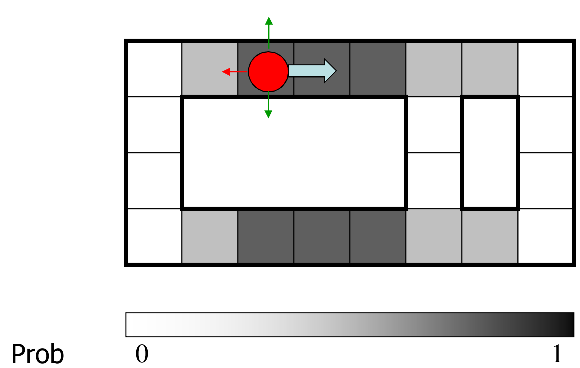
## Example: Robot Localization




t=1

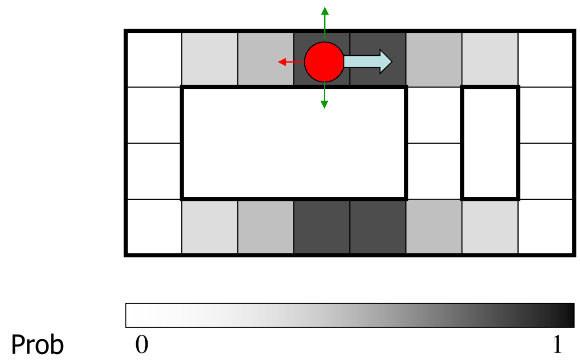
Lighter grey: was possible to get the reading, but less likely b/c required 1 mistake

## Example: Robot Localization



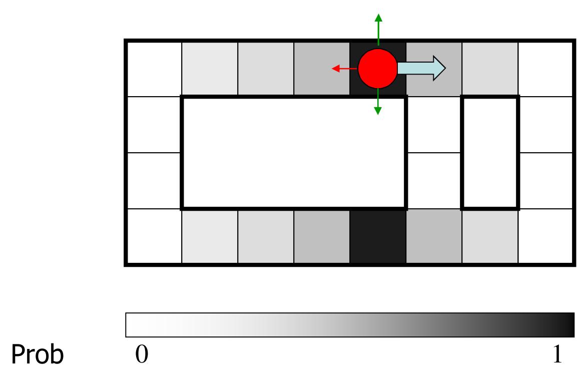

t=2

# Example: Robot Localization



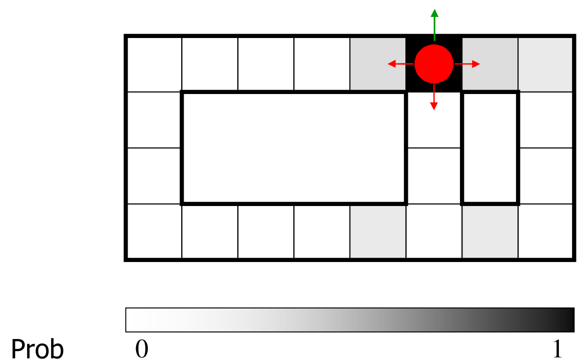

t=3

# Example: Robot Localization




t=4

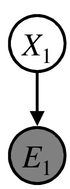
# Example: Robot Localization




t=5

# Inference: Base Cases

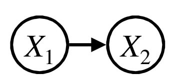
- **Observation**
  - Given: $P(X_1)$, $P(e_1 \mid X_1)$
  - Query: $P(x_1 \mid e_1)$ $\forall$ $x_1$



$$P(X_1|e_1)$$

$$\begin{aligned} P(x_1|e_1) &= P(x_1, e_1)/P(e_1) \\ &\propto_{X_1} P(x_1, e_1) \\ &= P(x_1)P(e_1|x_1) \end{aligned}$$

- **Passage of Time**
  - Given: $P(X_1)$, $P(X_2 \mid X_1)$
  - Query: $P(x_2)$ $\forall$ $x_2$



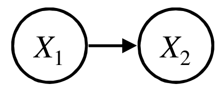
$$P(X_2)$$

$$\begin{aligned} P(x_2) &= \sum_{x_1} P(x_1, x_2) \\ &= \sum_{x_1} P(x_1)P(x_2|x_1) \end{aligned}$$

# Passage of Time

- Assume we have current belief P(X | evidence to date)

$$B(X_t) = P(X_t|e_{1:t})$$




- Then, after one time step passes:

$$P(X_{t+1}|e_{1:t}) = \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})$$

