# CS 188: Artificial Intelligence
# Fall 2011

Lecture 19: Particle Filtering

11/3/2011

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Presented by Woody Hoburg

## Announcements

- Project 4 out TONIGHT: due 11/16
- Pick up midterm:

  Jono's OH, SDH 730, 2-3:30pm

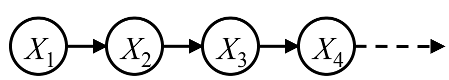
# Outline

- Particle Filtering
  - (sampling-based inference in HMMs)
- Dynamic Bayes Nets
- Viterbi Algorithm

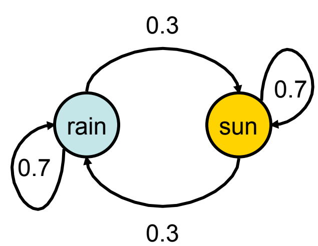


# Recap: Reasoning Over Time

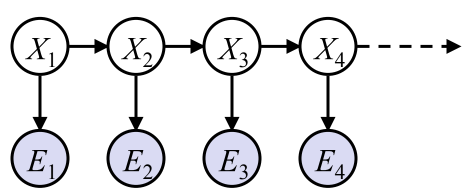
- Markov models




$P(X_1)$ $\quad$ $P(X|X_{-1})$




- Hidden Markov models




$P(E|X)$

| X | E | P |
|---|---|---|
| rain | umbrella | 0.9 |
| rain | no umbrella | 0.1 |
| sun | umbrella | 0.2 |
| sun | no umbrella | 0.8 |

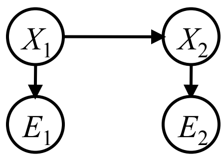
# Recap: Filtering

**Elapse time:** compute P( $X_t$ | $e_{1:t-1}$ )

$$P(x_t|e_{1:t-1}) = \sum_{x_{t-1}} P(x_{t-1}|e_{1:t-1}) \cdot P(x_t|x_{t-1})$$

**Observe:** compute P( $X_t$ | $e_{1:t}$ )

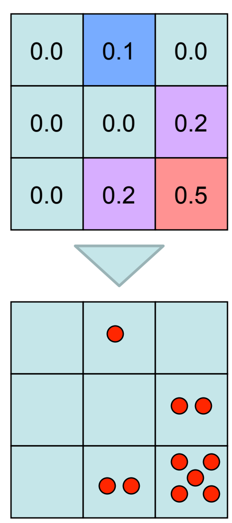
$$P(x_t|e_{1:t}) \propto P(x_t|e_{1:t-1}) \cdot P(e_t|x_t)$$




| | Belief: <P(rain), P(sun)> | |
|---|---|---|
| $P(X_1)$ | <0.5, 0.5> | *Prior on $X_1$* |
| $P(X_1 \mid E_1 = umbrella)$ | <0.82, 0.18> | *Observe* |
| $P(X_2 \mid E_1 = umbrella)$ | <0.63, 0.37> | *Elapse time* |
| $P(X_2 \mid E_1 = umb, E_2 = umb)$ | <0.88, 0.12> | *Observe* |

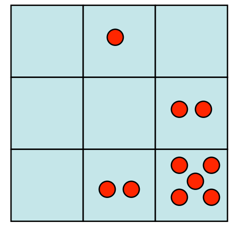
# Particle Filtering

- Sometimes |X| is too big to use exact inference
  - |X| may be too big to even store B(X)
  - E.g. X is continuous
  - $|X|^2$ may be too big to do updates
- Solution: approximate inference
  - Track samples of X, not all values
  - Samples are called particles
  - Time per step is linear in the number of samples
  - But: number needed may be large
- This is how robot localization works in practice



# Representation: Particles

- Our representation of P(X) is now a list of N particles (samples)
  - Generally, N << |X|
  - Storing map from X to counts would defeat the point
- P(x) approximated by number of particles with value x
  - So, many x will have P(x) = 0
  - More particles, more accuracy
- Initially, all particles have a weight of 1



Particles:
(3,3)
(2,3)
(3,3)
(3,2)
(3,3)
(3,2)
(2,1)
(3,3)
(3,3)
(2,1)

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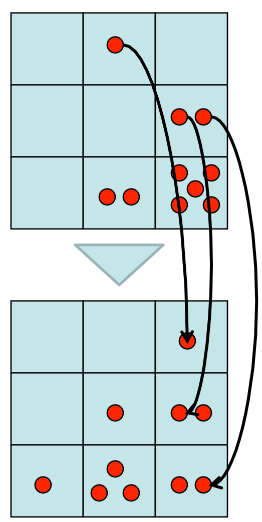
# Particle Filtering: Elapse Time

- Each particle is moved by sampling its next position from the transition model

$$x' = \text{sample}(P(X'|x))$$

  - This is like prior sampling – samples' frequencies reflect the transition probs
  - Here, most samples move clockwise, but some move in another direction or stay in place
- This captures the passage of time
  - If we have enough samples, close to the exact values before and after (consistent)



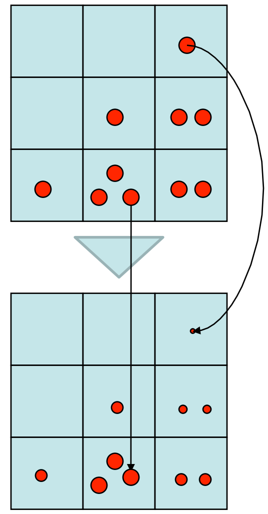
## Particle Filtering: Observe

- Slightly trickier:
  - Don't do rejection sampling (why not?)
  - We don't sample the observation, we fix it
  - As in likelihood weighting, downweight samples based on the evidence:

$$w(x) = P(e|x)$$

$$B(X) \propto P(e|X)B'(X)$$

  - Note that, as before, the probabilities don't sum to one, since most have been downweighted (in fact they sum to an approximation of P(e))

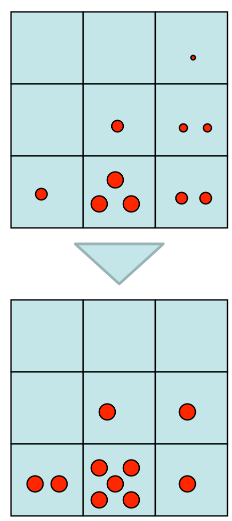


## Particle Filtering: Resample

- Rather than tracking weighted samples, we resample
- N times, we choose from our weighted sample distribution (i.e. draw with replacement)
- This is analogous to renormalizing the distribution
- Now the update is complete for this time step, continue with the next one

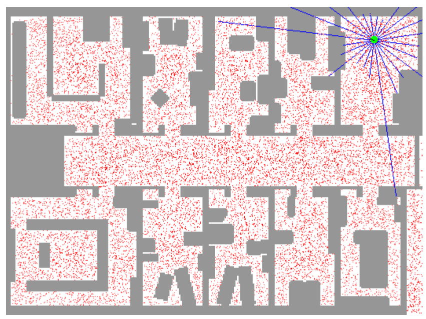
Old Particles:
(3,3) w=0.1
(2,1) w=0.9
(2,1) w=0.9
(3,1) w=0.4
(3,2) w=0.3
(2,2) w=0.4
(1,1) w=0.4
(3,1) w=0.4
(2,1) w=0.9
(3,2) w=0.3

New Particles:
(2,1) w=1
(2,1) w=1
(2,1) w=1
(3,2) w=1
(2,2) w=1
(2,1) w=1
(1,1) w=1
(3,1) w=1
(2,1) w=1
(1,1) w=1



# Robot Localization

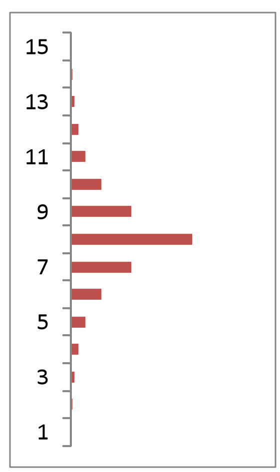
- In robot localization:
  - We know the map, but not the robot's position
  - Observations may be vectors of range finder readings
  - State space and readings are typically continuous (works basically like a very fine grid) and so we cannot store B(X)
  - Particle filtering is a main technique
- [Demo]