- Or, compactly:

$$B'(X_{t+1}) = \sum_{x_t} P(X_{t+1}|x_t)B(x_t)$$

- Basic idea: beliefs get “pushed” through the transitions
  - With the “B” notation, we have to be careful about what time step t the belief is about, and what evidence it includes

# Example: Passage of Time

- As time passes, uncertainty “accumulates”

| | | | | | |
|---|---|---|---|---|---|
| <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 |
| <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 |
| <0.01 | <0.01 | 1.00 | <0.01 | <0.01 | <0.01 |
| <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 |

T = 1

| | | | | | |
|---|---|---|---|---|---|
| <0.01 | <0.01 | <0.01 | <0.01 | <0.01 | <0.01 |
| <0.01 | <0.01 | 0.06 | <0.01 | <0.01 | <0.01 |
| <0.01 | 0.76 | 0.06 | 0.06 | <0.01 | <0.01 |
| <0.01 | <0.01 | 0.06 | <0.01 | <0.01 | <0.01 |

T = 2

| | | | | | |
|---|---|---|---|---|---|
| 0.05 | 0.01 | 0.05 | <0.01 | <0.01 | <0.01 |
| 0.02 | 0.14 | 0.11 | 0.35 | <0.01 | <0.01 |
| 0.07 | 0.03 | 0.05 | <0.01 | 0.03 | <0.01 |
| 0.03 | 0.03 | <0.01 | <0.01 | <0.01 | <0.01 |

T = 5

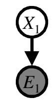
$$B'(X') = \sum_{x} P(X'|x)B(x)$$

Transition model: ghosts usually go clockwise

# Observation

- Assume we have current belief P(X | previous evidence):

$$B'(X_{t+1}) = P(X_{t+1}|e_{1:t})$$




- Then:

$$P(X_{t+1}|e_{1:t+1}) \propto P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$$

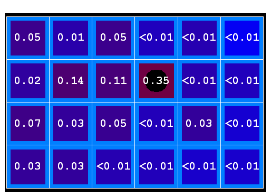
- Or:

$$B(X_{t+1}) \propto P(e|X)B'(X_{t+1})$$

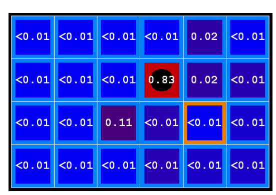
- Basic idea: beliefs reweighted by likelihood of evidence
- Unlike passage of time, we have to renormalize

# Example: Observation

- As we get observations, beliefs get reweighted, uncertainty “decreases”



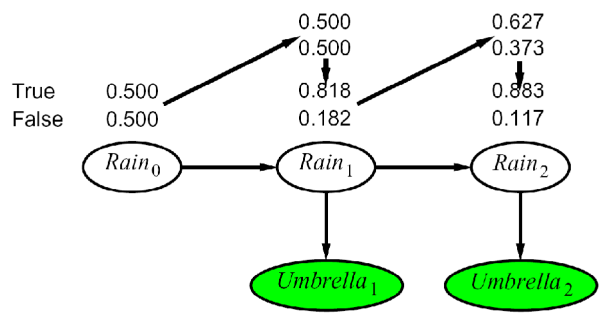

Before observation




After observation

$$B(X) \propto P(e|X)B'(X)$$

# Example HMM




# The Forward Algorithm

- We are given evidence at each time and want to know

$$B_t(X) = P(X_t|e_{1:t})$$

- We can derive the following updates

We can normalize as we go if we want to have P(x|e) at each time step, or just once at the end…

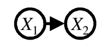
$$\begin{aligned} P(x_t|e_{1:t}) &\propto_X P(x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, x_t, e_{1:t}) \\ &= \sum_{x_{t-1}} P(x_{t-1}, e_{1:t-1}) P(x_t|x_{t-1}) P(e_t|x_t) \\ &= P(e_t|x_t) \sum_{x_{t-1}} P(x_t|x_{t-1}) P(x_{t-1}, e_{1:t-1}) \end{aligned}$$

- = exactly variable elimination in order $X_1$, $X_2$, …

# Online Belief Updates

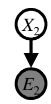
- Every time step, we start with current P(X | evidence)
- We update for time:

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$




- We update for evidence:

$$P(x_t|e_{1:t}) \propto_X P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$




- The forward algorithm does both at once (and doesn't normalize)
- Problem: space is |X| and time is $|X|^2$ per time step