# P4: Ghostbusters 2.0 (beta)

- **Goal:** Find and kill ghosts
- Agent is blind, but can hear the ghosts' banging and clanging.
- **Transition Model:** All ghosts move randomly, but are sometimes biased
- **Emission Model:** Sensor gives a "noisy" distance to each ghost

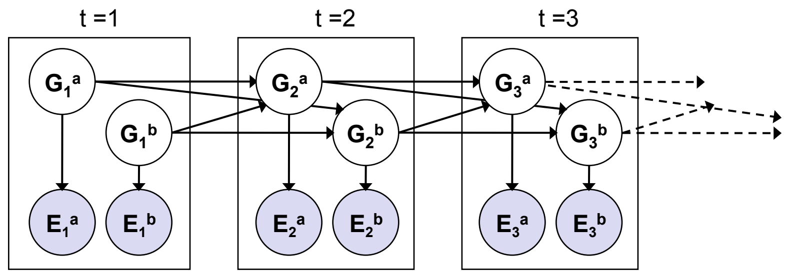
**Noisy distance prob**
True distance = 8




[Demo]

## Dynamic Bayes Nets (DBNs)

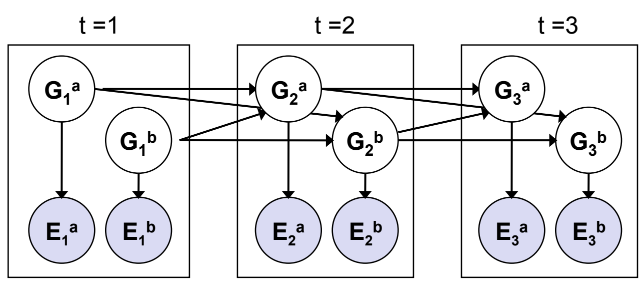
- We want to track multiple variables over time, using multiple sources of evidence
- Idea: Repeat a fixed Bayes net structure at each time
- Variables from time *t* can condition on those from *t-1*




- DBNs with evidence at leaves are (in principle) HMMs

## Exact Inference in DBNs

- Variable elimination applies to dynamic Bayes nets
- Procedure: "unroll" the network for T time steps, then eliminate variables until $P(X_T|e_{1:T})$ is computed




- Online belief updates: Eliminate all variables from the previous time step; store factors for current time only

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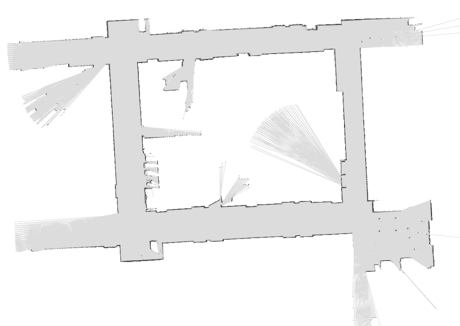
# DBN Particle Filters

- A particle is a **complete** sample for a time step
- **Initialize**: Generate prior samples for the t=1 Bayes net
  - Example particle: $\mathbf{G_1}^a = (3,3)$ $\mathbf{G_1}^b = (5,3)$
- **Elapse time**: Sample a successor for each particle
  - Example successor: $\mathbf{G_2}^a = (2,3)$ $\mathbf{G_2}^b = (6,3)$
- **Observe**: Weight each entire sample by the likelihood of the evidence conditioned on the sample
  - Likelihood: $P(\mathbf{E_1}^a \mid \mathbf{G_1}^a) * P(\mathbf{E_1}^b \mid \mathbf{G_1}^b)$
- **Resample:** Select new particles (tuples of values) in proportion to their likelihood

# SLAM

- SLAM = Simultaneous Localization And Mapping
  - We do not know the map or our location
  - Our belief state is over maps and positions!
  - Main techniques: Kalman filtering (Gaussian HMMs) and particle methods
- [DEMOS]



DP-SLAM, Ron Parr

# Outline

- Particle Filtering
  - (sampling-based inference in HMMs)
- Dynamic Bayes Nets
- Viterbi Algorithm

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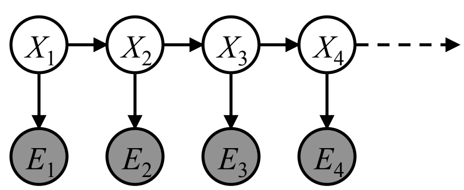
# HMMs: MLE Queries

- HMMs defined by
  - States X
  - Observations E
  - Initial distr: $P(X_1)$
  - Transitions: $P(X|X_{-1})$
  - Emissions: $P(E|X)$




- Query: most likely explanation:

$$\arg\max_{x_{1:t}} P(x_{1:t}|e_{1:t})$$

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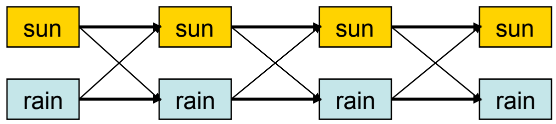
# State Path Trellis

- State trellis: graph of states and transitions over time




$X_1 \quad X_2 \quad \cdots \quad X_N$

- Each arc represents some transition $x_{t-1} \rightarrow x_t$
- Each arc has weight $P(x_t|x_{t-1})P(e_t|x_t)$
- Each path is a sequence of states
- The product of weights on a path is the seq's probability
- Can think of the Forward (and now Viterbi) algorithms as computing sums of all paths (best paths) in this graph

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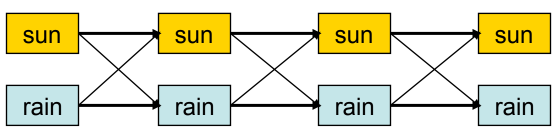
# Viterbi Algorithm




$$x^*_{1:T} = \arg\max_{x_{1:T}} P(x_{1:T}|e_{1:T}) = \arg\max_{x_{1:T}} P(x_{1:T}, e_{1:T})$$

$$m_t[x_t] = \max_{x_{1:t-1}} P(x_{1:t-1}, x_t, e_{1:t})$$

$$= \max_{x_{1:t-1}} P(x_{1:t-1}, e_{1:t-1})P(x_t|x_{t-1})P(e_t|x_t)$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) \max_{x_{1:t-2}} P(x_{1:t-1}, e_{1:t-1})$$

$$= P(e_t|x_t) \max_{x_{t-1}} P(x_t|x_{t-1}) m_{t-1}[x_{t-1}]$$

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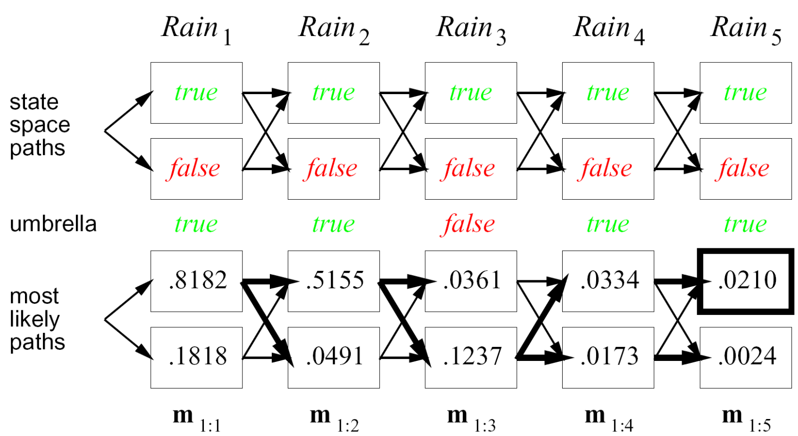
# Example



Rain 1
Rain 2
Rain 3
Rain 4
Rain 5
state space paths
true
false
true
false
true
false
true
false
true
false
umbrella
true
true
false
true
true
most likely paths
.8182
.1818
.5155
.0491
.0361
.1237
.0334
.0173
.0210
.0024
m 1:1
m 1:2
m 1:3
m 1:4
m 1:5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